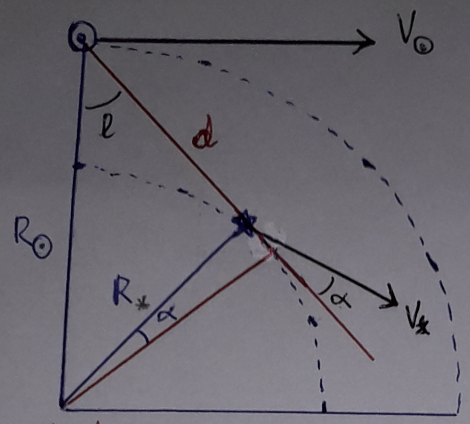


* órbitas circulares
* plano medio de la galaxia

Respecto del Sol:

$$V_{rel,radial} = V_{*,r} - V_{\odot,r} = V_* \cos\alpha - V_\odot \operatorname{sen} \ell$$

$$V_{rel,tang} = V_{*,t} - V_{\odot,t} = V_* \operatorname{sen}\alpha - V_\odot \cos \ell$$

Usando movimiento circular

$$\boxed{\begin{aligned} V_{rel,radial} &= \Omega_* R_* \cos\alpha - \Omega_\odot R_\odot \operatorname{sen} \ell \\ V_{rel,tang} &= \Omega_* R_* \operatorname{sen}\alpha - \Omega_\odot R_\odot \cos \ell \end{aligned}}$$

Mirando el vértice rojo: $R_* \cos\alpha = R_\odot \operatorname{sen} \ell$ (I)

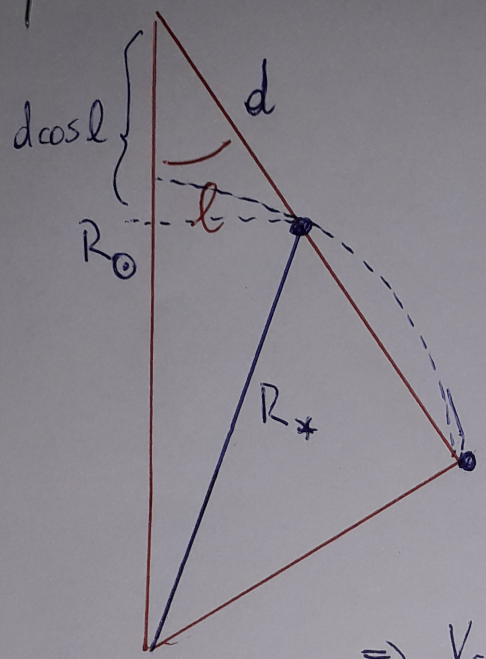
$$R_* \operatorname{sen}\alpha + d = R_\odot \cos \ell$$

Vuelvo a las expresiones recuadradas:

$$V_{rel,radial} = \Omega_* \overbrace{R_\odot \operatorname{sen} \ell}^{(I)} - \Omega_\odot R_\odot \operatorname{sen} \ell = (\Omega_* - \Omega_\odot) R_\odot \operatorname{sen} \ell$$

Como $R_\odot >> d$ y $R_* >> d$, entonces:

$$\Omega_* - \Omega_\odot \simeq (R_* - R_\odot) \left.\frac{d\Omega}{dR}\right|_{R_\odot} + \cdots$$



$$R_* - R_\odot = -d\cos l$$

$$\Rightarrow \Omega_* - \Omega_\odot \simeq -d\cos l \left.\frac{d\Omega}{dR}\right|_{R_\odot}$$

$$\Rightarrow V_{rel,radial} \simeq -R_\odot \left.\frac{d\Omega}{dR}\right|_{R_\odot} d\cos l \operatorname{sen} l$$

$$V_{rel,radial} = -R_\odot \left.\frac{d\Omega}{dR}\right|_{R_\odot} \frac{d}{2}\operatorname{sen}(2l)$$

$$R_\odot \frac{d\Omega}{dR} = \frac{d(R_\odot \Omega)}{dR} - \Omega \Rightarrow V_{rel,radial} = A\, d \operatorname{sen}(2l)$$

donde $$A = \frac{1}{2}\left[\Omega - \frac{d(R\Omega)}{dR}\right]\Bigg|_{R=R_\odot}$$