Lecture Series Buenos Aires

18-3-2024 until 22-3-2024

Lecture F2 – Oscillators and CEP

Max-Born-Institut

Federico Furch

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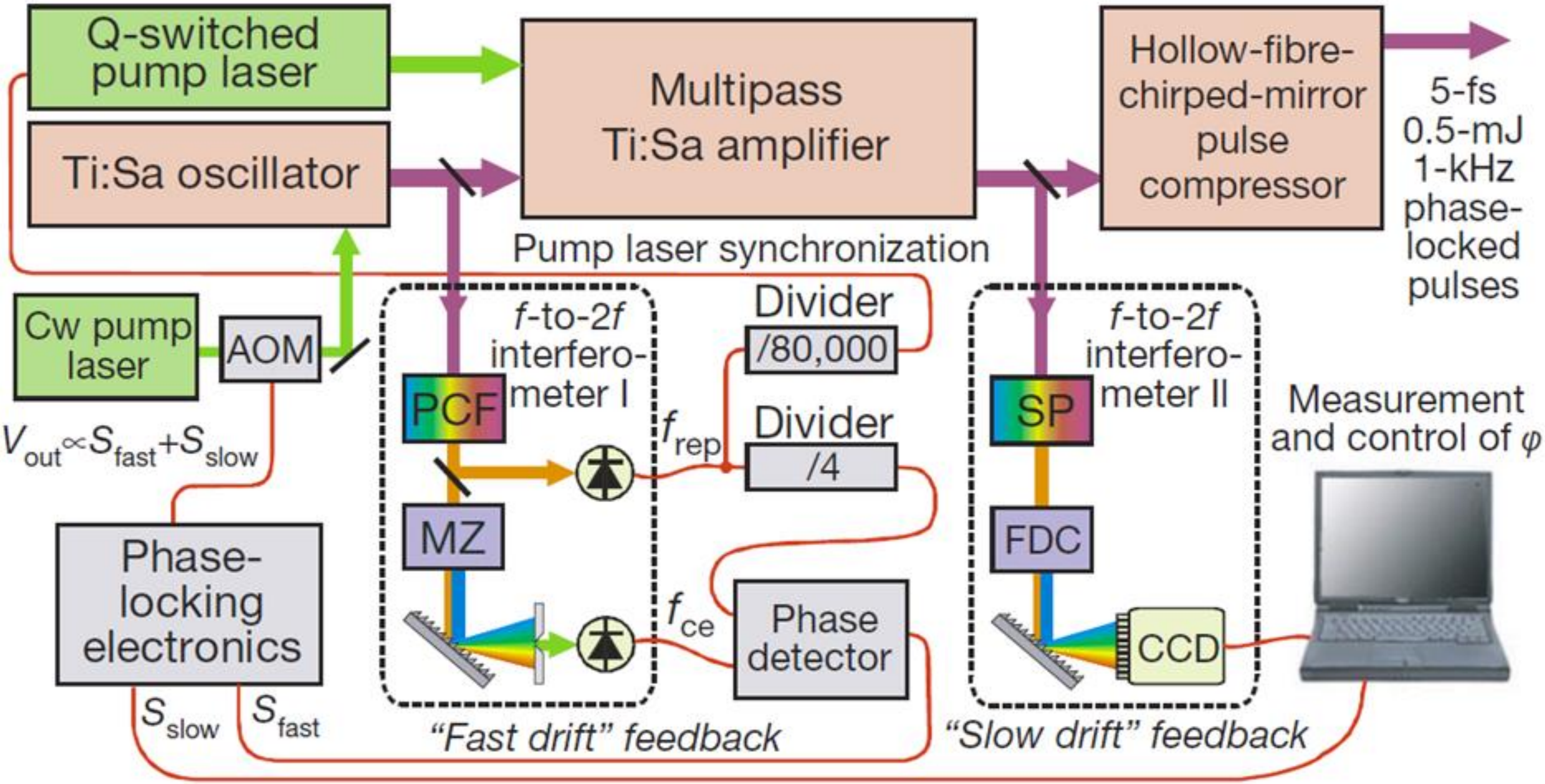
Laser oscillators and Carrier-Envelope Phase (CEP) stability

A state-of-the-art laser system for attosecond science



A. Baltuska et al., Nature 421, 611 (2003)

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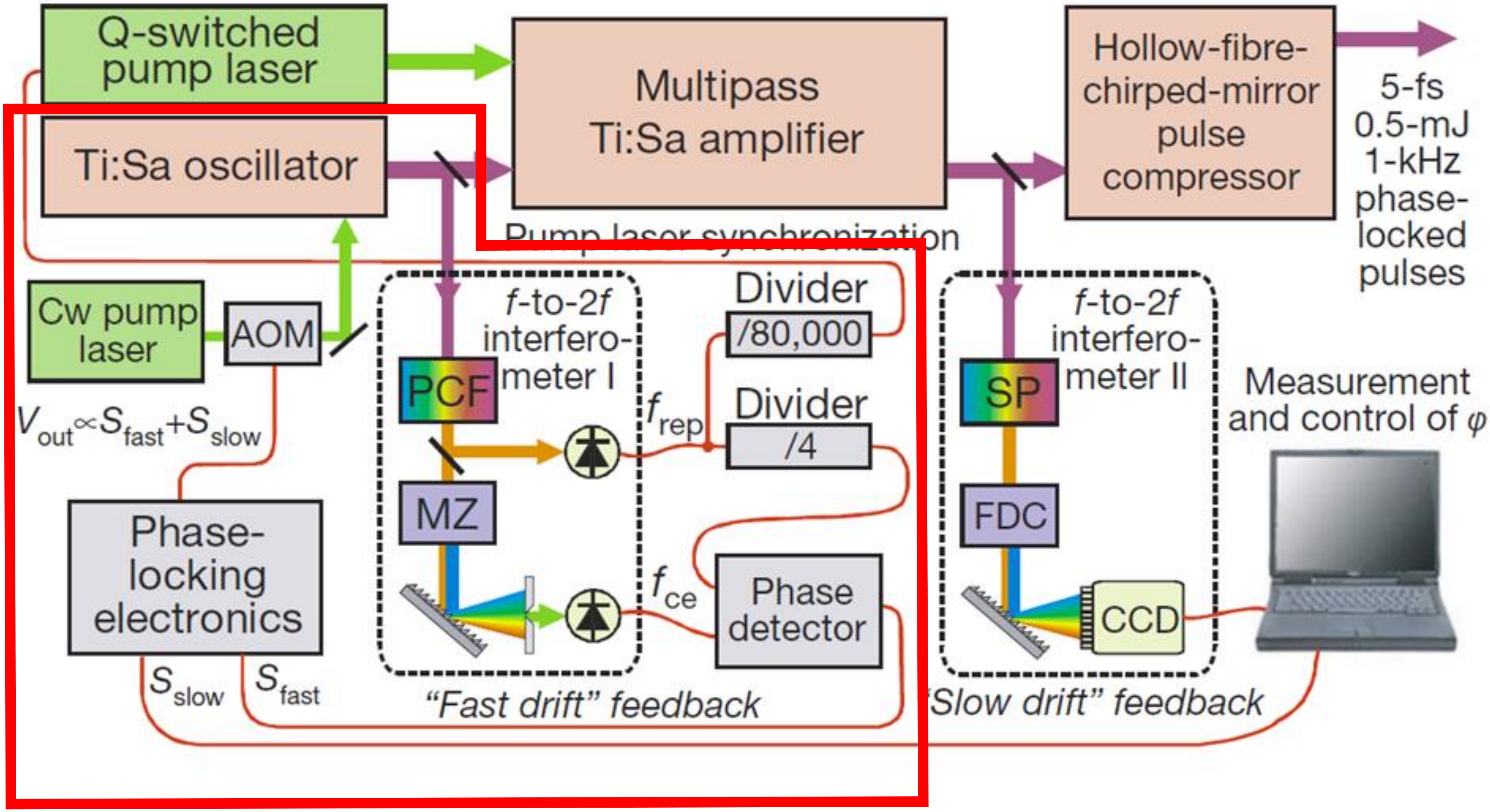

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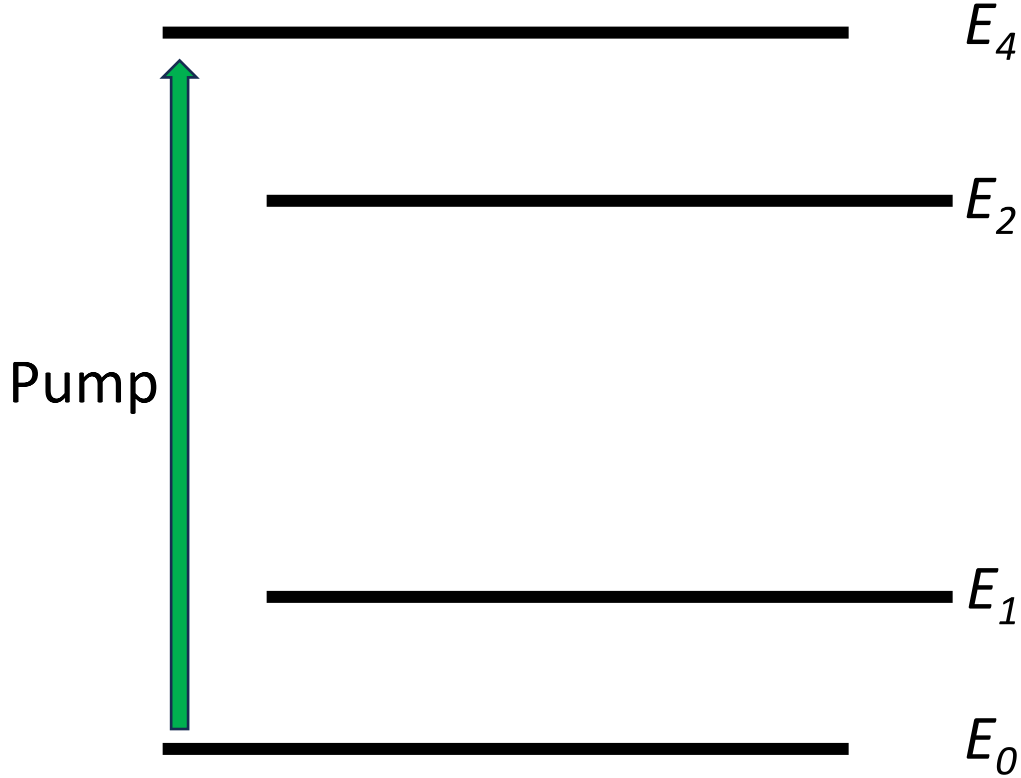
Topics to be discussed

- How do we generate short pulses that can later be amplified?
    - *Basics of Lasers and mode-locking*
    - *dispersion compensation to achieve the largest bandwidth*
- How to lock the carrier envelope phase of these pulses?

Light Ampflication by Stimulated Emission of Radiation (LASER)

Energy levels in LASER material

e.g. Levels of Ti^{3+} ion in Al_2O_3 (sapphire) matrix



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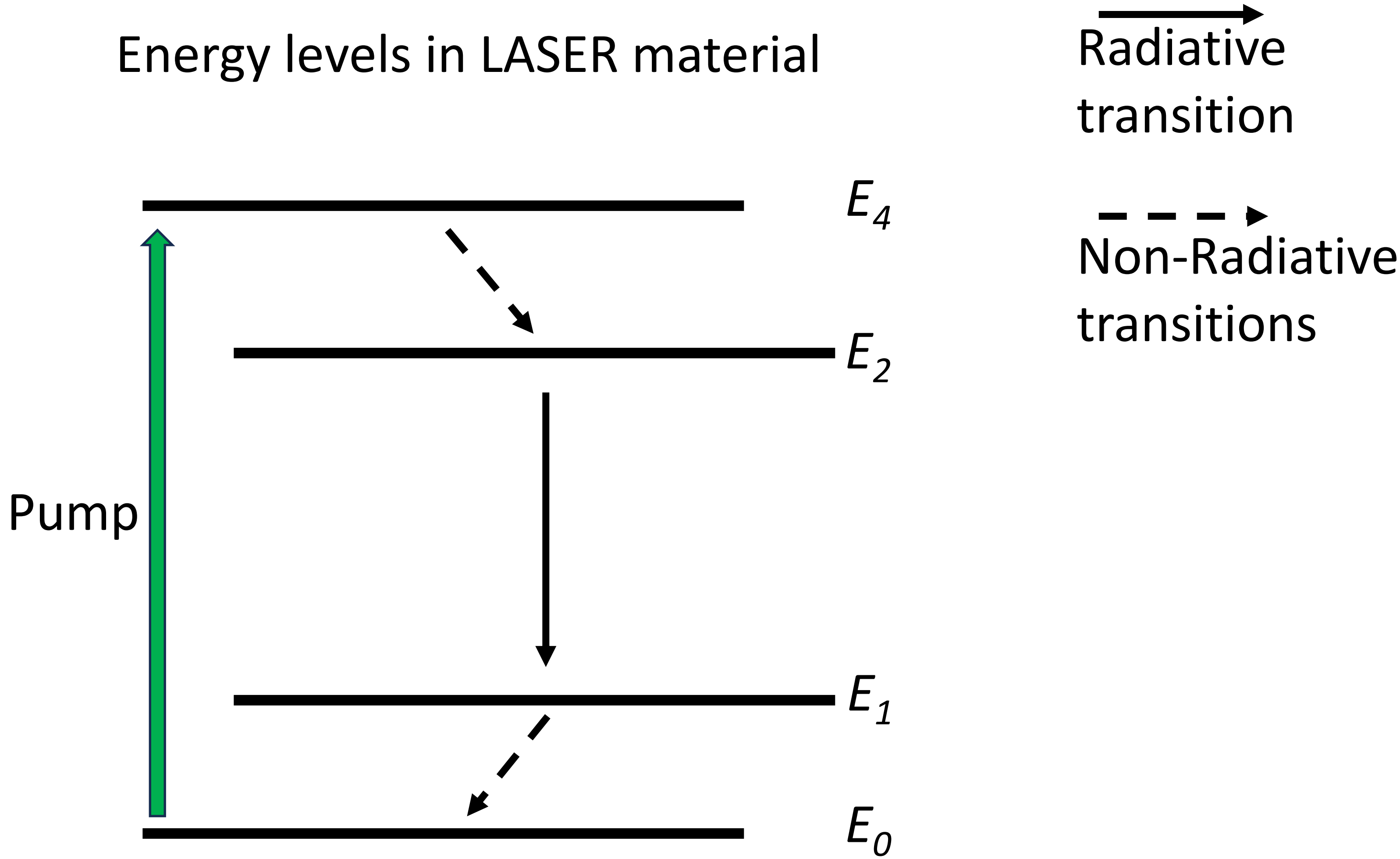

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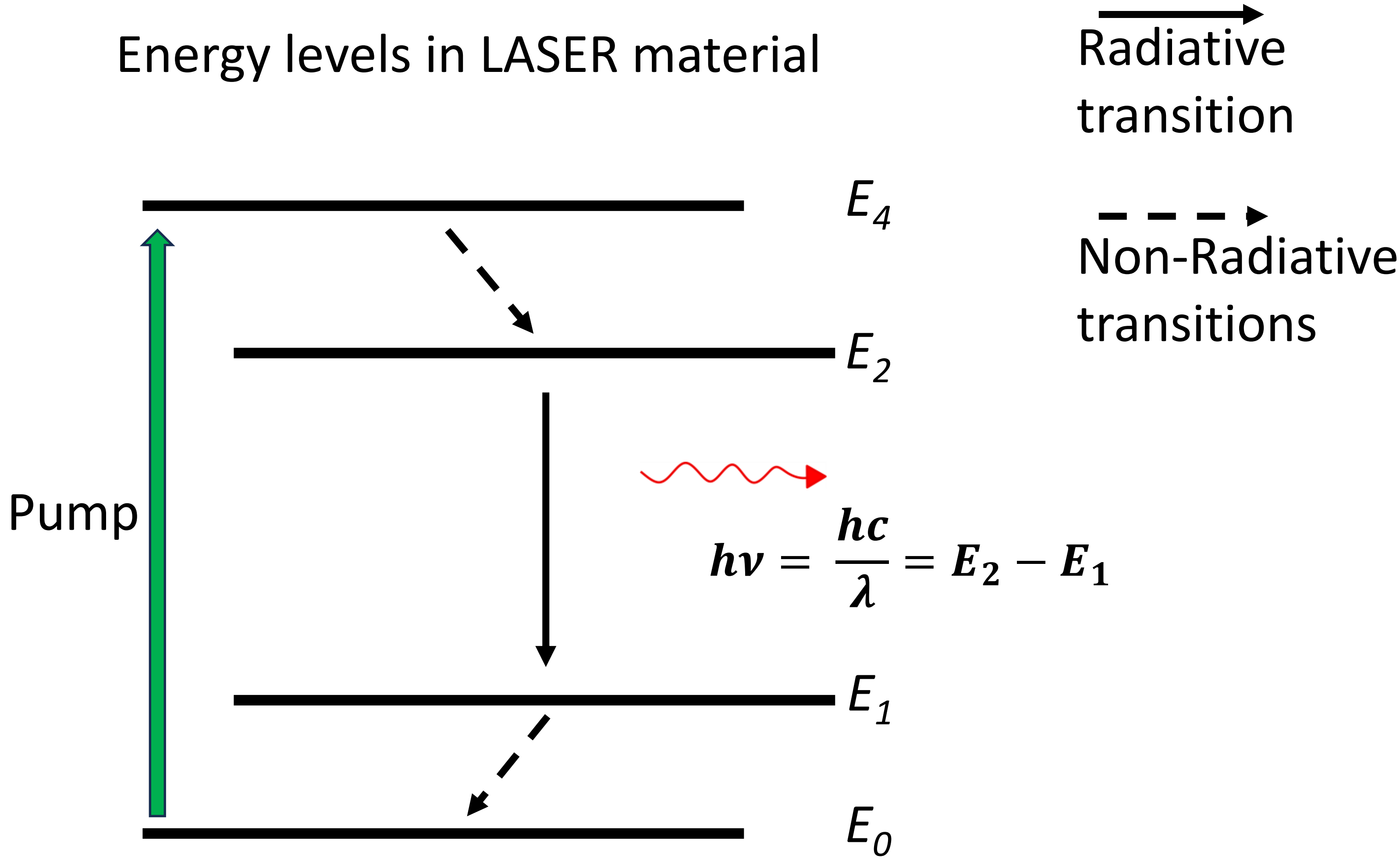
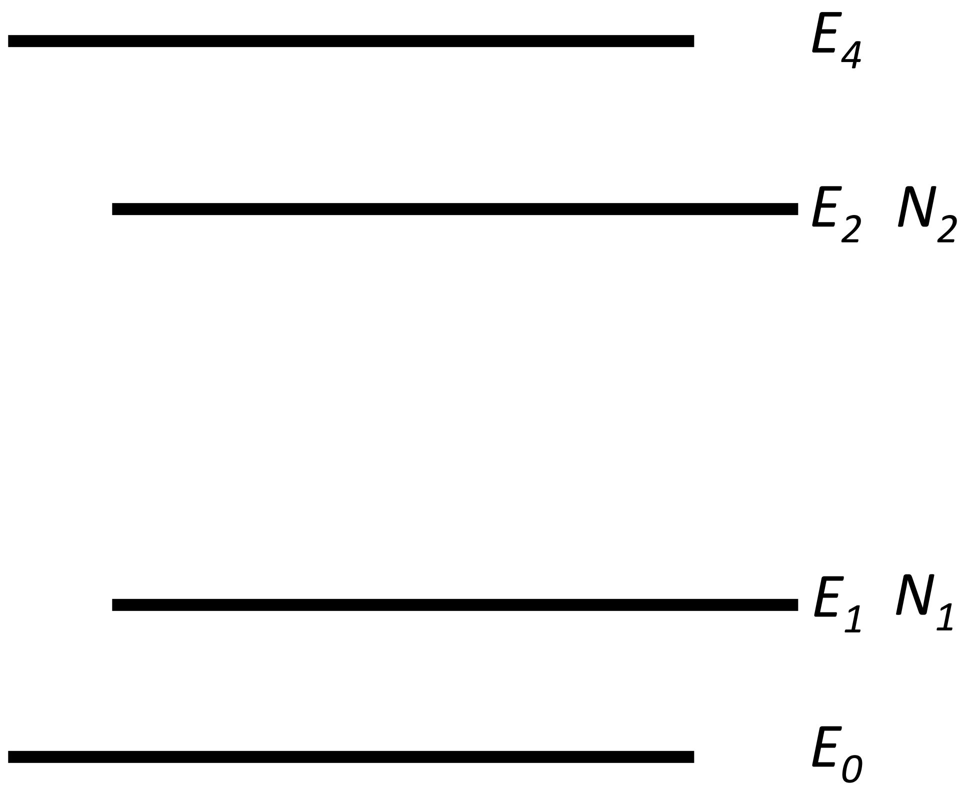


Light Ampflication by Stimulated Emission of Radiation (LASER)

Energy levels in LASER material

Incident photon resonant with transition

$$h\nu = E_2 - E_1$$



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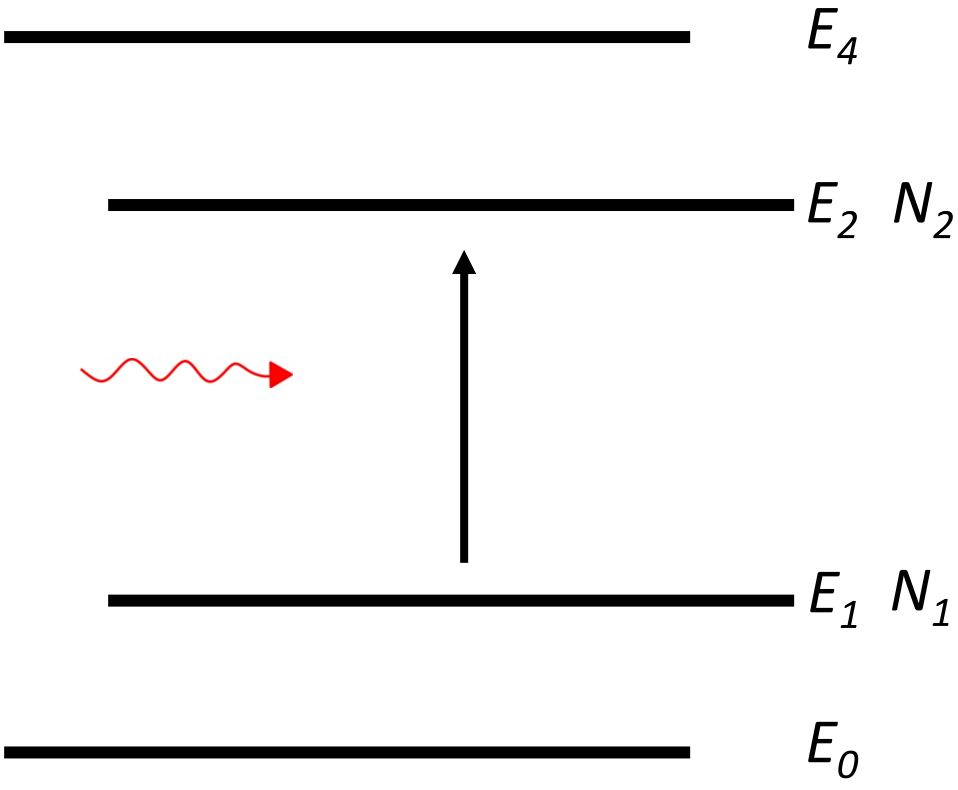

If $N_1 > N_2$

absorption

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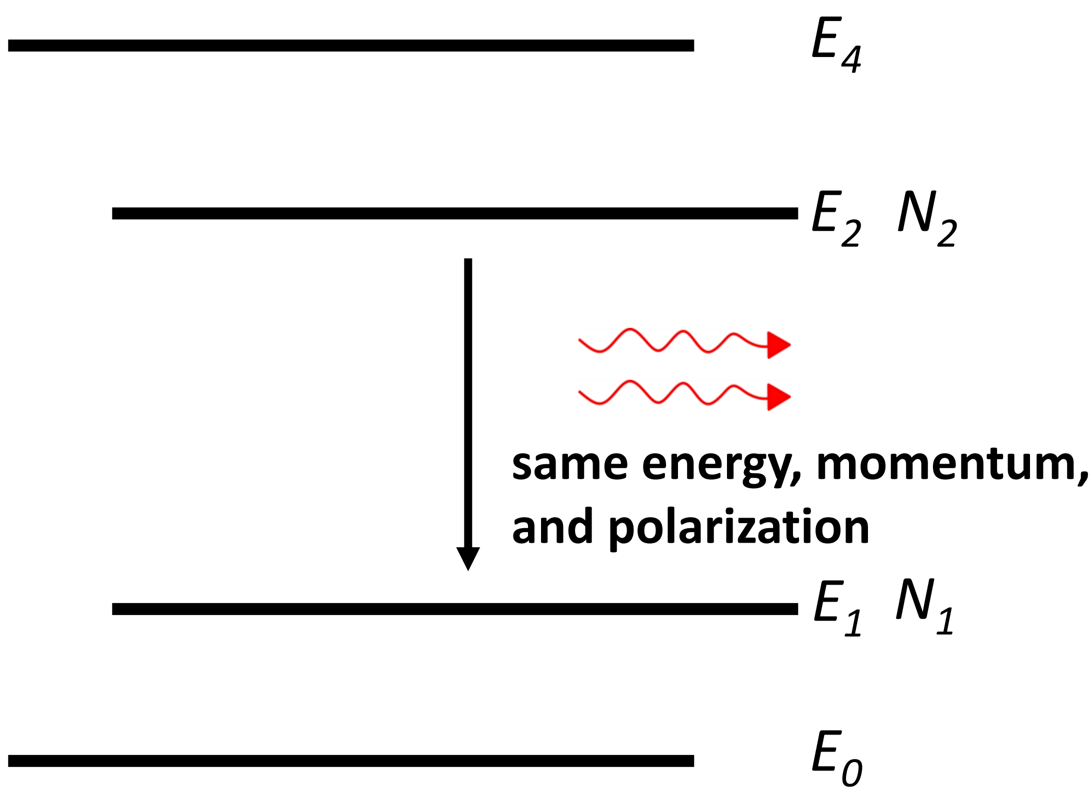
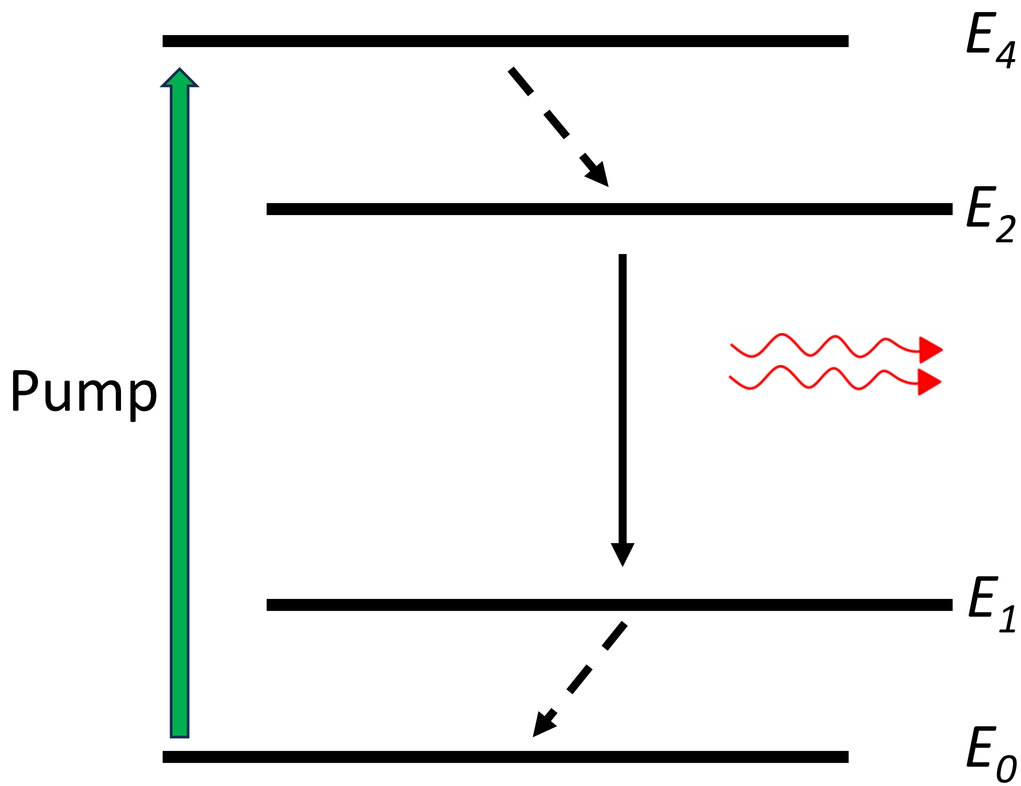


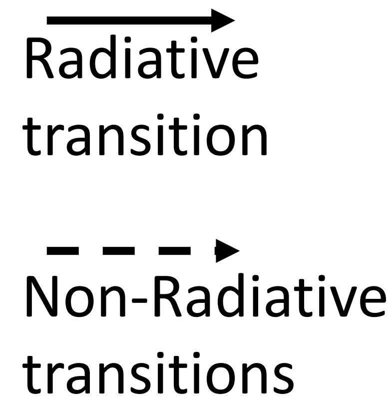
If $N_2 > N_1$ **stimulated emission = amplification**

$N_2 > N_1$: population inversion

Light Ampflication by Stimulated Emission of Radiation (LASER)

Energy levels in LASER material



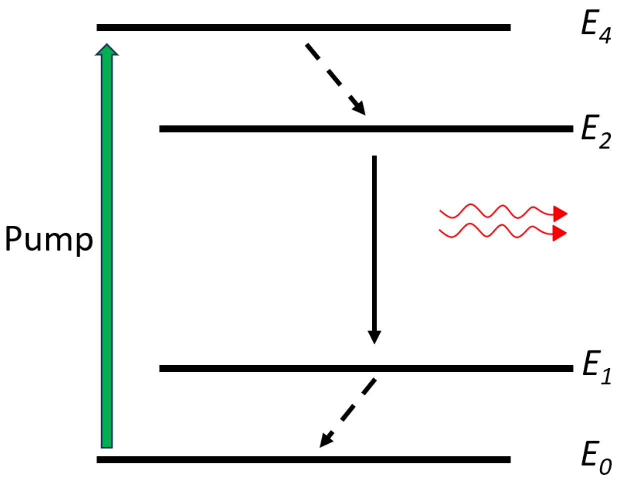


How is the population inversion created?

Typically: fast non-radiative decays.

Lifetime of level 2 >> Lifetime of level 1

Laser transition

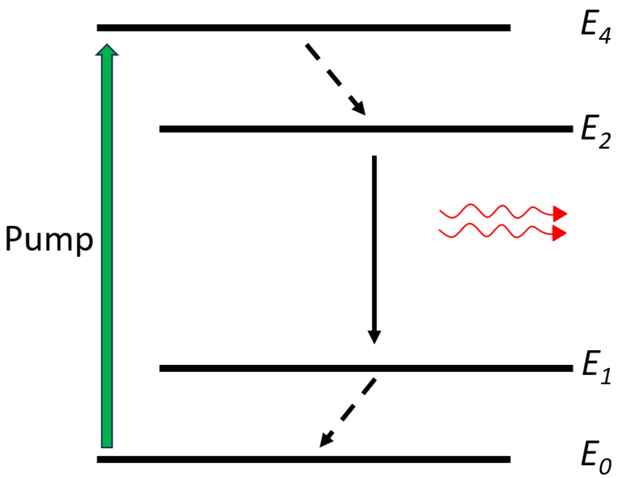


$$h\nu = \frac{hc}{\lambda} = E_2 - E_1$$

How monochromatic is the laser transition?

Figure from https://micro.magnet.fsu.edu/primer/java/lasers/tsunami/index.html

Laser transition



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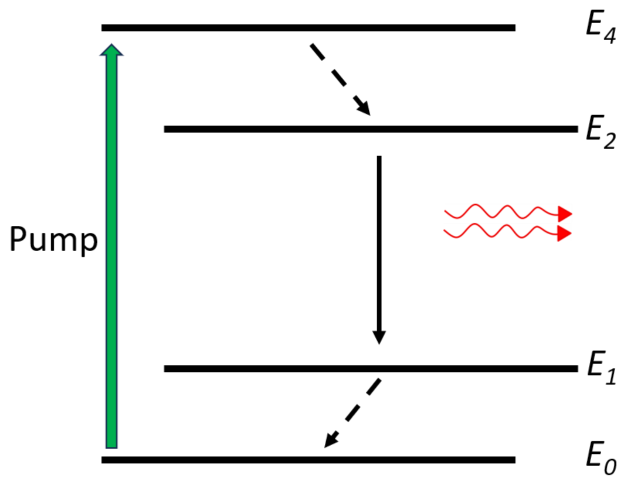

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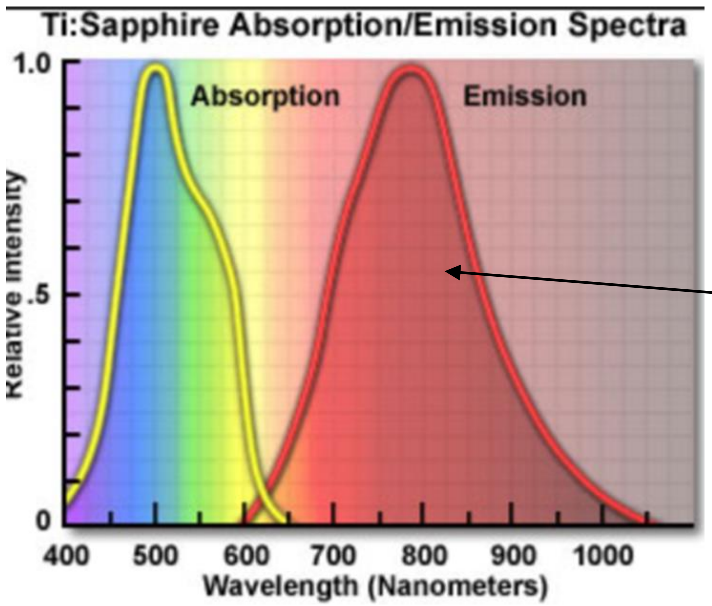
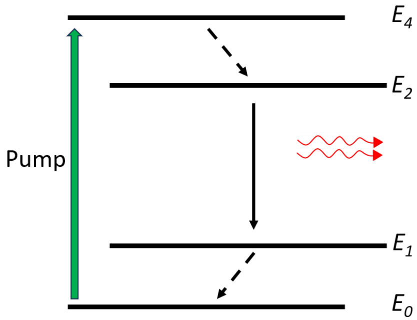
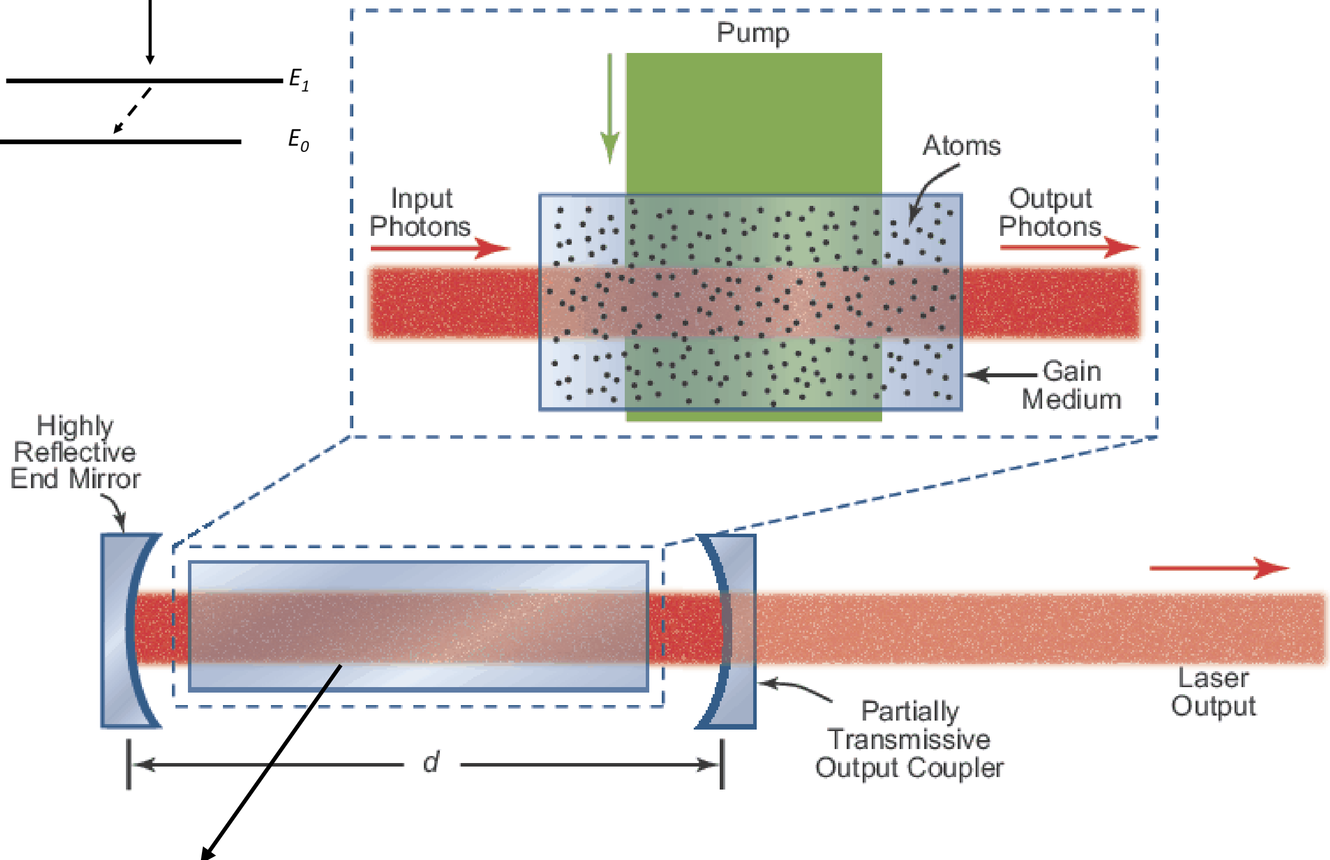


Ti:Sapphire can emit in any of these wavelengths (frequencies)

Figure from https://micro.magnet.fsu.edu/primer/java/lasers/tsunami/index.html

Laser oscillator

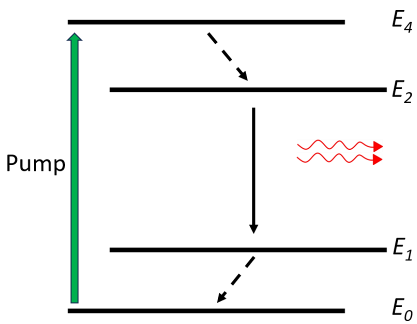




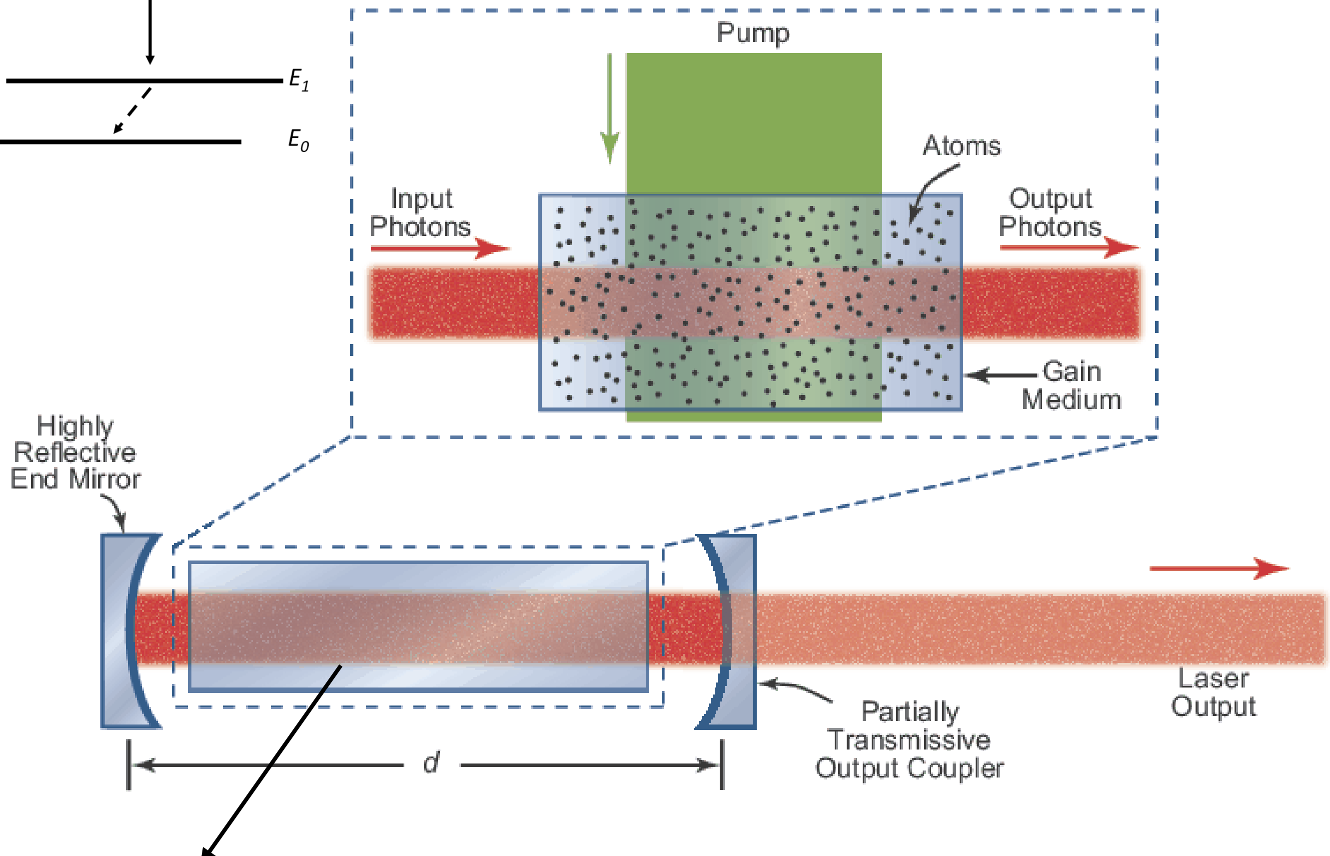
Gain (laser) material
e.g. Ti:Sapphire

Figure from https://www.newport.com/n/critical-laser-components

Laser oscillator



Intracavity power increases until steady state is reached: gain equal losses



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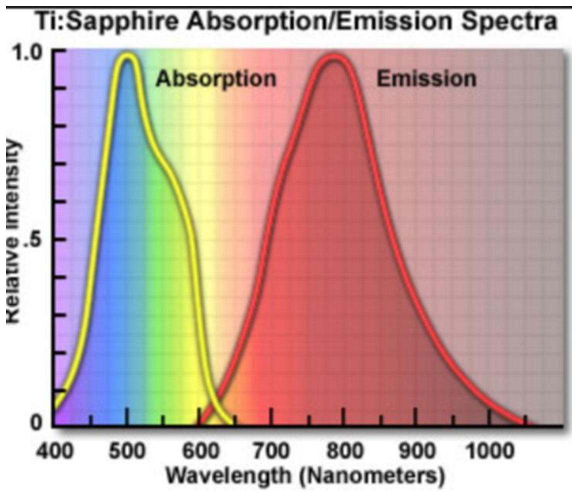
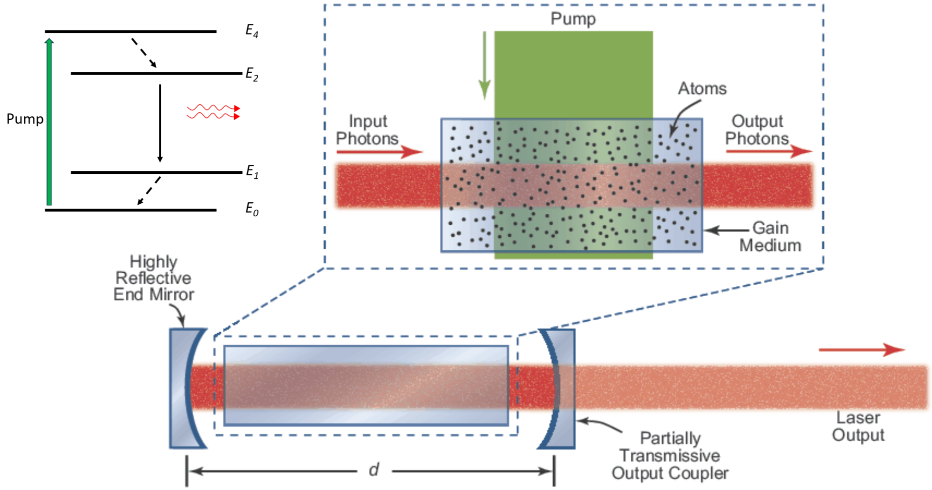
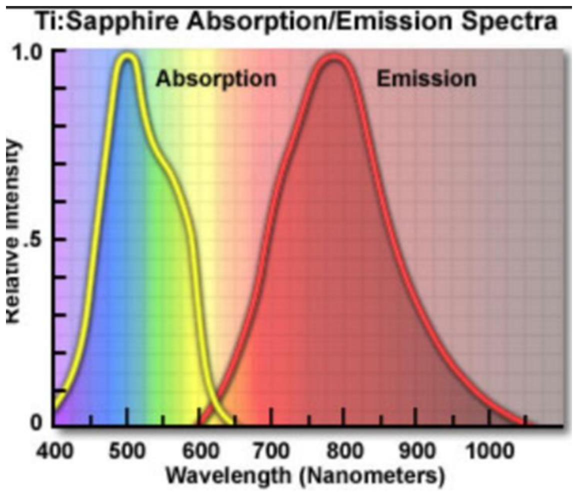
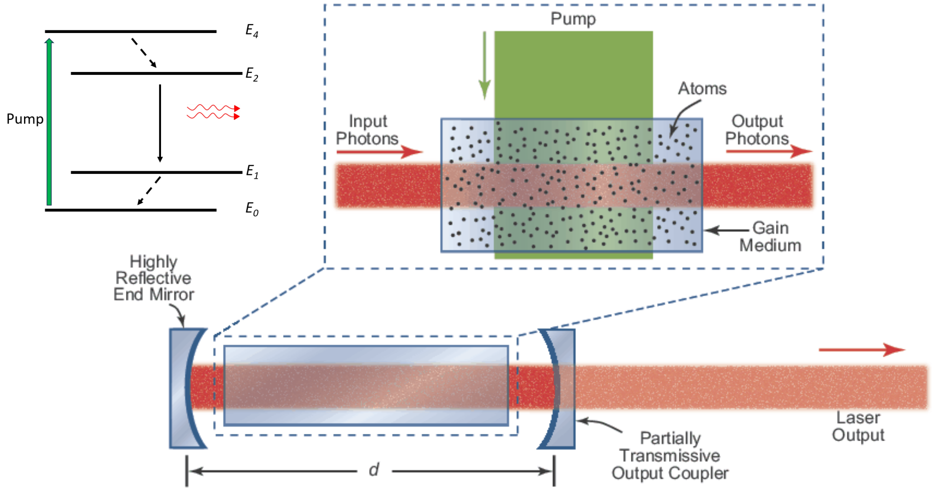




Does the laser emit in all possible frequencies within the emission cross section?

Laser oscillator





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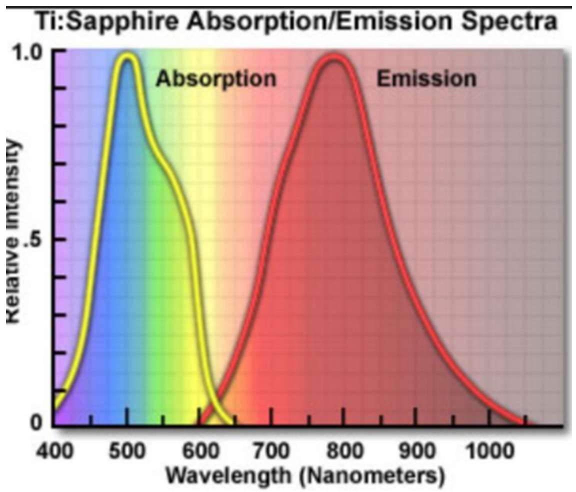
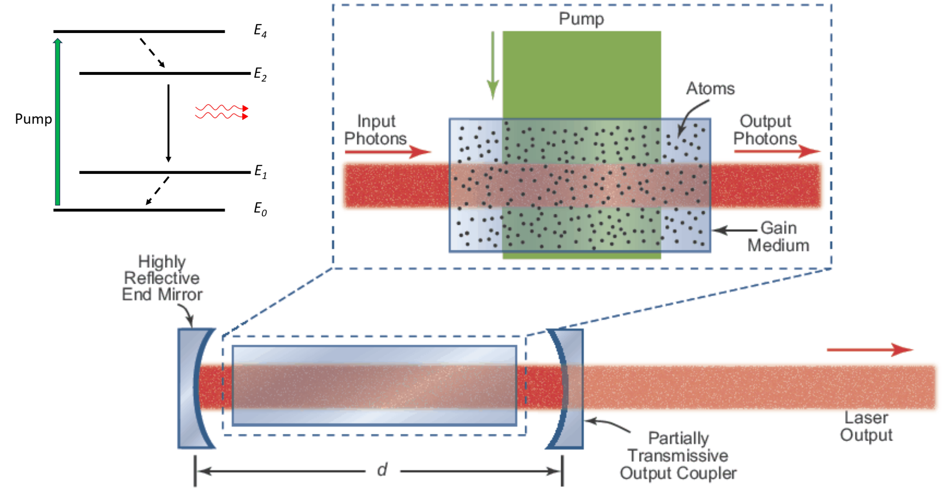
No…

Mirrors form resonant cavity. Only certain frequencies are allowed

So-called longitudinal modes

Laser oscillator





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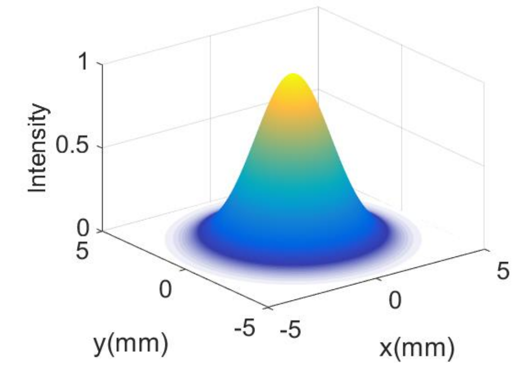
So-called longitudinal modes

It also gives rise to transverse modes



See **A. Siegman, Lasers, (University Science Books, 1986)**

Longitudinal modes and cw operation

Longitudinal modes in a laser cavity in vacuum: $v_m = \frac{mc}{2L}$

Mode spacing in vacuum: $\Delta v = v_{m+1} - v_m = c/(2L)$

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Diff. optical components (laser crystal, air)

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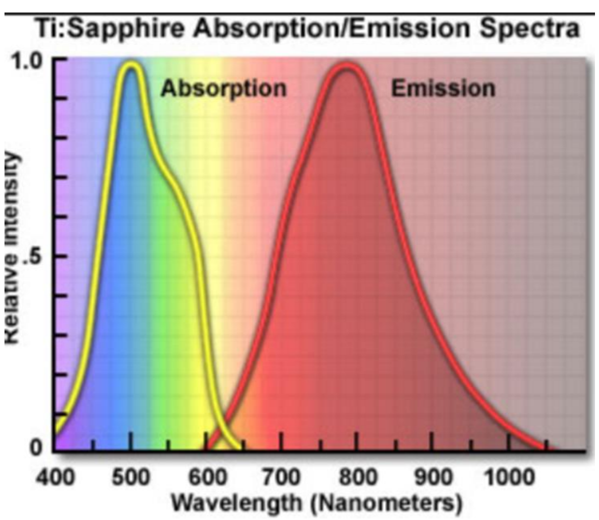

gain$\sim e^{g(\omega)L}$

$g(\omega) \propto \sigma_{21}$ → Emission cross section

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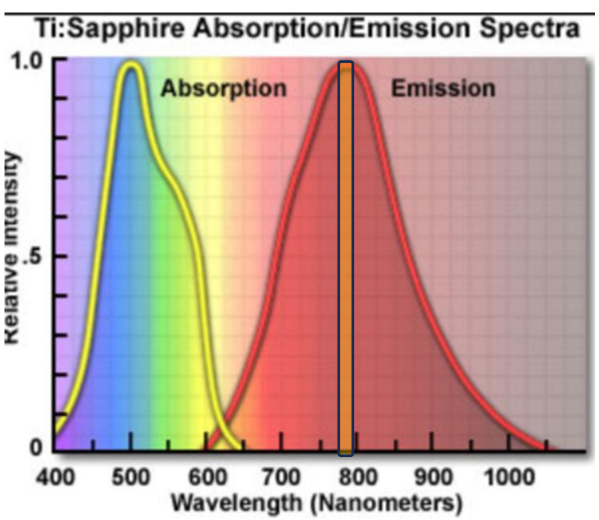

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A few longitudinal modes take over the gain: narrowband laser emission

Other modes: gain – losses < 0

Longitudinal modes and cw operation

Electric field from superposition of adjacent longitudinal modes

Random phase relation between modes

$$E^{+}(t) = \frac{1}{2}\varepsilon(t)e^{i\varphi(t)}e^{i\omega_l t} = \frac{1}{2}\varepsilon_0 e^{i\omega_l t}\sum_m e^{i(2m\pi\Delta\upsilon t+\varphi_m)}$$

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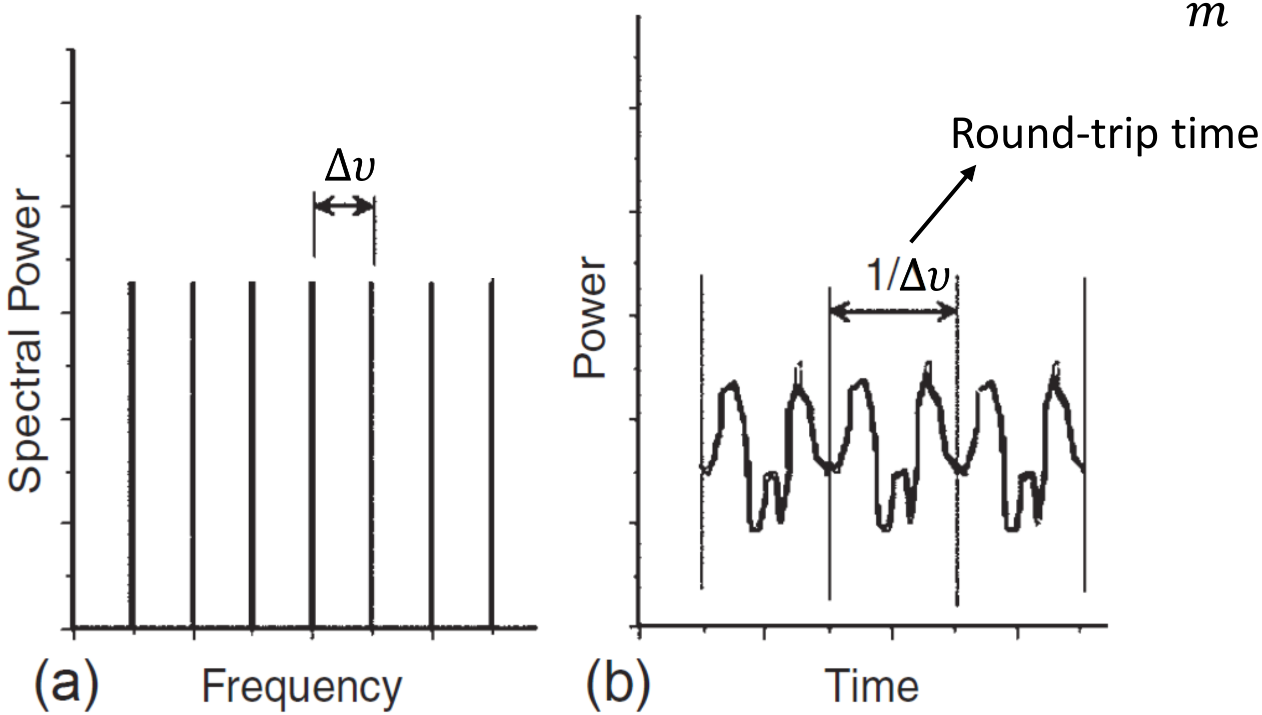

CW: Continuous Wave operation

Mode-locking

What happens when several modes operate in phase?

Electric field from superposition of M adjacent modes with same amplitude and phase $\varphi_m = \varphi_0$

$$E^{+}(t) = \frac{1}{2}\varepsilon_0 e^{i\omega_l t} \sum_m e^{i(2m\pi\Delta\upsilon t + \varphi_0)} = \frac{1}{2}\varepsilon_0 e^{i\varphi_0} e^{i\omega_l t} \frac{\sin(M\pi\Delta\upsilon t)}{\sin(\pi\Delta\upsilon t)}$$

J.-C. Diels and W. Rudolph, *Ultrashort Laser Pulse Phenomena,* (Academic Press, 2006)

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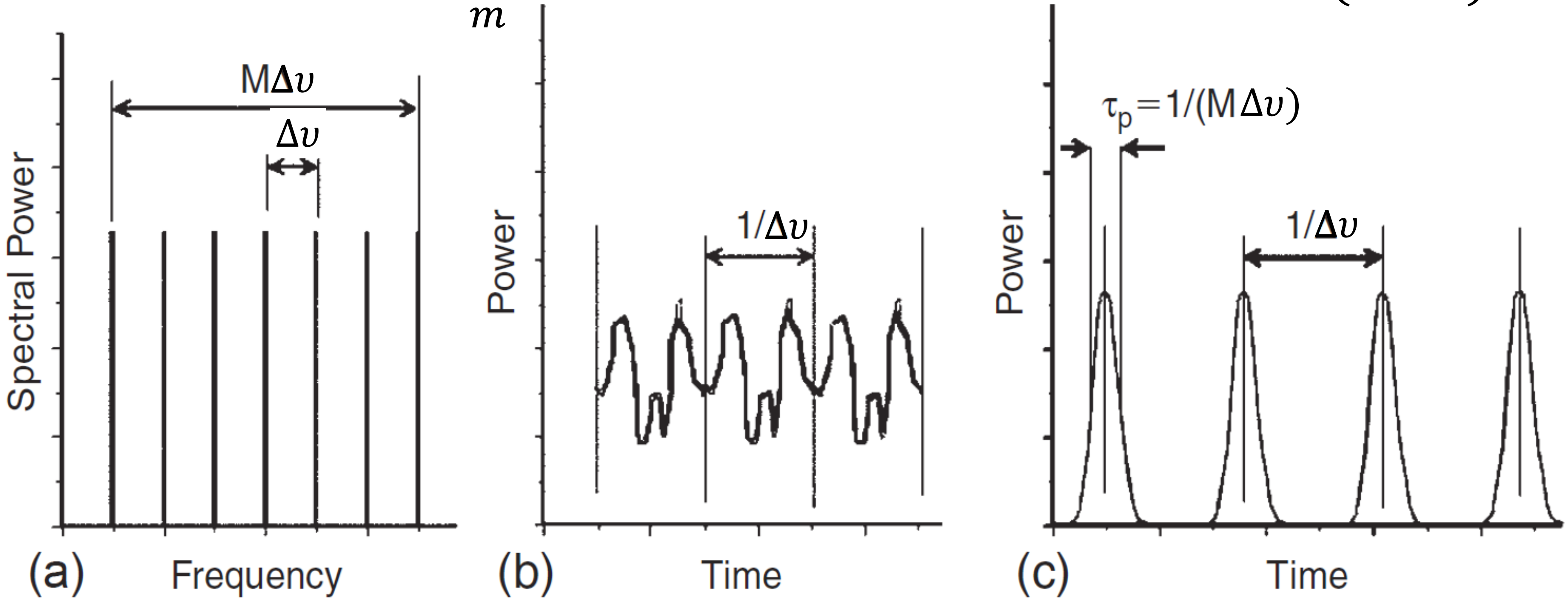

The cw and the mode-locked laser have approximately the same **average power**

The mode-locked laser has an M-fold higher **peak power ($M\sim10^6$)**

J.-C. Diels and W. Rudolph, *Ultrashort Laser Pulse Phenomena,* (Academic Press, 2006)

Mode-locking: Kerr lens

How to we make the modes to operate in phase?

Under normal circumstances a few modes saturate the gain

→Create conditions such that pulsed operation is favored over cw

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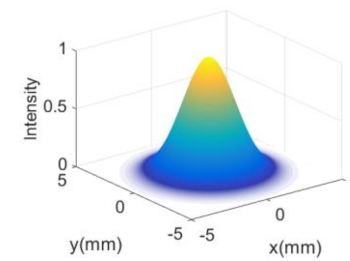
Kerr-lens mode-locking exploits the non-linearity of the refractive index

$$n(I) = n_0 + n_2 I$$



Focal length of the Kerr lens $f = \frac{w_0^2}{4 n_2 d I_0}$

w_0 = beam radius

d = thickness

Mode-locking: Kerr lens

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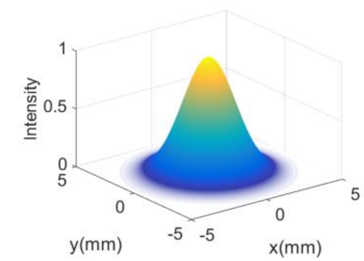


The mode-locked laser has an M-fold higher **peak power (thus intensity)**

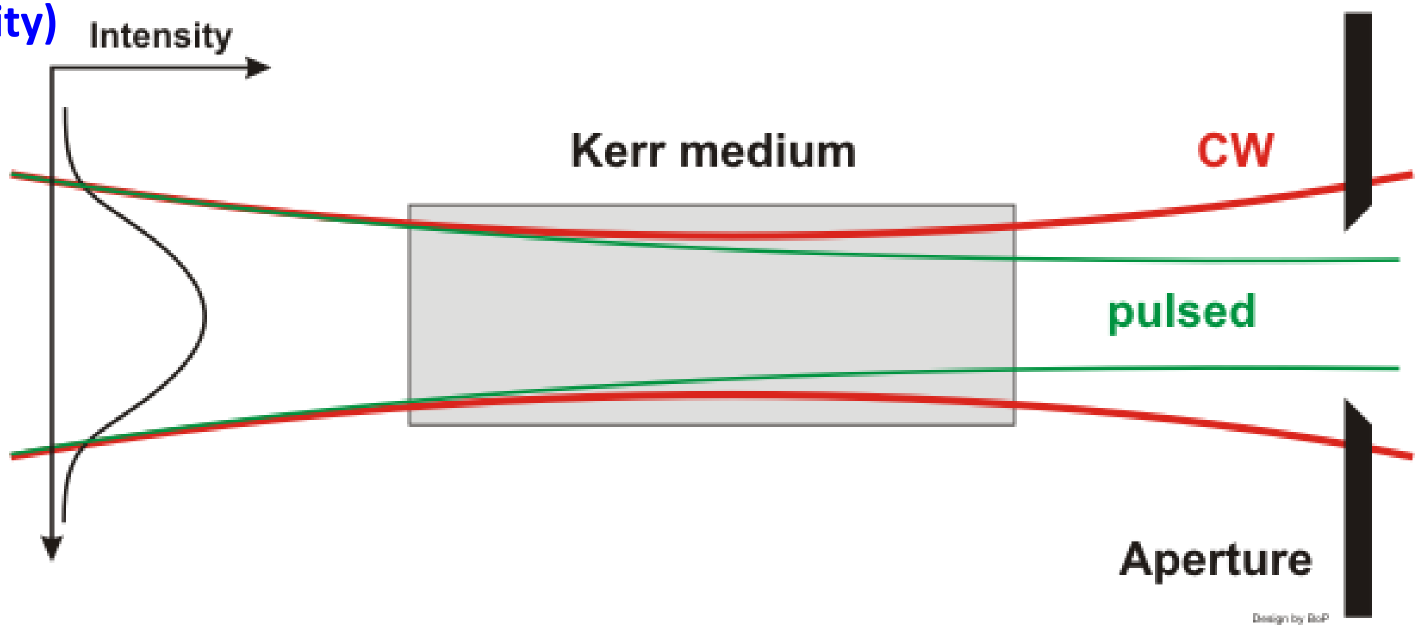
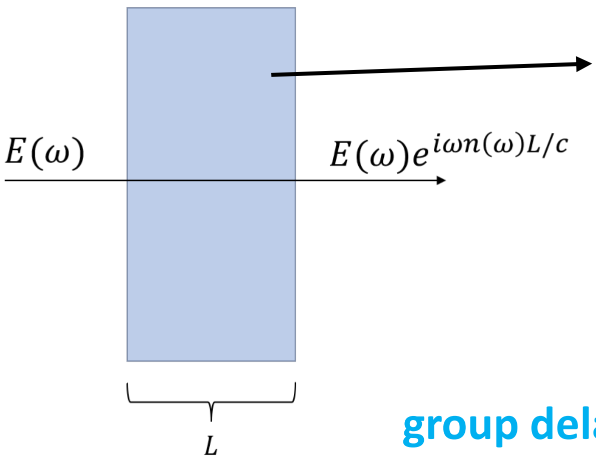


Image from http://www.rp-photonics.com/kerr_lens.html

Mode-locking: Large bandwidths



Optical elements in the cavity
e.g. laser crystal

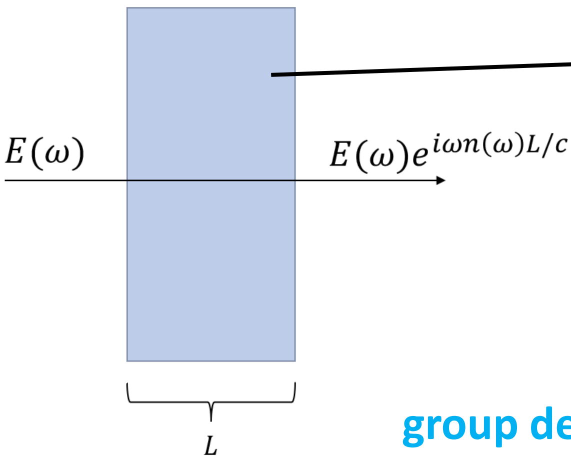
group delay (GD)

group delay dispersion (GDD)

$$\varphi(\omega) = \varphi_0 + \left(\frac{\partial\varphi(\omega)}{\partial\omega}\right)_{\omega=\omega_0}(\omega-\omega_0) + \frac{1}{2}\left(\frac{\partial^2\varphi(\omega)}{\partial\omega^2}\right)_{\omega=\omega_0}(\omega-\omega_0)^2 + \frac{1}{6}\left(\frac{\partial^3\varphi(\omega)}{\partial\omega^3}\right)_{\omega=\omega_0}(\omega-\omega_0)^3 + \cdots$$

Third Order Dispersion (TOD)

Mode-locking: Large bandwidths



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Third Order Dispersion (TOD)

As the pulse travels in the cavity it accumulates a phase due to dispersion and stretches, reducing its intensity:
The larger the bandwidth, the more it is affected by dispersion

Mode-locking: Large bandwidths

group delay (GD)

group delay dispersion (GDD)

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Third Order Dispersion (TOD)

Before 1984: no method for (low-loss) negative GDD in a laser cavity

Mode-locking: Large bandwidths

group delay (GD) — group delay dispersion (GDD)

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Third Order Dispersion (TOD)

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150 OPTICS LETTERS / Vol. 9, No. 5 / May 1984

Negative dispersion using pairs of prisms

R. L. Fork, O. E. Martinez, and J. P. Gordon

AT&T Bell Laboratories, Holmdel, New Jersey 07733

Received December 12, 1983; accepted February 22, 1984

We show that pairs of prisms can have negative group-velocity dispersion in the absence of any negative material dispersion. A prism arrangement is described that limits losses to Brewster-surface reflections, avoids transverse displacement of the temporally dispersed rays, permits continuous adjustment of the dispersion through zero, and yields a transmitted beam collinear with the incident beam.

Large bandwidths with prism pairs

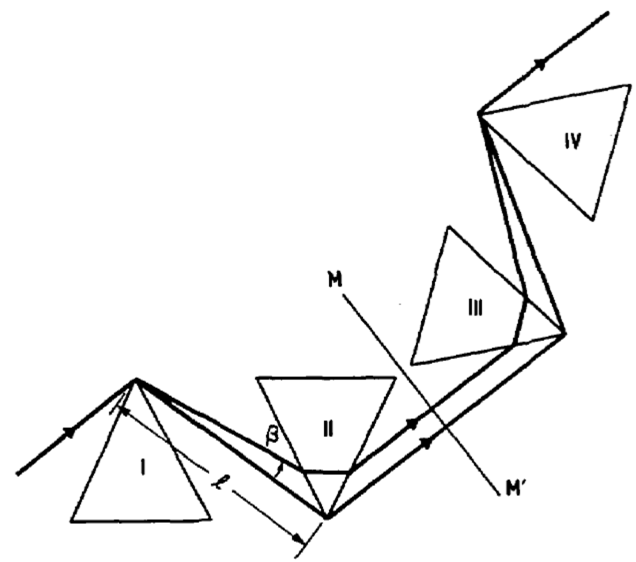
It was the dominant technique for chirp management in oscillator cavities for over 20 years (and still very much in use)

Prism pair dispersion

$$P(\omega) = -\frac{c}{\omega}\varphi(\omega) = n(\omega) l \cos\theta$$

$$GDD = -\frac{d^2\varphi}{d\omega^2} = \frac{\lambda^3}{2\pi c^2}\frac{d^2P}{d\lambda^2} = -l\frac{\lambda^3}{2\pi c^2}\left(\frac{d\theta}{d\lambda}\right)^2.$$

$$TOD = -\frac{d^2\varphi}{d\omega^3} = -\frac{\lambda^4}{(2\pi)^2 c^3}\left(3\frac{d^2P}{d\lambda^2} + \lambda\frac{d^3P}{d\lambda^3}\right)$$

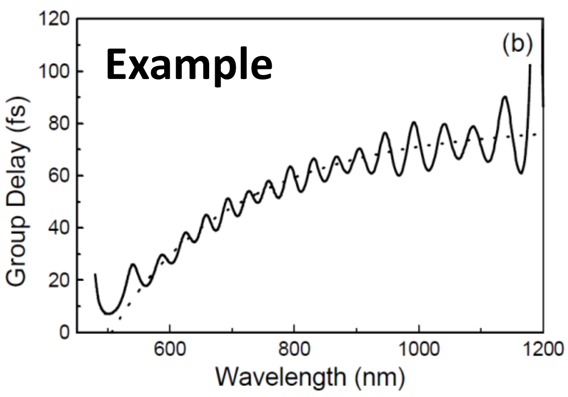


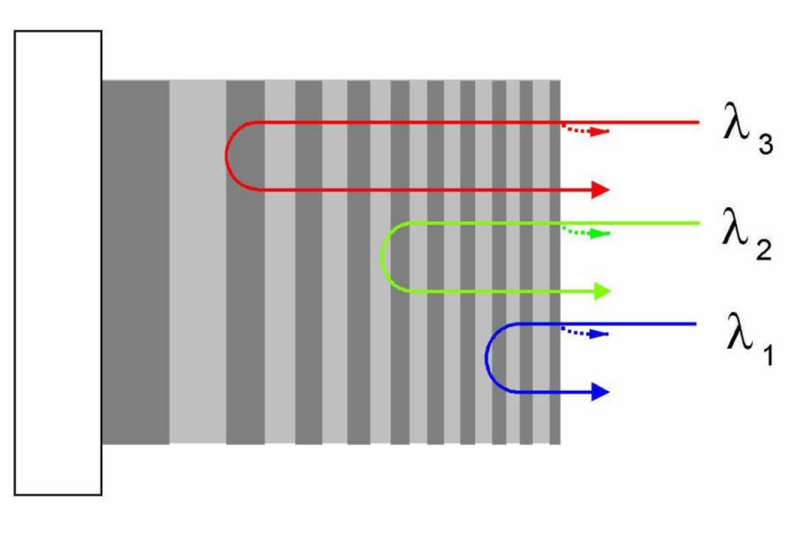
Fork, Martinez, Gordon, Opt. Lett. 9, 150 (1984)

Large bandwidths with dispersive mirrors

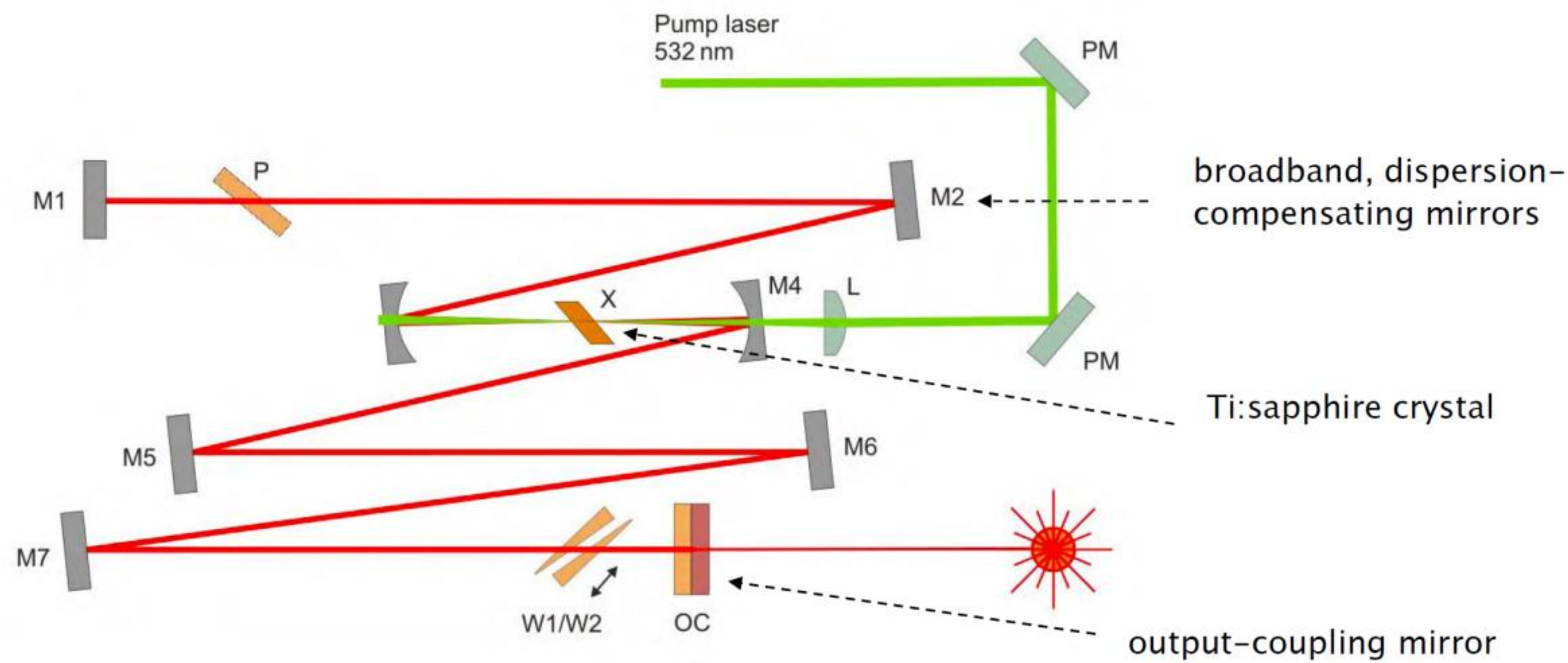
Modern oscillators incorporate mirrors designed to compensate the dispersion inside the cavity

Custom designed dispersive mirrors



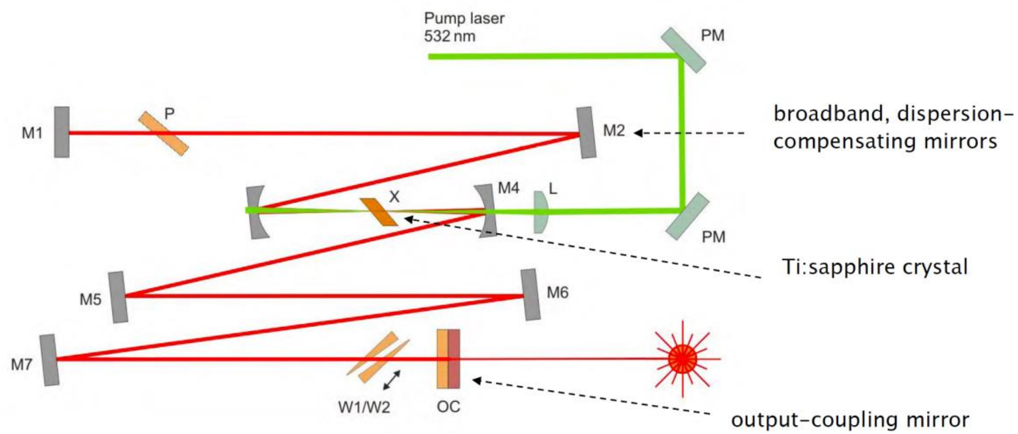


Large bandwidths with dispersive mirrors

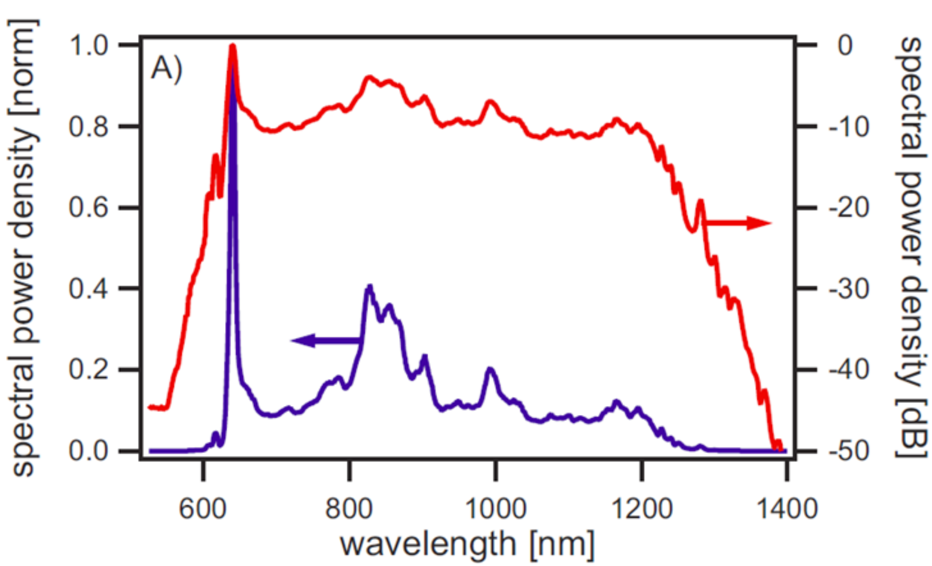


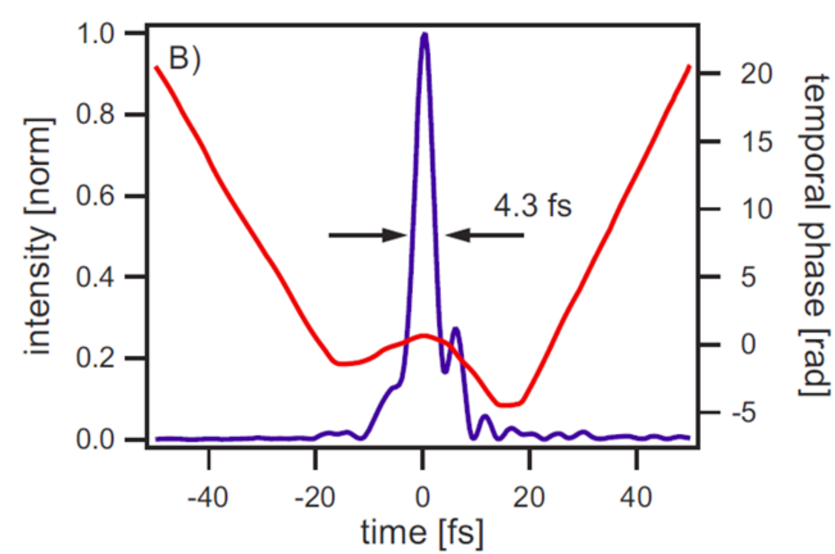
S. Rausch et al., Opt. Expr. 16, 9739 (2008)

Large bandwidths with dispersive mirrors



Using an octave-spanning oscillator 4-fs pulses can be generated **and the stage is set for stabilization of the carrier envelope phase**

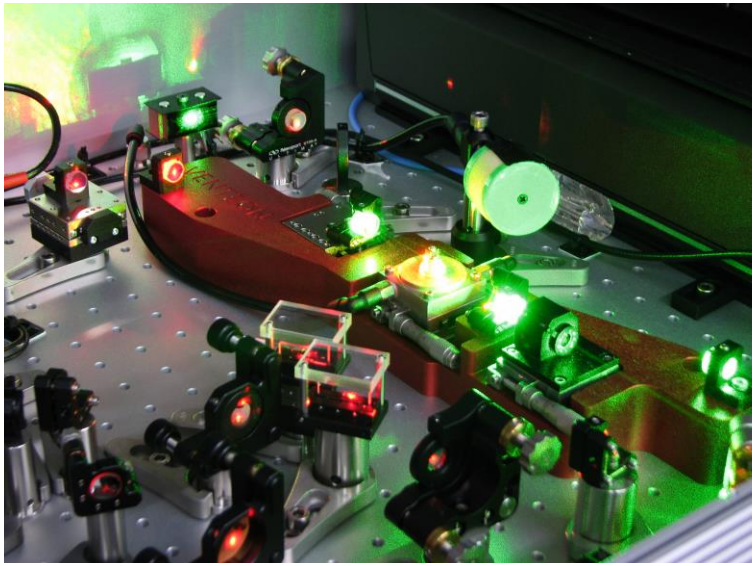




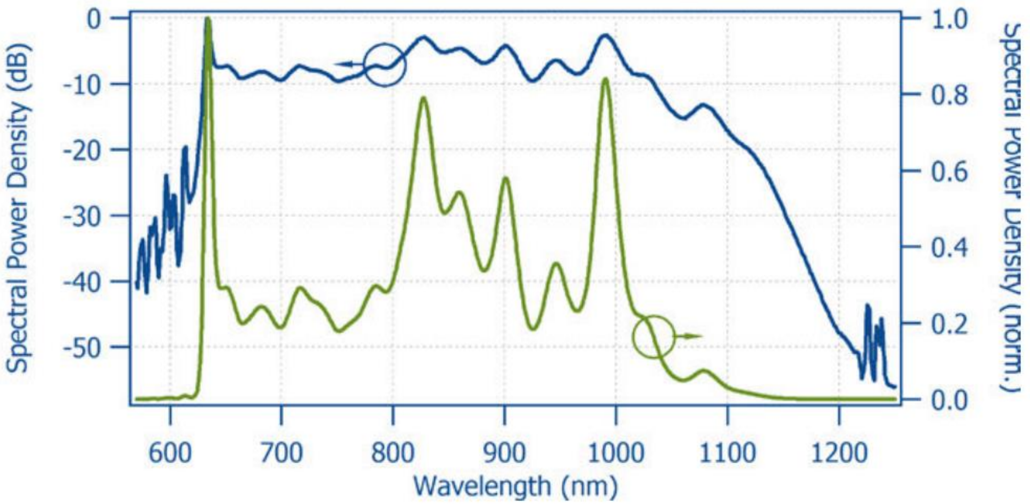
S. Rausch et al., Opt. Expr. 16, 9739 (2008)

Large bandwidths with dispersive mirrors

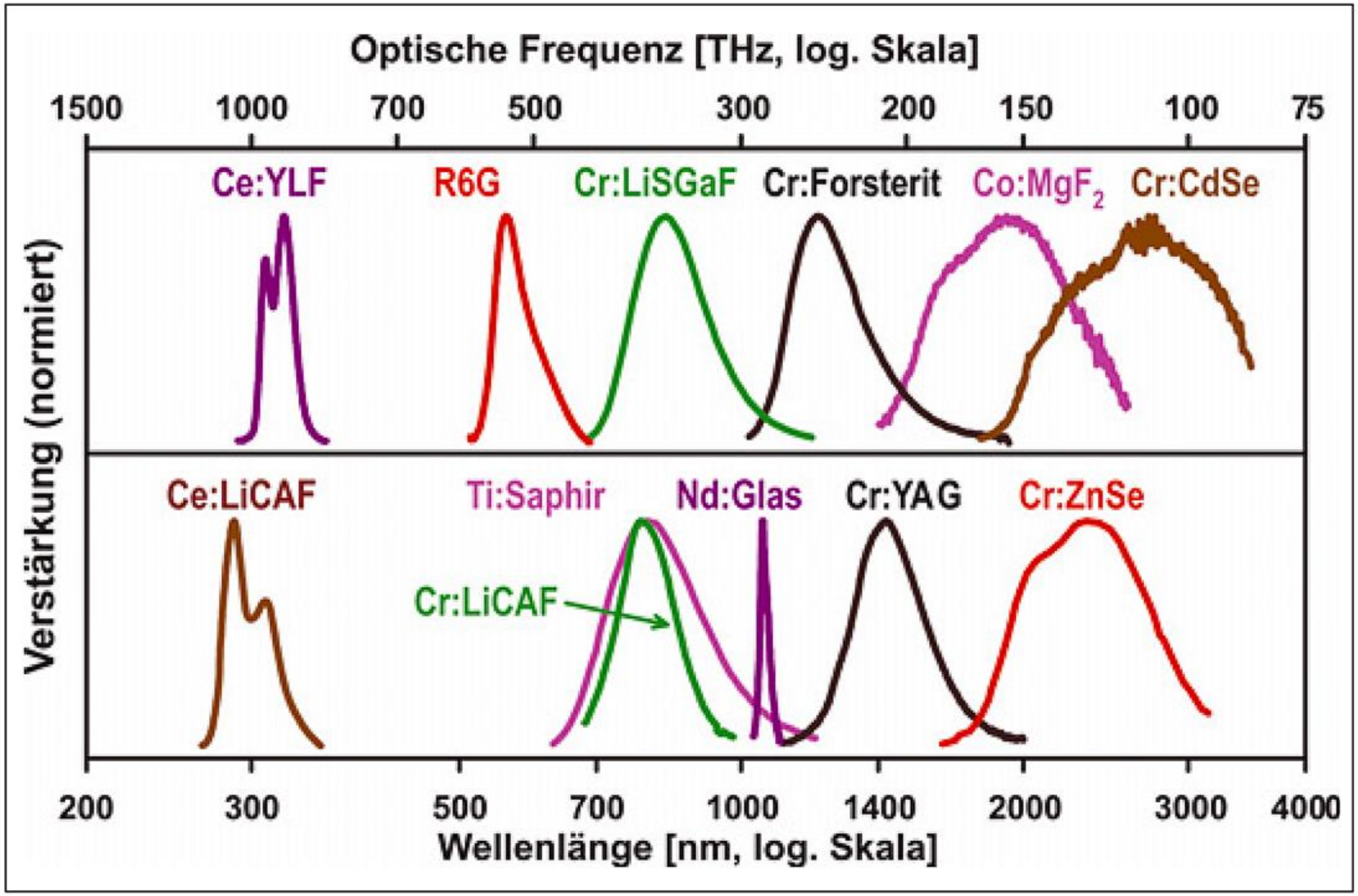
Ti:Sapphire oscillator at MBI



Venteon, Pulse: One
(from Novanta website)



Materials for short pulse generation



Attractive features of Ti:Sapphire:

- extremely broad bandwidth
- long fluorescence lifetime (3μs)
- Pumping at readily available green wavelengths (527, 532 nm)
- high thermal conductivity
- good optical/mechanical properties

Polynomial expansion of phase

carrier envelope phase — instantaneous frequency — linear chirp — N.B. positive chirp means dω/dt > 0

$$\varphi(t) = \varphi_0 + \left(\frac{\partial\varphi(t)}{\partial t}\right)_{t=t_0}(t - t_0) + \frac{1}{2}\left(\frac{\partial^2\varphi(t)}{\partial t^2}\right)_{t=t_0}(t - t_0)^2 + \frac{1}{6}\left(\frac{\partial^3\varphi(t)}{\partial t^3}\right)_{t=t_0}(t - t_0)^3 + \cdots$$

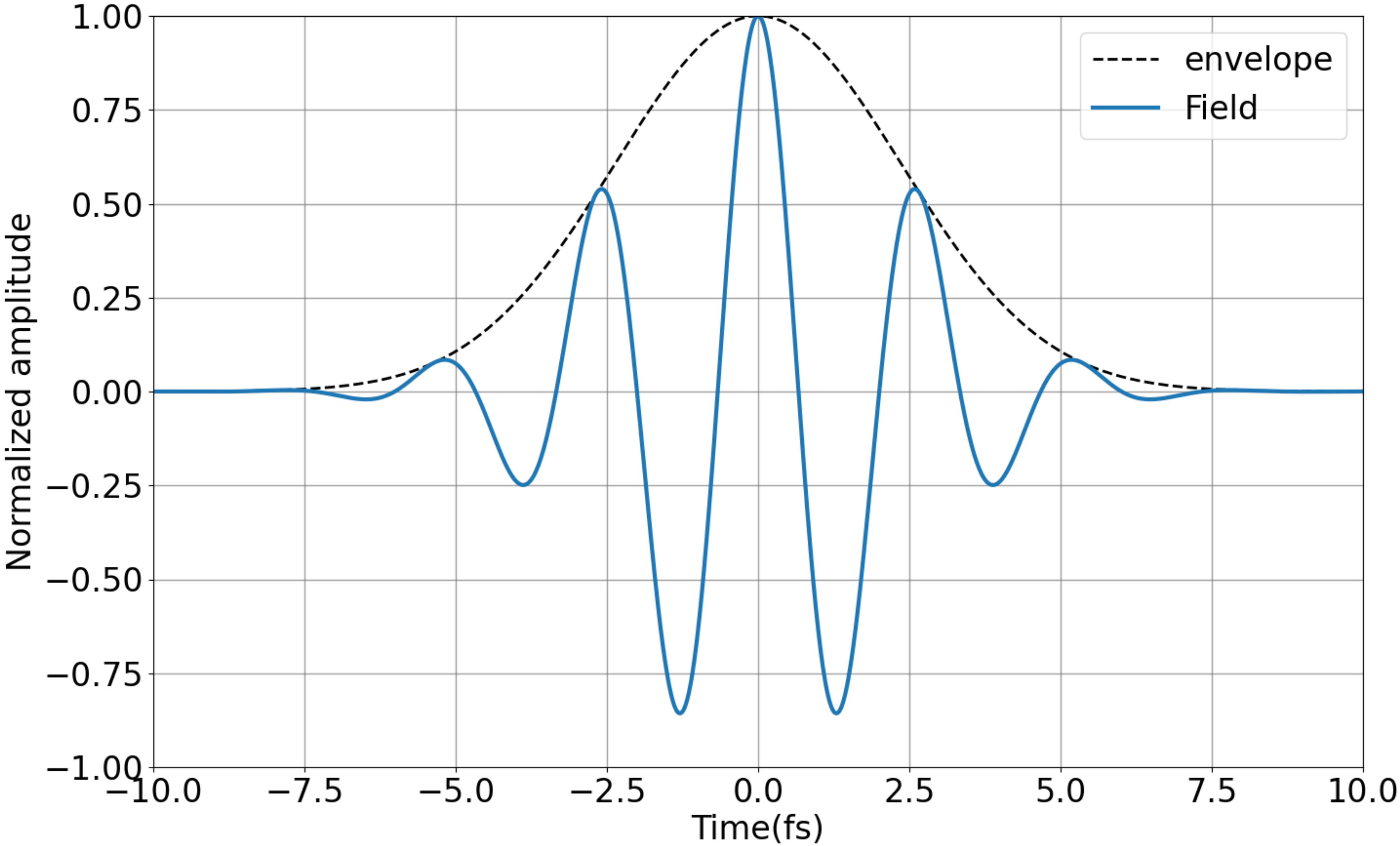
carrier envelope phase — group delay — group delay dispersion (GDD)

$$\varphi(\omega) = \varphi_0 + \left(\frac{\partial\varphi(\omega)}{\partial \omega}\right)_{\omega=\omega_0}(\omega - \omega_0) + \frac{1}{2}\left(\frac{\partial^2\varphi(\omega)}{\partial \omega^2}\right)_{\omega=\omega_0}(\omega - \omega_0)^2 + \frac{1}{6}\left(\frac{\partial^3\varphi(\omega)}{\partial \omega^3}\right)_{\omega=\omega_0}(\omega - \omega_0)^3 + \cdots$$

N.B. positive GDD; red before blue

The Carrier-Envelope Phase (CEP)

$$\varphi_0 = \varphi_{CEP}$$



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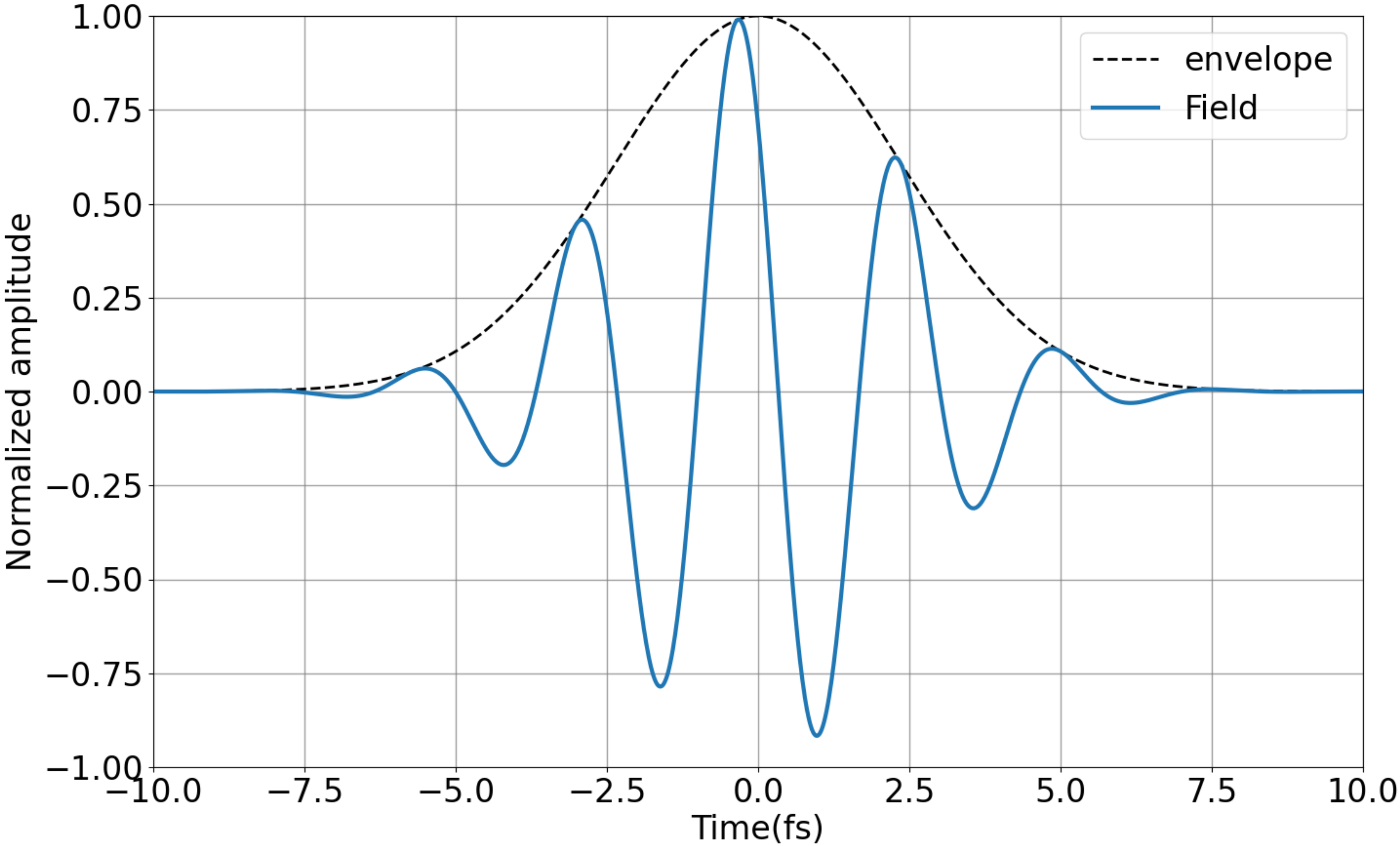

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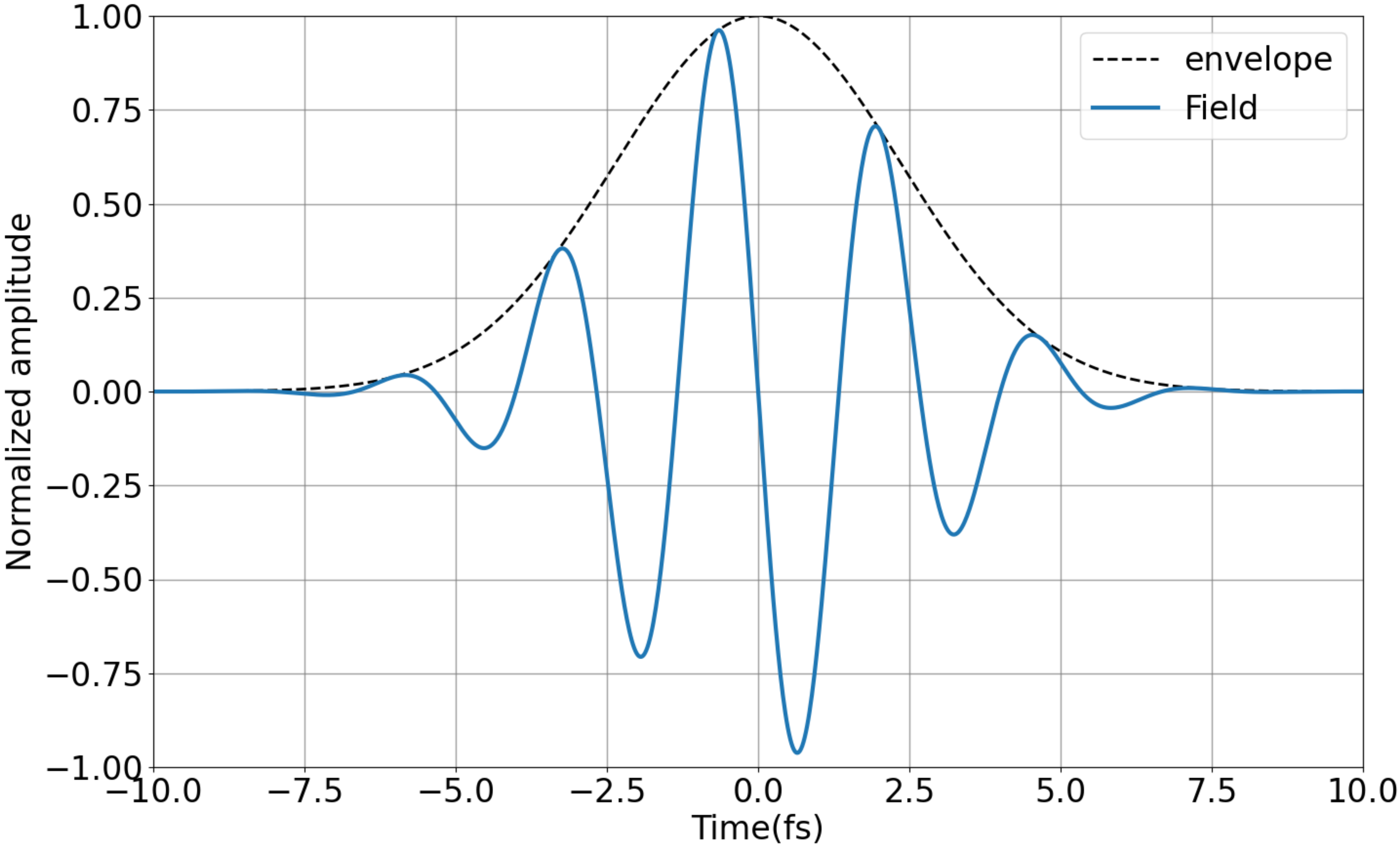
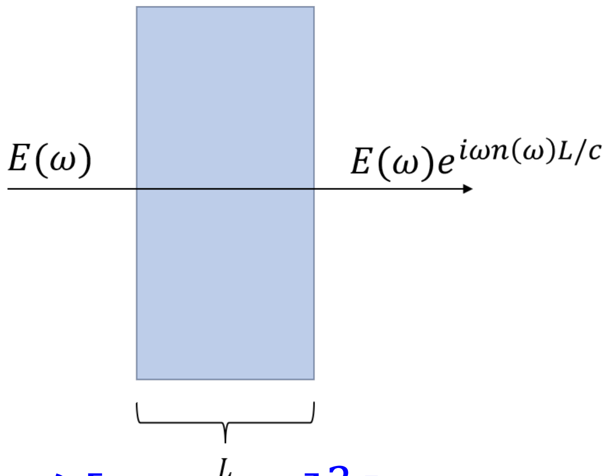


CEP in the oscillator

What happens during propagation of the pulse in the cavity of the oscillator?
Example: passing through laser crystal



$$\varphi(\omega) = \frac{\omega n(\omega)}{c} L = k(\omega) L$$

$$= k(\omega_0)L + k'(\omega_0)[\omega - \omega_0]L + \frac{1}{2}k''(\omega_0)[\omega - \omega_0]^2 L + \cdots$$

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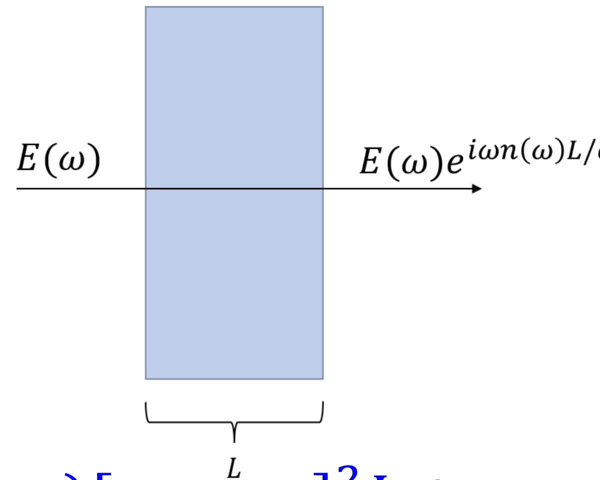

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$$k(\omega_0) = \frac{\omega_0}{v_{phase}(\omega_0)} \qquad k'(\omega_0) = \frac{1}{v_{group}(\omega_0)}$$

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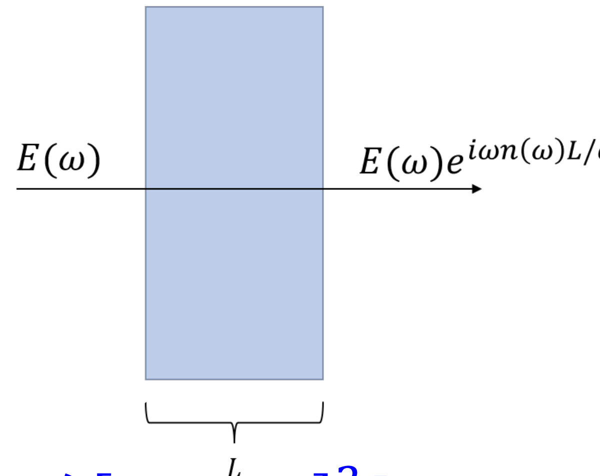

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$$k(\omega_0) = \frac{\omega_0}{v_{phase}(\omega_0)} \qquad k'(\omega_0) = \frac{1}{v_{group}(\omega_0)}$$

$$v_{phase} = \frac{c}{n(\omega_0)}$$

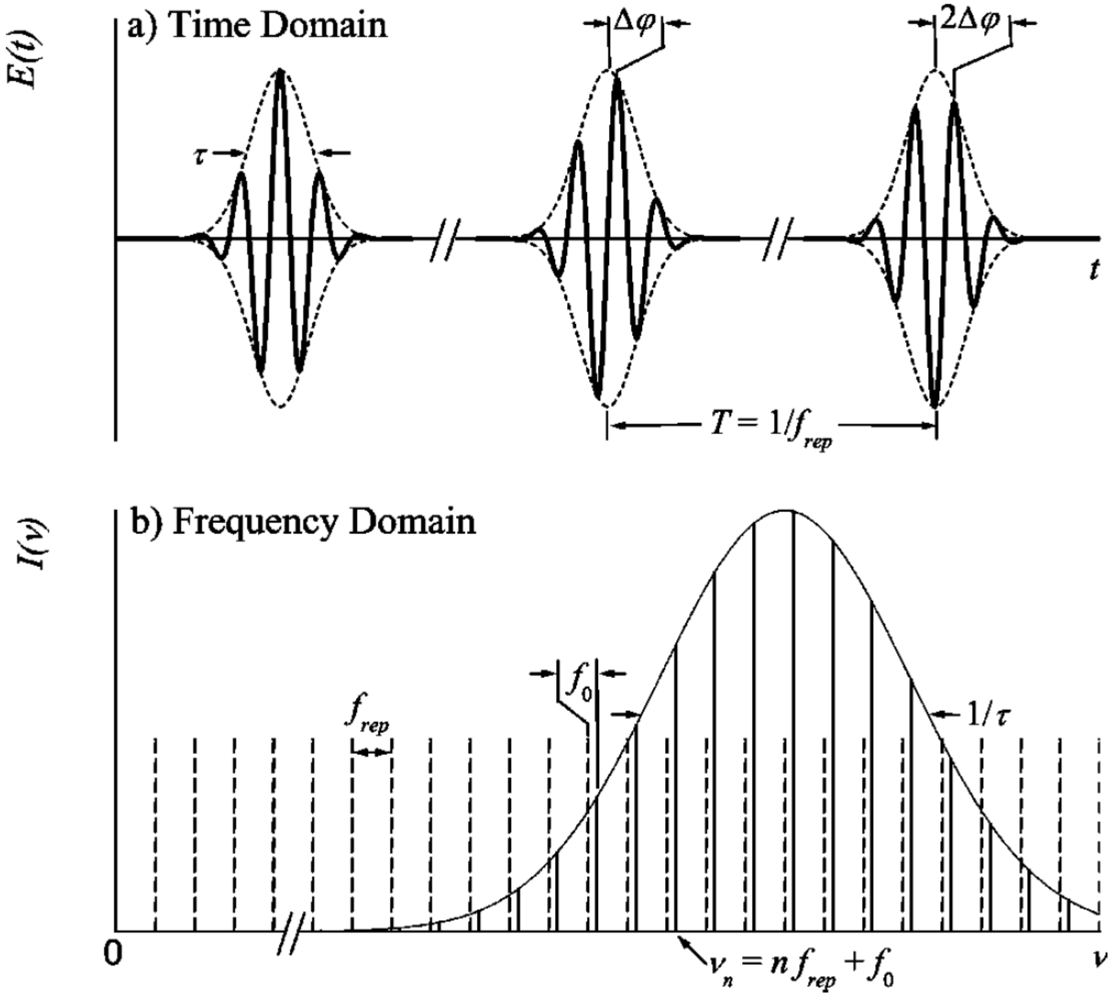
$$v_{group} = \left.\frac{dk}{d\omega}\right|_{\omega_0}$$

In general $v_{phase} \neq v_{phase}$

And that implies that the carrier moves with respect to the pulse envelope…

That means that CEP changes

CEP in the oscillator



Carrier-Envelope Offset Frequency

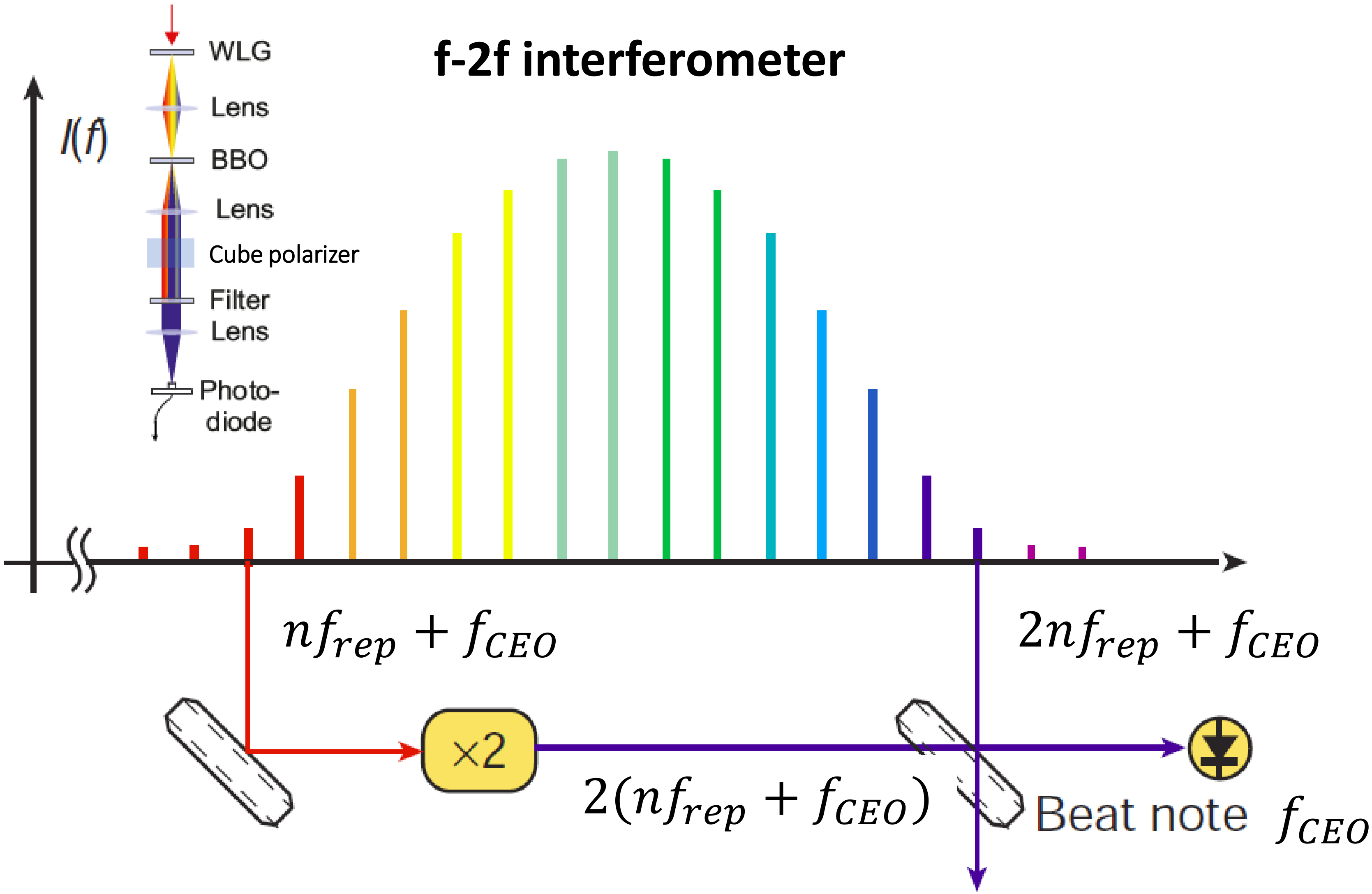
$$f_0 = \frac{1}{2\pi} f_{rep} \Delta\varphi$$

nth line or comb in the frequency comb

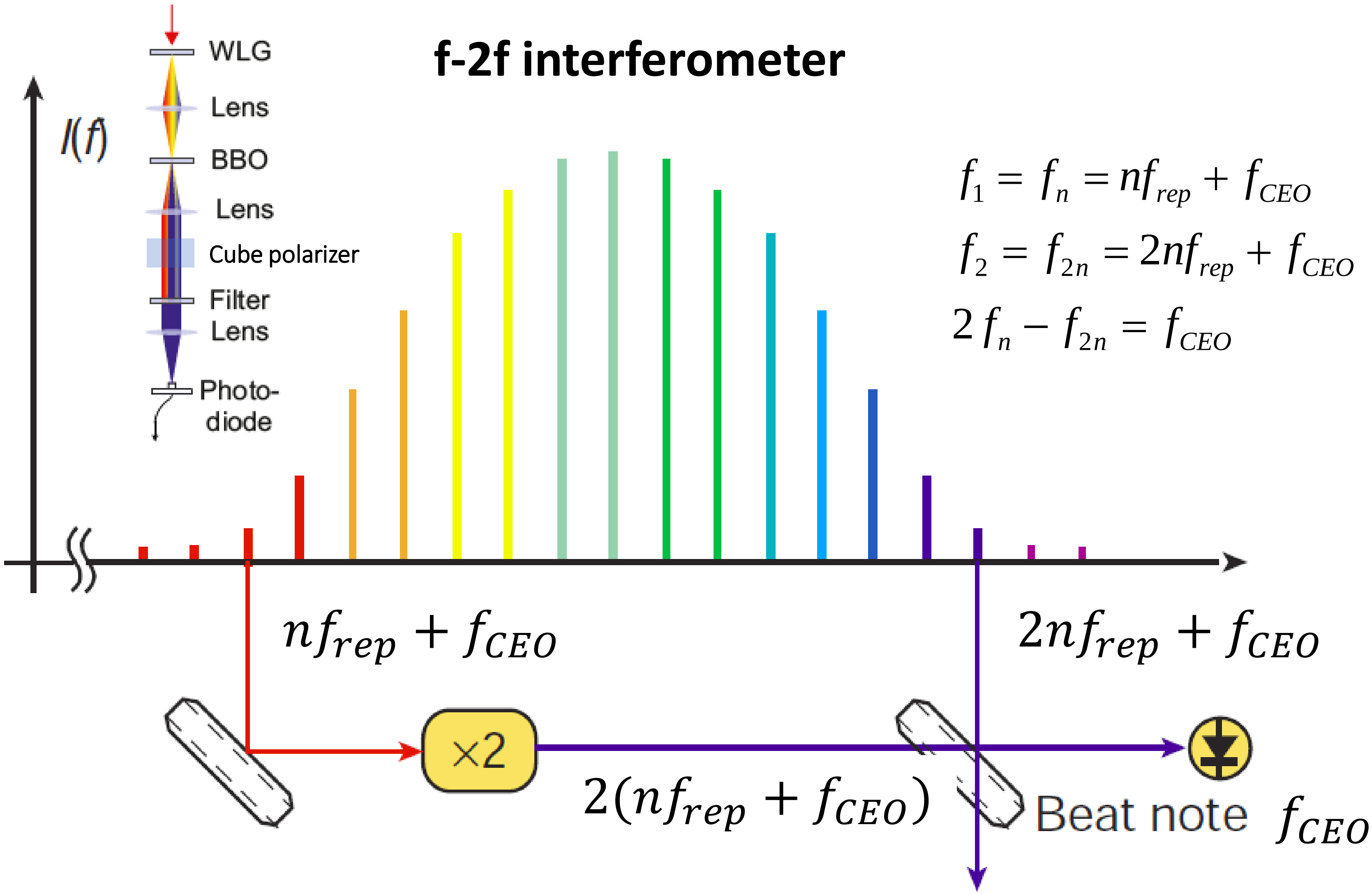
$$\nu_n = n f_{rep} + f_0$$

S. Cundiff and J. Ye, Rev. Mod. Phys. 75, 325 (2003)

How to measure/control the CEP offset



How to measure/control the CEP offset

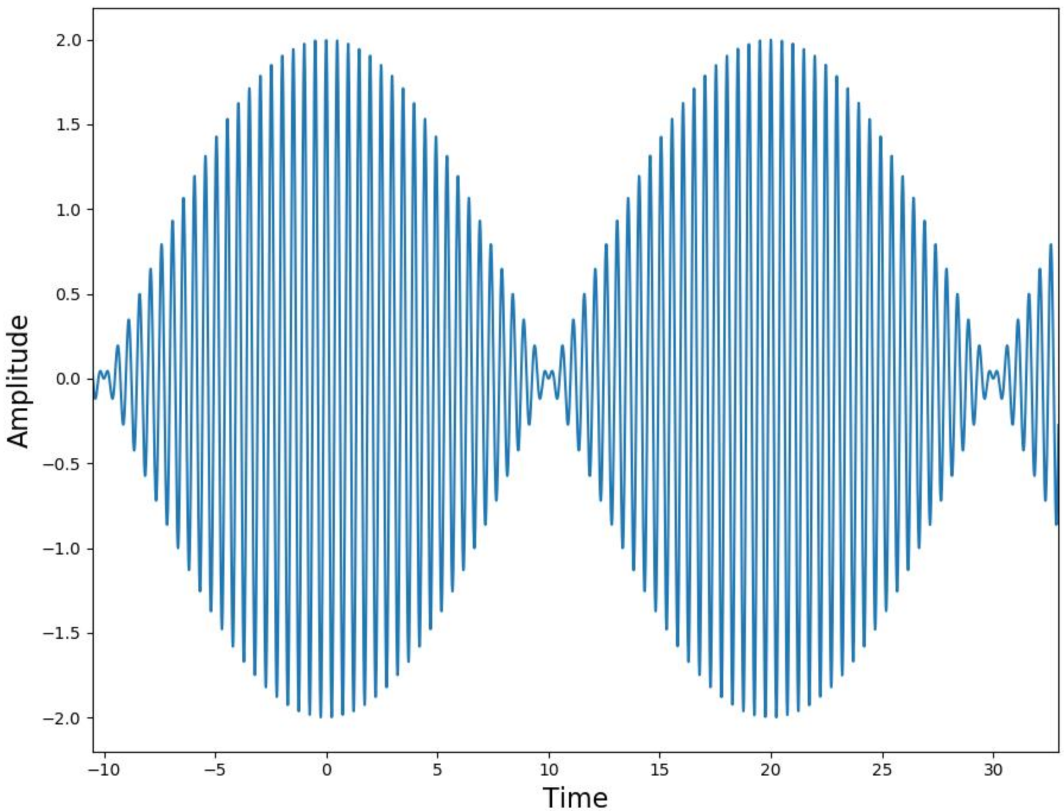


$$f_1 = f_n = nf_{rep} + f_{CEO}$$
$$f_2 = f_{2n} = 2nf_{rep} + f_{CEO}$$
$$2f_n - f_{2n} = f_{CEO}$$

How to measure/control the CEP offset

f-2f interferometer

$$T = 1/(f_2 - f_1)$$



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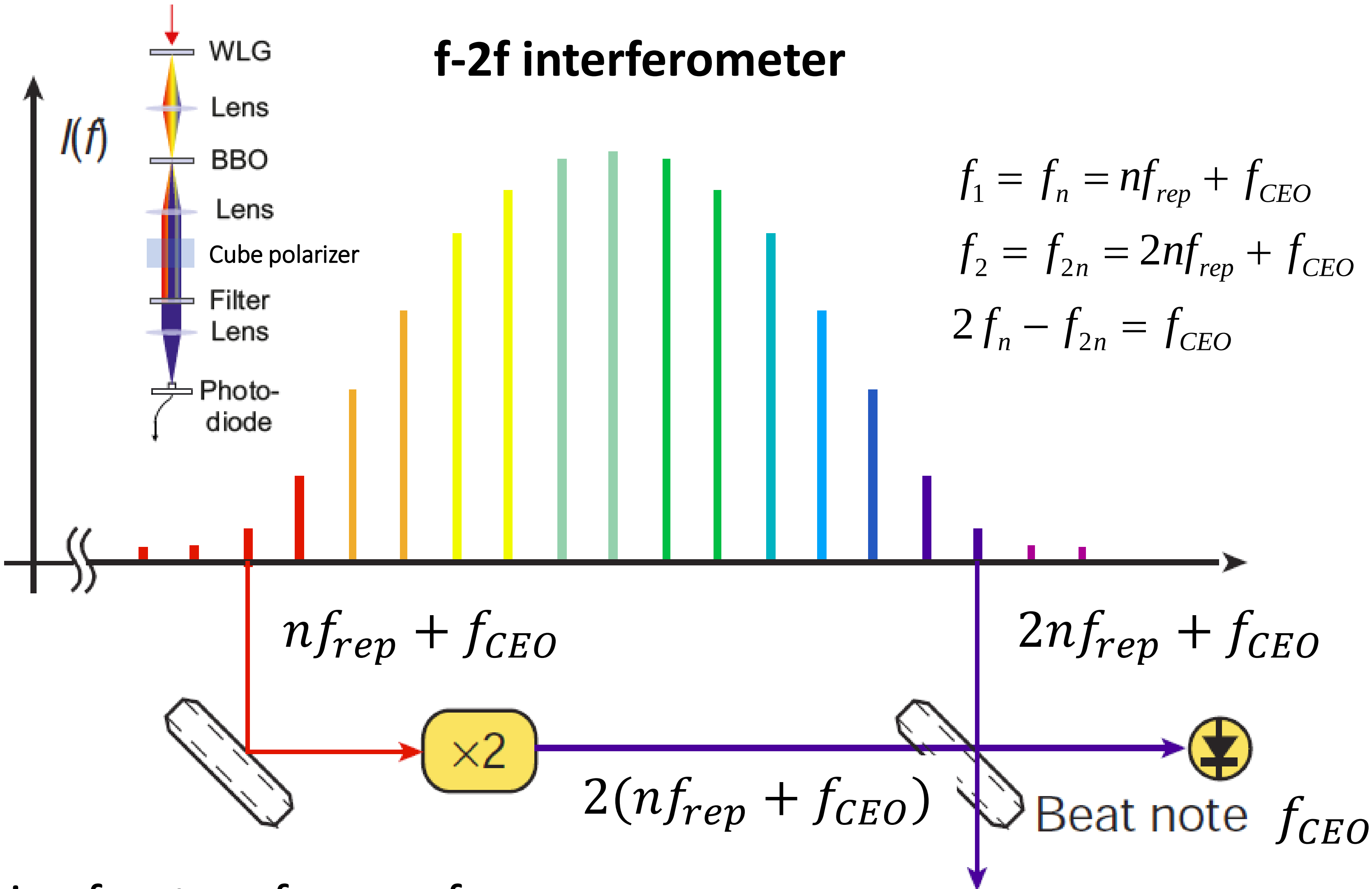

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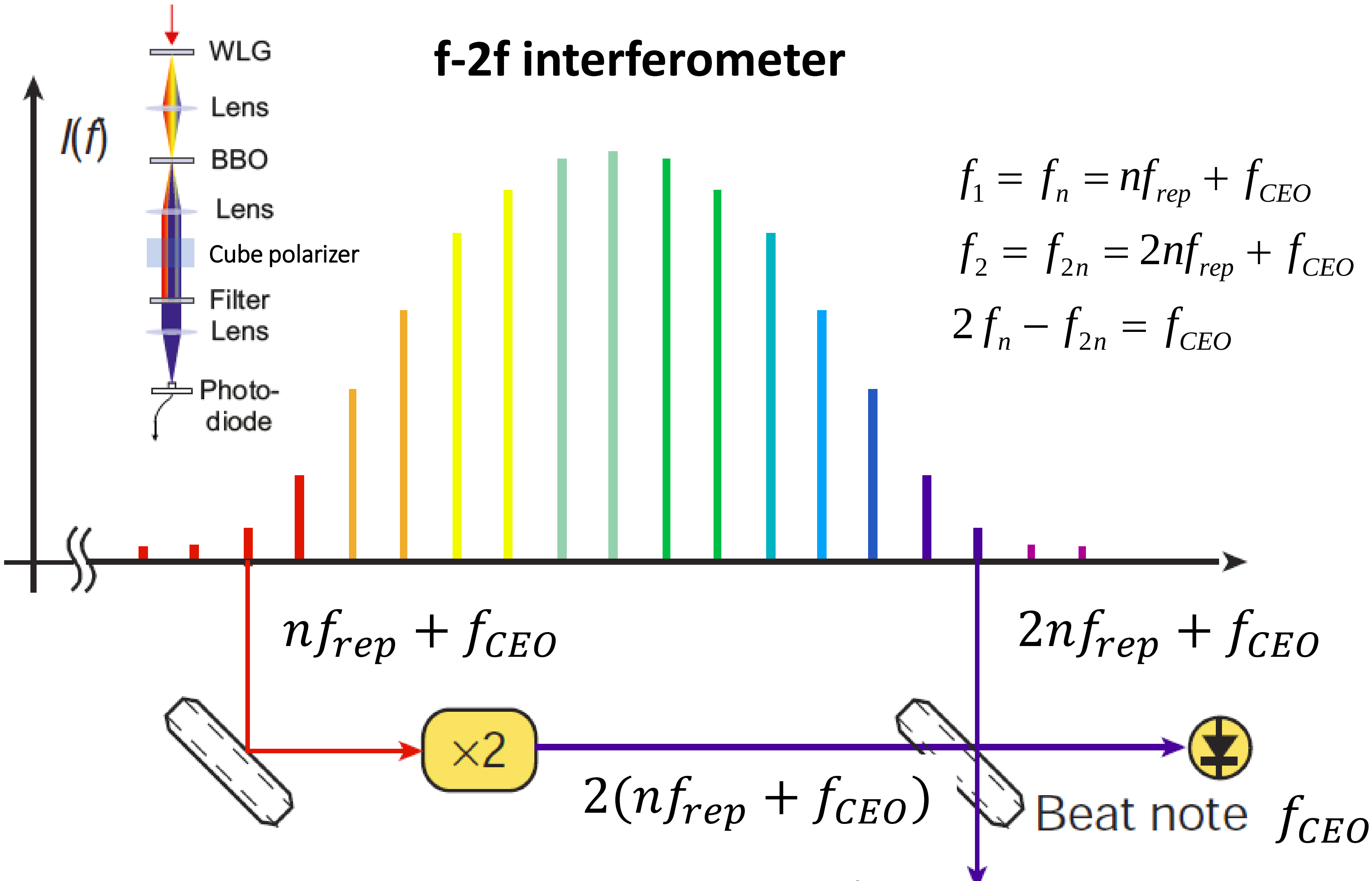
Locking f_{CEO} to reference frequency

H.R. Telle et al., Appl. Phys. B 69, 327 (1999)

How to measure/control the CEP offset



Locking f_{CEO} to reference frequency: e.g. $f_{rep}/4$, by varying the power of the oscillator pump laser, **ensures that every 4-th pulse is identical**

H.R. Telle et al., Appl. Phys. B 69, 327 (1999)

Frequency combs

The Nobel Prize in Physics 2005

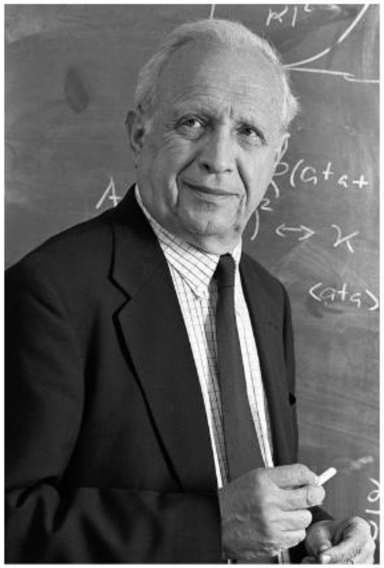


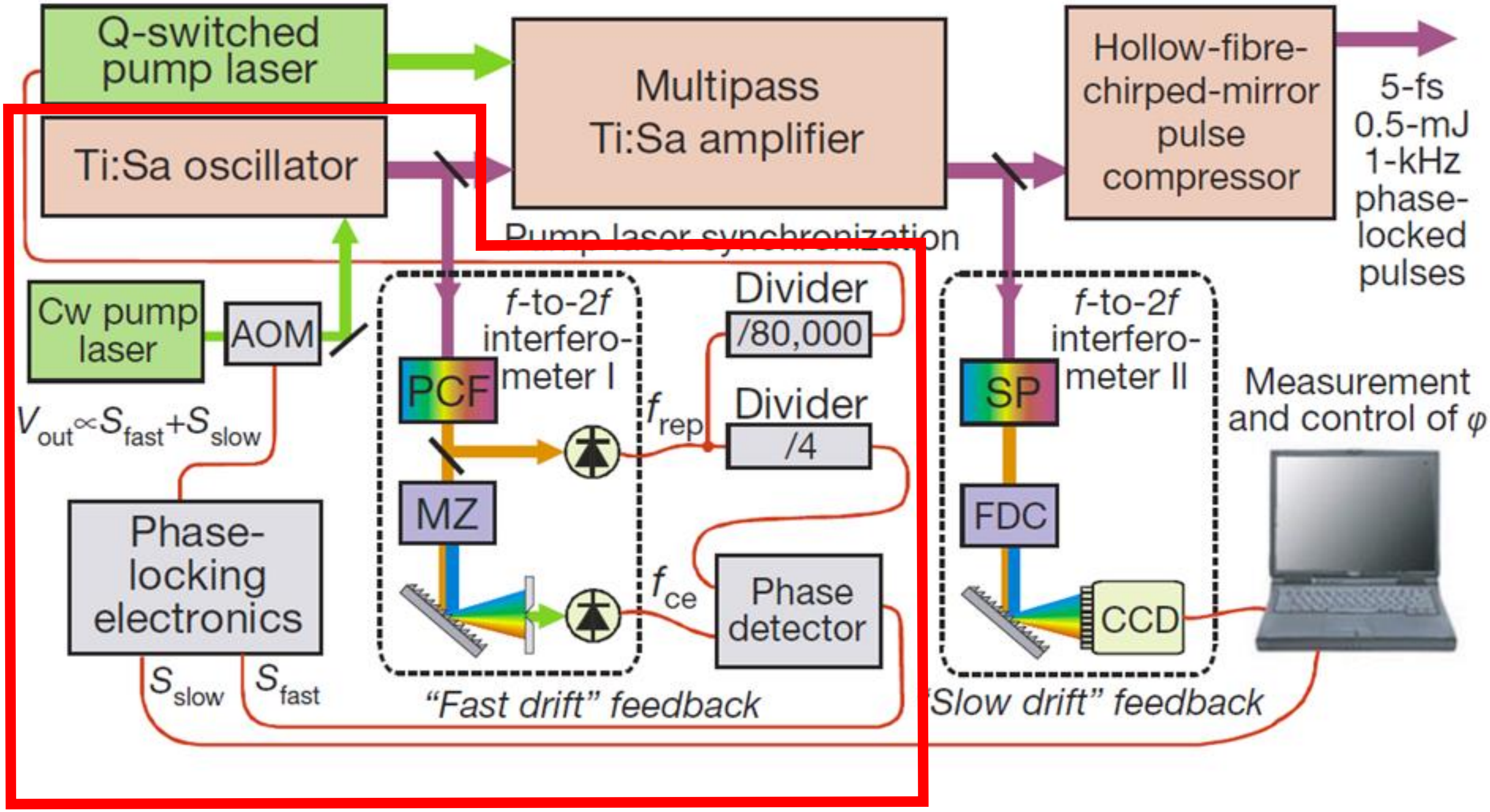
Photo: J.Reed
Roy J. Glauber
Prize share: 1/2

Photo: Sears.P.Studio
John L. Hall
Prize share: 1/4

Photo: F.M. Schmidt
Theodor W. Hänsch
Prize share: 1/4

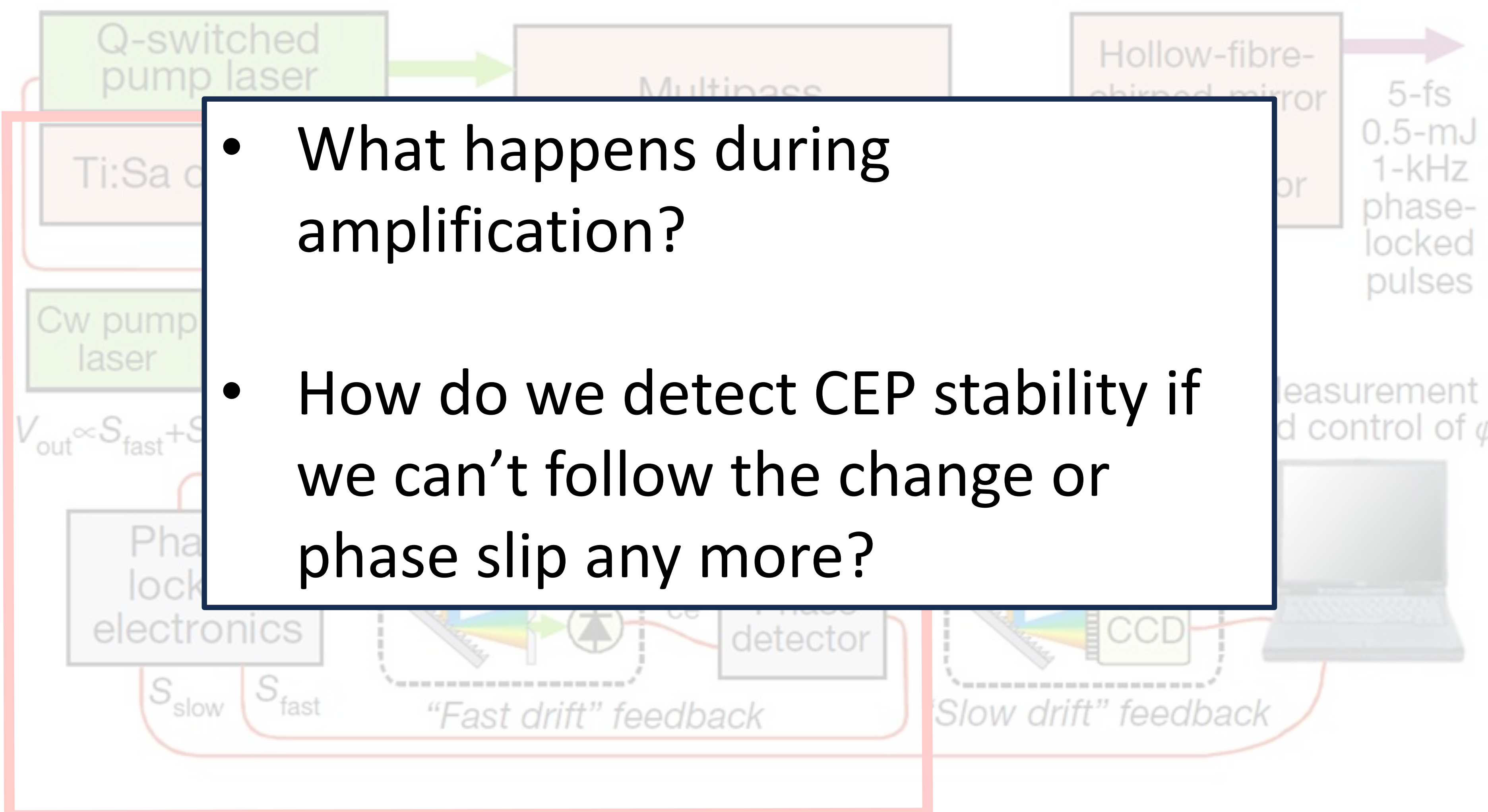
The Nobel Prize in Physics 2005 was divided, one half awarded to Roy J. Glauber "for his contribution to the quantum theory of optical coherence", the other half jointly to **John L. Hall and Theodor W. Hänsch "for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"**

A state-of-the-art laser system for attosecond science



A. Baltuska et al., Nature 421, 611 (2003)

A state-of-the-art laser system for attosecond science



- What happens during amplification?

- How do we detect CEP stability if we can't follow the change or phase slip any more?

A. Baltuska et al., Nature 421, 611 (2003)

Suggested literature

J.-C. Diels and W. Rudolph, *Ultrashort Laser Pulse Phenomena,* (Academic Press, 2006)

Z. Cheng, Fundamental of Attosecond Optics (CRC Press, 2011)

S. Cundiff and J. Ye, Rev. Mod. Phys. 75, 325 (2003)

A. Baltuska et al., Nature 421, 611 (2003)

B. Siegman, Lasers, (University Science Books 1986)

U. Keller, Ultrafast Lasers (Springer 2021)