

Ec. continuidad:

$$\partial_t \rho + \bar{\nabla}(\rho \bar{u}) = 0$$

$$\boxed{\rho = \rho_0 + \rho_1 \quad \frac{\rho_1}{\rho_0} \ll 1} \quad (1)$$

$$\underbrace{\partial_t \rho_0 + \partial_t \rho_1}_{(I)} + \underbrace{\bar{\nabla}(\rho_0 \bar{u}_0)}_{(II)} + \bar{\nabla}(\rho_0 \bar{u}_1) + \bar{\nabla}(\rho_1 \bar{u}_0) + \bar{\nabla}(\rho_1 \bar{u}_1) = 0$$

$$(I) + (II) = 0 \quad \text{a orden cero}$$

$$\partial_t \rho_1 + \bar{\nabla}(\rho_0 \bar{u}_1) + \bar{\nabla}(\rho_1 \bar{u}_0) + \bar{\nabla}(\rho_1 \bar{u}_1) = 0$$

$$\downarrow \quad \underbrace{\hspace{2cm}}_{\text{orden 2}} \\ 0 \quad \text{fue}$$

$$\partial_t \rho_1 + \bar{\nabla}(\rho_0 \bar{u}_1) = 0$$

$$f \sim e^{i(\vec{k}\vec{r} - \omega t)}$$

$$\boxed{-i\omega \rho_1 + iK_x U_{1x} \rho_0 + iK_y U_{1y} \rho_0 + iK_z U_{1z} \rho_0 = 0} \quad (a)$$

Ley de Ohm:

$$\vec{E} + \frac{1}{c} \bar{u} \times \vec{B} = 0$$

$$\underbrace{\vec{E}_0 + \vec{E}_1}_{(I)} + \underbrace{\frac{1}{c} \bar{u}_0 \times \vec{B}_0 + \frac{1}{c} \bar{u}_1 \times \vec{B}_0}_{(II)} + \frac{1}{c} \bar{u}_0 \times \vec{B}_1 + \frac{1}{c} \underbrace{\bar{u}_1 \times \vec{B}_1}_{\text{orden 2}} = 0$$

$$0 = (I) + (II) \quad \text{a orden cero}$$

$$\downarrow \\ 0$$

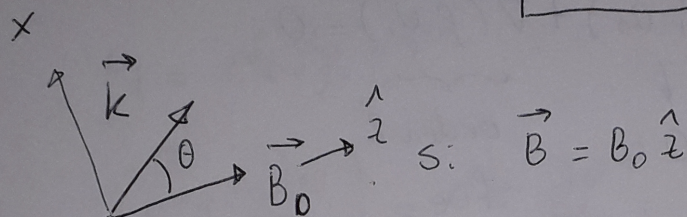
$$\boxed{\vec{E}_1 + \frac{1}{c} \bar{u}_1 \times \vec{B}_0 = 0} \quad (b)$$

(2)

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{B} = \vec{B}_0 + \vec{B}_1 = 0 \Rightarrow \vec{\nabla} \cdot \vec{B}_1 = 0$$

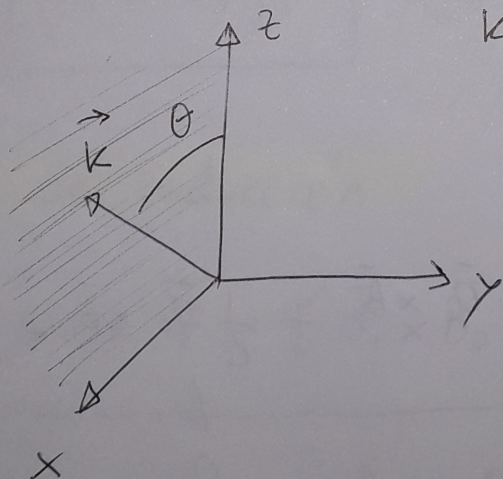
$$-i \vec{k} \cdot \vec{B}_1 = 0 \Rightarrow k_x B_{1x} + k_y B_{1y} + k_z B_{1z} = 0$$



Elijo $\vec{k}$ en el plano x - z

$$k \cos \theta B_{1x} +$$

$$\vec{k} = k \sin \theta \hat{x} + k \cos \theta \hat{z}$$



$$\vec{k} \cdot \vec{B}_1 = k \sin \theta B_{1x} + k \cos \theta B_{1z}$$

$$\Rightarrow \vec{B}_1 = B_{1x} \cos \theta \hat{x} + B_{1y} \hat{y} - B_{1x} \sin \theta \hat{z} \quad (c)$$

Politrópe

(3)

$$p = \text{cte } \rho^\gamma$$

$$p_0 + p_1 = \text{cte } (\rho_0 + \rho_1)^\gamma = \text{cte } \rho_0^\gamma \left(1 + \frac{\rho_1}{\rho_0}\right)^\gamma$$

$$p_1 = \text{cte } \rho_0^\gamma \rho_1^{\gamma-1}$$

$$C_s^2 = \left. \frac{\partial p}{\partial \rho} \right|_s \xrightarrow{\text{adiabático}} = \text{cte } \frac{\partial \rho^\gamma}{\partial \rho} = \text{cte } \gamma \rho^{\gamma-1} = \frac{p}{\rho}$$

$$\Rightarrow p = C_s^2 \rho$$

$$\Rightarrow \boxed{p_1 = C_s^2 \rho_1} \quad (d)$$

Ec. de inducción

$$\partial_t \bar{B}_1 = \bar{v} \times (\bar{v}_1 \times \bar{B}_0) \quad (\text{aprox. ideal})$$

$$-\omega B_{1x} = -\omega B_1 \cos \theta = k B_0 v_{1x} \cos \theta$$

$$-\omega B_{1y} = -\omega B_1 \sin \theta = k B_0 v_{1y} \cos \theta$$

$$-\omega B_{1z} = +\omega B_1 \sin \theta = -k B_0 v_{1x} \sin \theta$$

(e) x 3

$$\vec{U}_1 \times \vec{B}_0 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ U_{1x} & U_{1y} & U_{1z} \\ 0 & 0 & B_0 \end{vmatrix} = (U_{1y} B_0, -U_{1x} B_0, 0)$$

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$$\vec{\nabla} \times (\vec{U}_1 \times \vec{B}_0) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ U_{1y} B_0 & -U_{1x} B_0 & 0 \end{vmatrix} =$$

$$= [\partial_z (U_{1x} B_0), + \partial_z (U_{1y} B_0), -\partial_x (U_{1x} B_0) - \partial_y (U_{1y} B_0)]$$

$$= i k B_0 [U_{1x} \cos \theta, U_{1y} \cos \theta, -U_{1x} \sin \theta]$$

- *) Queda un sistema de 7×7
- *) pero U_y y B_y queda desacoplada

Así que queda un sistema de 6×6

Vamos a la ecuación de Navier-Stokes

$\rho_0 \partial_t \bar{U}_i = - \bar{\nabla} p_i + \frac{1}{4\eta} (\bar{\nabla} \times \bar{B}_i) \times \bar{B}_0$	3 ecuaciones
$\partial_t \bar{B}_i = \bar{\nabla} \times (\bar{U}_i \times \bar{B}_0)$	2 ecuaciones efectivas
$\partial_t p_i + \bar{\nabla} (\rho_0 \bar{U}_i) = 0$	1 ecuación

¿Y la ecuación de Ohm?

No la uso porque no me interesa $\vec{E}_i$

Me queda un sistema de 6×6

$$\begin{pmatrix} 2 \times 2 & \dots & \dots \\ \dots & 4 \times 4 & \dots \end{pmatrix} \begin{pmatrix} U_{iy} \\ B_{iy} \\ U_{ix} \\ B_{ix} \\ p_i \\ p_i \end{pmatrix} = \vec{0}$$

$p_i, p_i, U_{ix}, U_{iy}, \cancel{U_{iz}}, \overbrace{B_{ix}, B_{iy}, B_{iz}}^{B_{iz}}$