Nano-óptica.
Guía de problemas

Cátedra: A. Bragas

2^{do} cuatrimestre 2013

Guía 1: Repaso electrodinámica

1. (Ej 2.2 Novotny). Consider an interface between two media 1 and 2 with dielectric constants $\varepsilon_1 = 2{,}25$ and $\varepsilon_2 = 1$, respectively. The magnetic permeabilities are equal to one. A p-polarized plane wave with wavelength $\lambda = 532$ nm is incident from medium 1 at an angle of incidence of θ_1. Express the Fresnel reflection coefficient in terms of amplitude A and phase Φ. Plot A and as a function of θ_1. What are the consequences for the reflected wave?

2. Grafique los coeficientes de fresnel para los dos casos de polarización, en función del ángulo de incidencia, para una interfase

   a) aire-oro

   b) agua-vidrio

3. (Ej 2.4 Novotny). Show that the z-component of the time-averaged Poynting vector $\langle \mathbf{S} \rangle_z$ vanishes for an evanescent field propagating in the x-direction. Considere la microscopía por reflexión total interna, ¿cómo se excitan los fluorósforos cercanos a la interfase si no hay flujo de energía a través de la superficie?

4. (Ej 2.5 Novotny). Analyze the polarization state of an evanescent field propagating in the x-direction created by total internal reflection of a p-polarized plane wave. Calculate the time-dependent electric field $\mathbf{E}_2(x,t) = (E_{2,x}(x,t), 0, E_{2,z}(x,t))$ just on top of the interface ($z = 0$). For a fixed position x, the electric field vector $\mathbf{E}_2$ defines a curve in the (x,z) plane as the time runs from 0 to λ/c. Determine and plot the shape of these curves as a function of the position x. For numerical values choose $\theta_1 = 60°$, $n = 1{,}5$.

Guía 2: Propagación y enfoque de campos ópticos

1. (Ej 3.1 Novotny). The paraxial Gaussian beam is not a rigorous solution of Maxwell's equations. Its field is therefore not divergence free ($\nabla \cdot \mathbf{E} = 0$). By requiring $\nabla \cdot \mathbf{E} = 0$ one can derive an expression for the longitudinal field E_z . Assume that $E_y = 0$ everywhere and derive E_z to lowest order for which the solution is non-zero. Sketch the distribution of $|\mathbf{E}|^2$ in the focal plane.

2. (Ej 3.2 Novotny). Determine the decomposition of an arbitrary optical field into transverse electric (TE) and transverse magnetic (TM) fields. The longitudinal field E_z vanishes for the TE field, whereas H_z vanishes for the TM field.

3. (Ej 3.5 Novotny). Consider a small circular aperture with radius a_0 in an infinitely thin and ideally conducting screen which is illuminated by a plane wave at normal incidence and polarized along the x-axis. In the long wavelength limit ($\lambda \ll a_0$) the electric field in the aperture ($z = 0$, $x^2 + y^2 \leq a_0^2$) has been derived by Bouwkamp [22] as

$$E_x(x, y) = -\frac{4ikE_0}{3\pi} \frac{2a_0^2 - x^2 - 2y^2}{\sqrt{a_0^2 - x^2 - y^2}} \tag{1}$$

$$E_y(x, y) = -\frac{4ikE_0}{3\pi} \frac{xy}{\sqrt{a_0^2 - x^2 - y^2}} \tag{2}$$

where $E0$ is the incident field amplitude. The corresponding spatial Fourier spectrum has been calculated by Van Labeke et al. [23] as

$$\hat{E}_x(x, y) = -\frac{2ika_0^2 E_0}{3\pi^2} \left[\frac{3k_y^2 \cos(a_0 k_\rho)}{a_0^2 k_\rho^4} - \frac{(a_0^2 k_x^4 + 3k_y^2 + a_0^2 k_x^2 k_y^2) \sin(a_0 k_\rho)}{a_0^3 k_\rho^5} \right] \tag{3}$$

$$\hat{E}_x(x, y) = -\frac{2ika_0^2 E_0}{3\pi^2} \left[\frac{3k_x k_y \cos(a_0 k_\rho)}{a_0^2 k_\rho^4} - \frac{k_x k_y (3 - a_0^2 k_\rho^2) \sin(a_0 k_\rho)}{a_0^3 k_\rho^5} \right] \tag{4}$$

with $k_\rho = (k_x^2 + k_y^2)^{1/2}$ being the transverse wavenumber.

a) Derive the Fourier spectrum of the longitudinal field component E_z.

b) Find expressions for the field $\mathbf{E} = (E_x, E_y, E_z)$ at an arbitrary field point (x, y, z).

c) Calculate the far-field and express it in spherical coordinates (r, θ, ϕ) and spherical vector components $\mathbf{E} = (E_r, E_\theta, E_\phi)$. Expand in powers of ka_0 and retain only the lowest orders. What does this field look like?

4. (*) (Ej 3.8 Novotny). In order to correct for the aberrations introduced by the reflection of a strongly focused beam from an interface we design a pair of phase plates. By using a polarizing beamsplitter, the collimated reflected beam (cf. Fig. 3.18 and Eq. (3.100)) is split into two purely polarized light paths. The phase distortion in each light path is corrected by a phase plate. After correction, the two light paths are recombined and refocused on the image plane. Calculate and plot the phase distribution of each phase plate if the incident field is a Gaussian beam ($f_0 \to \infty$) focused by an NA $= 1{,}4$ objective on a glass air interface ($z_0 = 0$) and incident from the optically denser medium with $n_1 = 1{,}518$. What happens if the focus is displaced from the interface ($z_0 = 0$)?

5. (*) Campos fuertemente enfocados cerca de superficies: Sec. 3.9 Novotny.

Guía 3: Resolución en microscopías

1. (Ej 4.1 Novotny). A continuously fluorescing molecule is located at the focus of a high NA objective lens. The fluorescence is imaged onto the image plane as described in Section 4.1. Although the molecule's position is fixed (no translational diffusion) it is rotating in all three dimensions (rotational diffusion) with high speed. Calculate and plot the averaged field distribution in the image plane using the paraxial approximation.

2. (Ej 4.2 Novotny). Consider the set-up of Fig. 4.1. Replace the single dipole emitter by a pair of incoherently radiating dipole emitters separated by a distance $x = \lambda/2$ along the x-axis. The two dipoles radiate at $\lambda = 500$ nm and they have the same dipole strength. One of the dipoles is oriented transverse to the optical axis whereas the other dipole is parallel to the optical axis. The two dipoles are scanned in the object plane and for each position of their center coordinate a signal is recorded in the image plane using a NA = 1,4 ($n = 1{,}518$), $M = 100$x objective lens.

   a) Determine the total integrated field intensity (s_1) in the image plane.

   b) Calculate and plot the recorded image (s_2) if a confocal detector is used. Use the paraxial approximation.

   c) Discuss what happens in 1 and 2 if the dipoles are scanned at a constant height $z = \lambda/4$ above the image plane.

3. (*) (Ej 4.3 Novotny). Consider a sample with a uniform layer of dipolar particles with fixed dipole orientations along the x-axis. The layer is transverse to the optical axis and each element of the layer has a constant polarizability α_{xx} . The sample is illuminated by a focused Gaussian beam and is translated along the optical axis z. We use both non-confocal (s_1) and confocal (s_2) detection. The two signals are well approximated by Eqs. (4.47) and (4.48), respectively.

   a) Calculate the non-confocal signal as a function of z.

   b) Calculate the confocal signal as a function of z.

   c) What is the conclusion?

   Hint: Use the Bessel function closure relations of Eq. (3.112).

4. Ej 4.4 Novotny. Calculate the longitudinal fields corresponding to the Gaussian field distribution in Eq. (4.67). Assume that $E_y = 0$ everywhere in space. Show how the longitudinal field evolves in transverse planes $z =$const. State the result in cylindrical coordinates as in Eq. (4.68). Plot the longitudinal field strength in the planes $z = 0$ and $z = \lambda$.

5. (*) Ej 4.6 Novotny. In order to verify the validity of Eq. (4.64) perform a Monte-Carlo simulation of the fitting process. To this end simulate a large number (~ 1000) of point images by creating Gaussian peaks with uncorrelated Poissonian noise superimposed on the background and on the amplitude. In terms of Eq. (4.54), in the absence of the background B, this means that for each data point a random number drawn from a Poissonian distribution with maximum at $G(x, y)$ and width $\sqrt{G(x, y)}$ is added to the originally calculated $G(x, y)$. Now perform a nonlinear least-squares fit on each of the peaks using a suitable software package (the use of a Levenberg-Marquard algorithm is recommended). Plot the resulting distribution of positions $x_{0,min}$ and $y_{0,min}$ that result from the fits. Compare the width of this distribution with the value for σ obtained from Eq. (4.64).

6. (*) (Ej 4.7 Novotny). Determine analytical expressions for the uncertainties of the other parameters in Eq. (4.54) using the same analysis that led to Eq. (4.64).

Guía 4: Super-resolución y SNOM

1. (Ej 5.2 Novotny). Use the formalism of Section 3.6 to determine the diameter of the on-axis phase plate that should be used in STED microscopy in order to exactly cancel the total field in the geometrical focus. Discuss why it is important to really achieve zero field with a high degree of accuracy.

2. Leer paper: Appl Opt. 1992 Jun 1;31(16):3036-45. doi: 10.1364/AO.31.003036. Estudiar la resolución del SNOM en función del tamaño de la abertura de detección.

Guía 5: Emisores cuánticos

1. (Ej 8.2 Novotny). Derive the far-field Green's function $\overleftrightarrow{\mathbf{G}}_{FF}$ in spherical coordinates and Cartesian vector components. Calculate the radiation pattern $P(\theta, \phi)/P$ for a dipole μ which encloses an angle α with the z-axis.

2. (Ej 8.3 Novotny). Prove that the near-field and intermidiate-field terms of a dipole in free space do not contribute to radiation.

3. (*) (Ej 8.8 Novotny). A molecule with emission dipole moment in the direction of the x-axis is scanned in the x, y-plane. A spherical gold particle ($\varepsilon = -7{,}6 + 1{,}7i$) with radius $r_0 = 10$nm is placed above the x, y-plane. The emission wavelength is $\lambda = 575$ nm (DiI molecule). The center of the particle is located at the fixed position $(x, y, z) = (0, 0, 20\text{nm})$.

   a) Calculate the normalized decay rate γ/γ_0 as a function of x, y. Neglect retardation effects and draw a contour plot. What is the minimum value of γ/γ_0? How does the quenching rate scale with the sphere radius r_0?

   b) Repeat the calculation for a dipole oriented in the direction of the z-axis.

4. (Ej 8.9 Novotny). Two molecules, fluorescein (donor) and alexa green 532 (acceptor), are located in a plane centered between two perfectly conducting surfaces separated by the distance d. The emission spectrum of the donor (f_D) and the absorption spectrum of the acceptor (σ_A) are approximated by a superposition of two Gaussian distribution functions. Use the fit parameters from Section 8.6.2.

   a) Determine the Green's function for this configuration.

   b) Calculate the decay rate γ_0 of the donor in the absence of the acceptor.

5. (Ej 9.2 Novotny). The rate of energy dissipation (absorption) by a molecule with dipole moment μ can be written as $P_{abs}(\omega) = (\omega/2)\text{Im}[\mu \cdot \mathbf{E}(\omega)]$, with $\mathbf{E}$ being the local exciting field. The dipole moment μ can be considered to be induced by the same field according to $\mu = \overleftrightarrow{\alpha}\ \mathbf{E}$, where $\overleftrightarrow{\alpha}$ is the tensorial polarizability of the molecule defined by its dipole orientation. Derive Eqs. (9.3) and (9.4).

6. (*) ver algo con dda: cálculo del campo con interacción de a 4 dipolos o dipolo imagen.

Guía 6: Plasmónica

1. (Ej 12.1 Novotny). Study the effect of a complex dielectric function on the propagation of a plane wave. What happens if a plane wave is normally incident on a metal interface?

2. (*) (Ej 12.7 Novotny). Solve the Laplace equation (12.41) for a spherical particle and verify the results (12.45) and (12.46). Grafique los campos obtenidos y la intesidad.

Guía 7:Fuerzas ópticas y Espectroscopía Raman

1. (*) (Ej 13.1 Novotny). A spherical glass particle in water is trapped at the focus of a monochromatic paraxial Gaussian beam with $\lambda = 800$ nm and variable NA (see Section 3.2). The polarizability of the particle is

$$\alpha = 3\varepsilon_0 V_0 \frac{\varepsilon - \varepsilon_w}{\varepsilon + 2\varepsilon_w}$$

where V_0 is the volume of the particle, and the dielectric constants of glass and water are $\varepsilon = 2{,}25$ and $\varepsilon_w = 1{,}76$, respectively.

   a) Show that for small transverse displacements (x) from the focus the force is proportional to x. Determine the spring constant as a function of NA, d_0 , λ, and P_0, where d_0 is the particle diameter and P_0 the laser power.

   b) Is it possible to derive in the same way a spring constant for longitudinal displacements z? If yes, calculate the corresponding spring constant as a function of NA, d_0 , and P_0.

   c) Assume NA$=$ 1,2 and $d_0 = 100$ nm. What laser power is necessary in order to create a trapping potential $V > 10k_bT$, where k_b is Boltzmann's constant and $T = 300$ K is the ambient temperature? What is the restoring force for a transverse displacement of $x = 100nm$?

2. (Ej 13.2 Novotny). Consider the total internal reflection of a plane wave with wavelength $\lambda = 800$ nm incident at an angle $\theta = 70°$ from the normal of a glass/air interface ($\varepsilon = 2{,}25$). The plane wave is incident from the glass-side and is s-polarized. The normal of the interface is parallel to the gravitational axis and the air-side is pointing to the bottom. A tiny glass particle is trapped on the air-side in the evanescent field generated by the totally internally reflected plane wave. Calculate the minimum required intensity I of the plane wave to prevent the glass particle from falling down (α given by Eq. (13.65) with $\varepsilon_w = 1$). The specific density of glass is $\rho = 2{,}2\text{x}103\,\text{kg/m}^3$ and the particle diameter is $d_0 = 100$ nm. What happens if the particle size is increased?