Lecture Series Buenos Aires

18-3-2024 until 22-3-2024

Lecture F1 – Mathematical description of laser pulses

Max-Born-Institut

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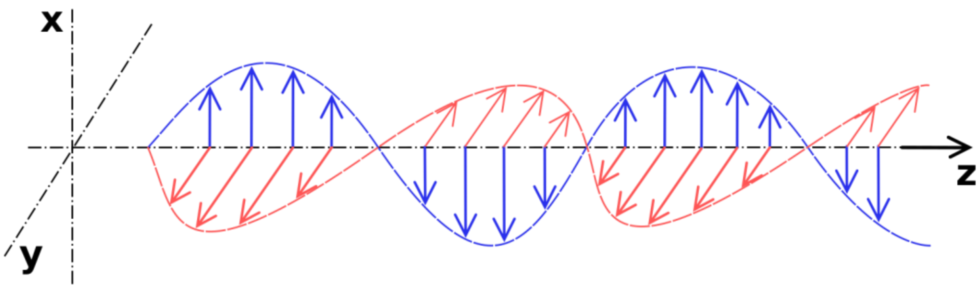
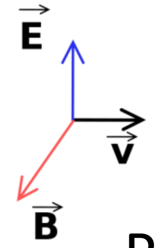
Mathematical description of Laser Pulses and some basic concepts in Ultrafast Optics

Description of EM waves and Laser fields





$$\vec{E} = E(x, y, z, t)\hat{x}$$
$$\vec{B} = B(x, y, z, t)\hat{y} \approx 0$$

Propagation direction in $\hat{z}$

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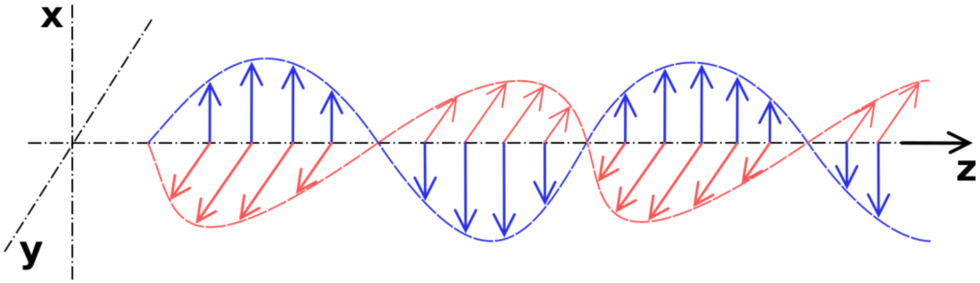

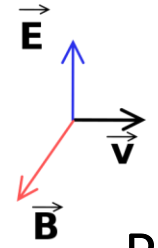

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$$E(x, y, z, t) = E_{spatial}(x, y, z)E_{temporal}(t)$$

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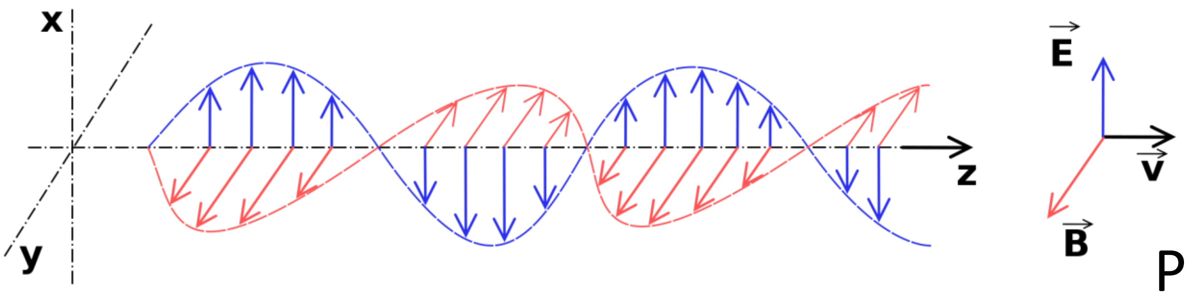

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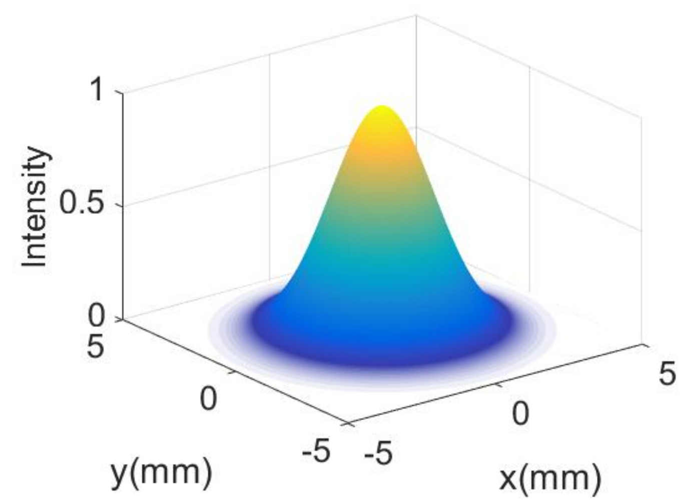
Spatial part: Gaussian beam

→ See **A. Siegman, Lasers, (University Science Books, 1986)**



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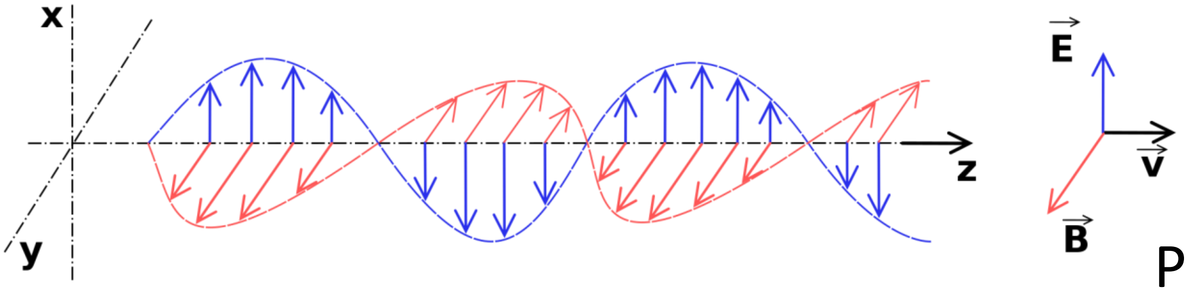

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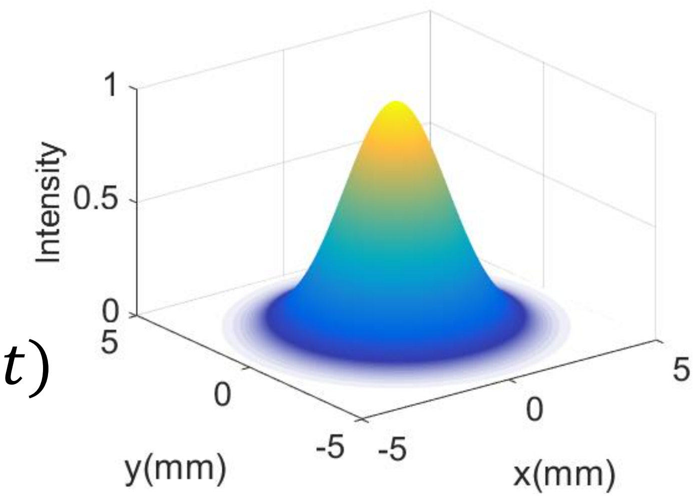
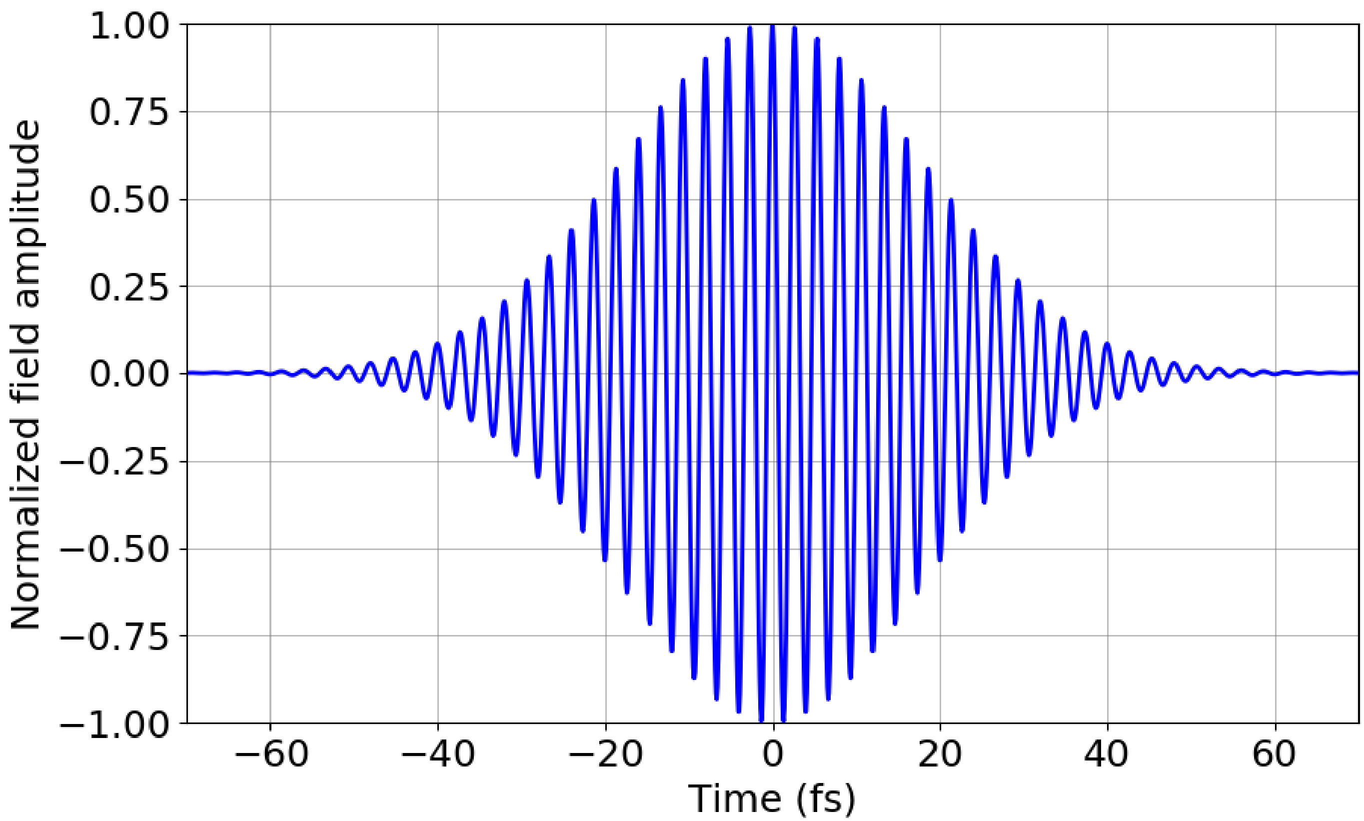


From now on we use $E_{temporal}(t) = E(t)$

Typical ultrashort laser pulse

$$E_{temporal}(t) = E(t)$$

Example: 30 fs pulse, at λ = 800 nm



Time and frequency representations

Frequency domain representation of laser field $E(t)$:

$$\tilde{E}(\omega) = \int_{-\infty}^{\infty} E(t)e^{-i\omega t}d\omega = \left|\tilde{E}(\omega)\right|e^{i\varphi(\omega)}$$

J.-C. Diels and W. Rudolph, *Ultrashort Laser Pulse Phenomena,* (Academic Press, 2006)

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$$E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}(\omega) e^{i\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\tilde{E}(\omega)| e^{i\omega t + i\varphi(\omega)} d\omega$$

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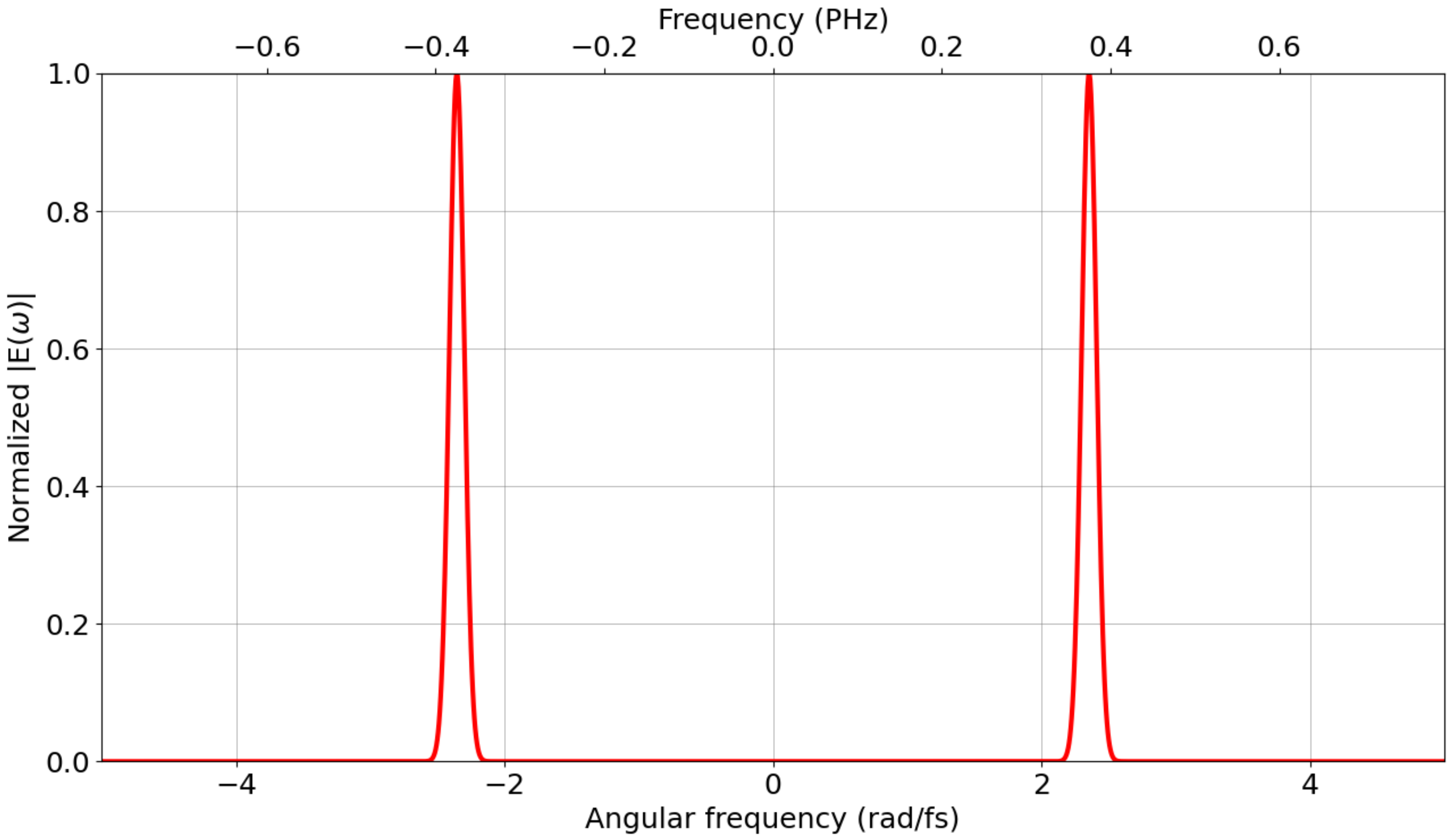

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$$\tilde{E}(\omega) = \tilde{E}^{+}(\omega) + \tilde{E}^{-}(\omega)$$

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Time and frequency representations

$$\tilde{E}(\omega) = \tilde{E}^{+}(\omega) + \tilde{E}^{-}(\omega)$$

Alternative description using only positive frequencies

$$\tilde{E}^{+}(\omega) = \begin{cases} \tilde{E}(\omega), & for\ \omega \geq 0 \\ 0, & for\ \omega < 0 \end{cases}$$

$|\tilde{E}^{+}(\omega)|^2 = S(\omega)$ pulse spectrum
(what we measure with spectrometer)

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$$\tilde{E}^{+}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}^{+}(\omega) e^{i\omega t} d\omega$$

$$\tilde{E}^{+}(\omega) = \int_{-\infty}^{\infty} \tilde{E}^{+}(t) e^{-i\omega t} d\omega$$

$\tilde{E}^{+}(t)$ → complex

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$$E(t) = \tilde{E}^{+}(t) + \tilde{E}^{-}(t) = \tilde{E}^{+}(t) + \mathrm{cc}$$

→ real

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Time and frequency representations

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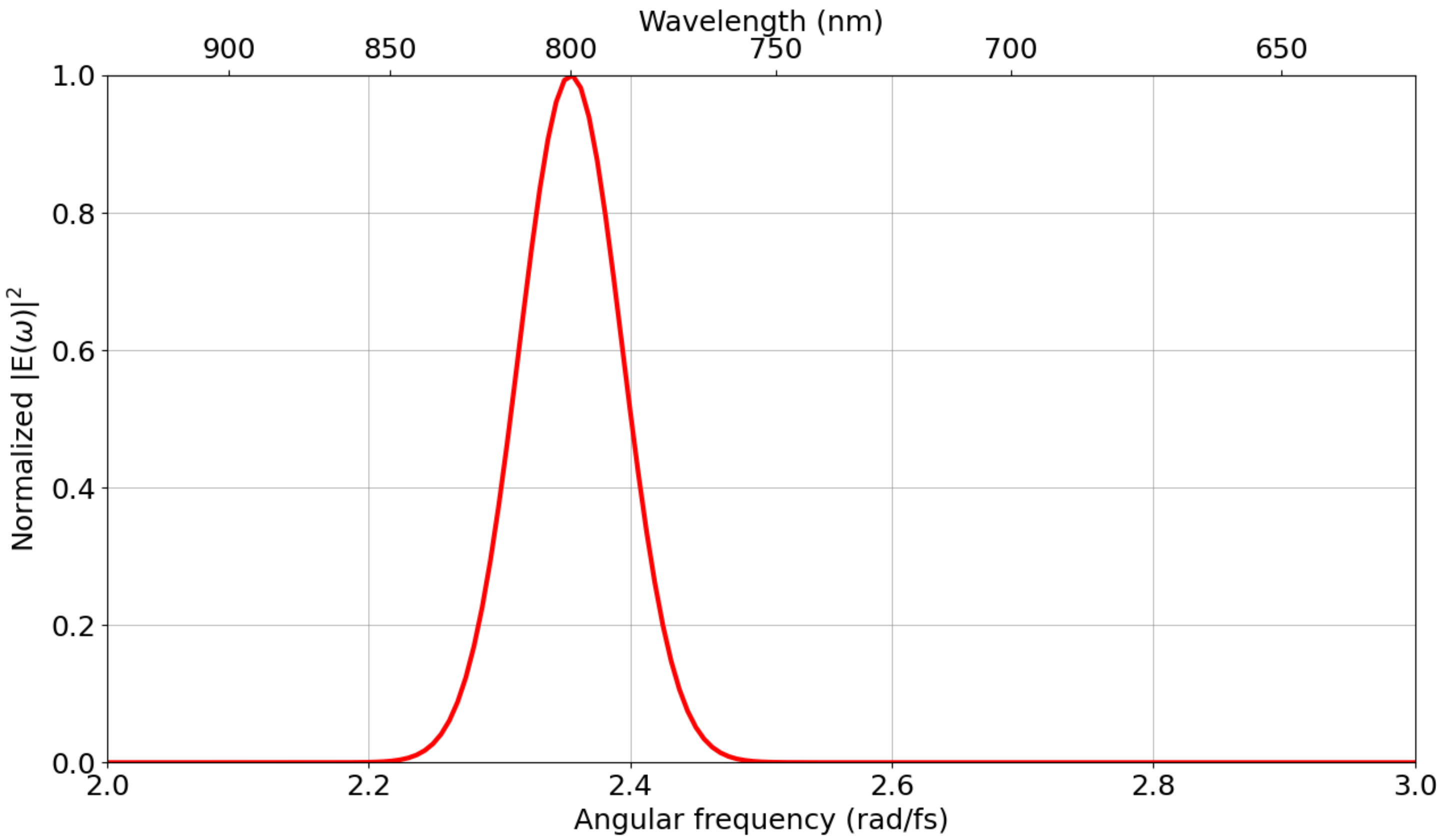

Time and frequency representations

Introduce ω_l as the carrier frequency of the laser pulse

$$E^+(t) = \frac{1}{2}\varepsilon(t)e^{i\varphi(t)}e^{i\omega_l t} = \frac{1}{2}\tilde{\varepsilon}(t)e^{i\omega_l t}$$

$\varphi(t)$ time-dependent phase

$\varepsilon(t), \tilde{\varepsilon}(t)$ real and complex field envelope

Carrier and envelope

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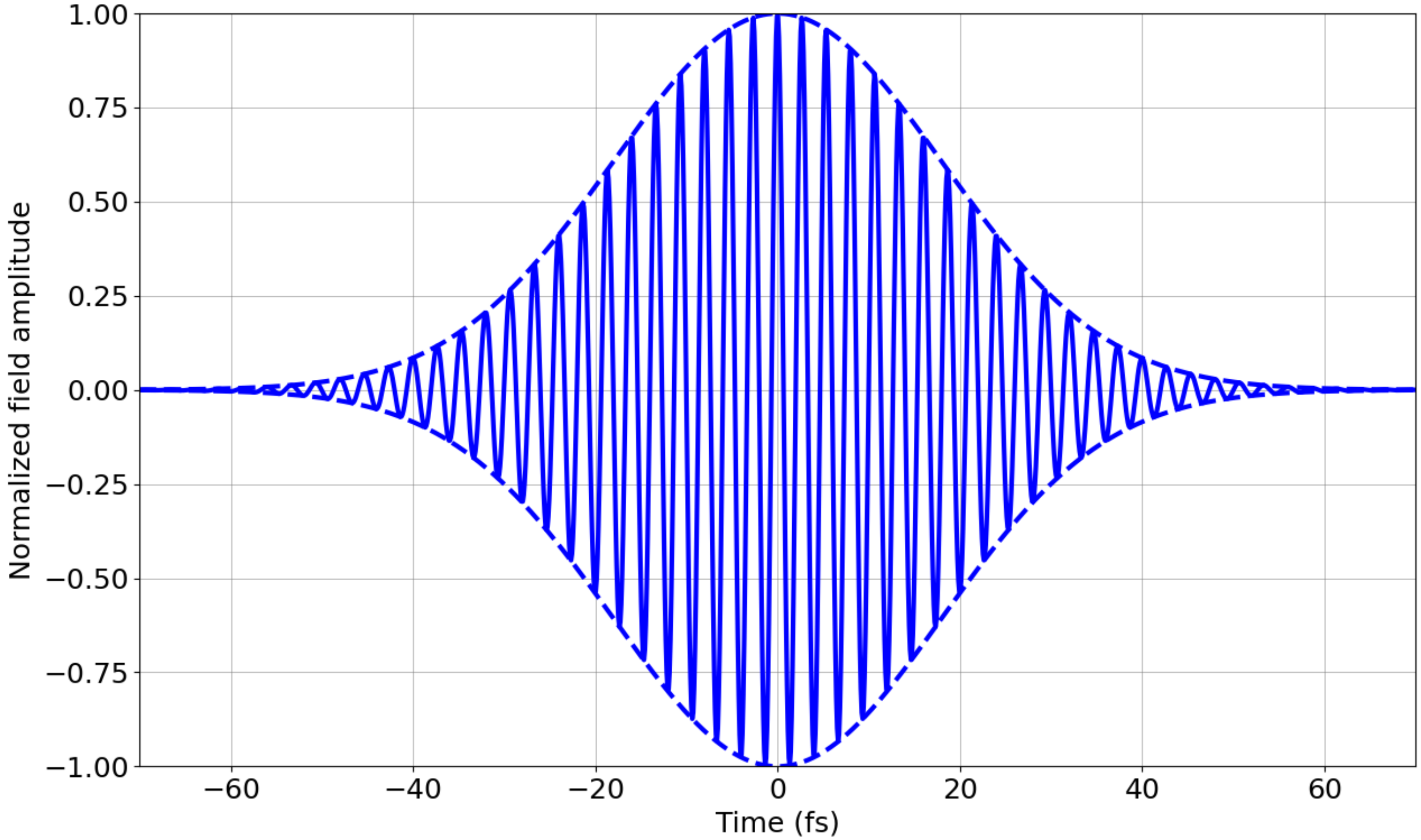

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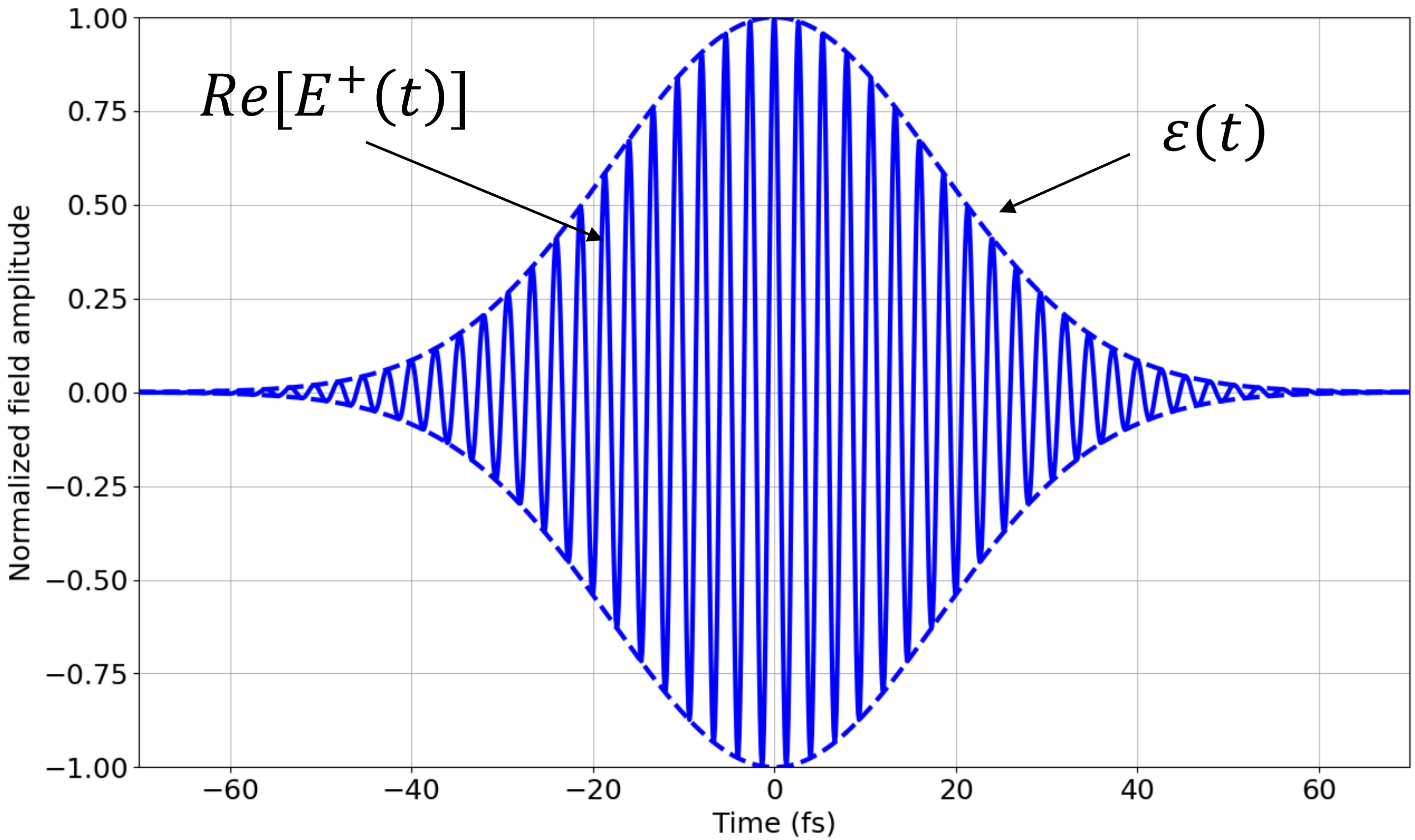

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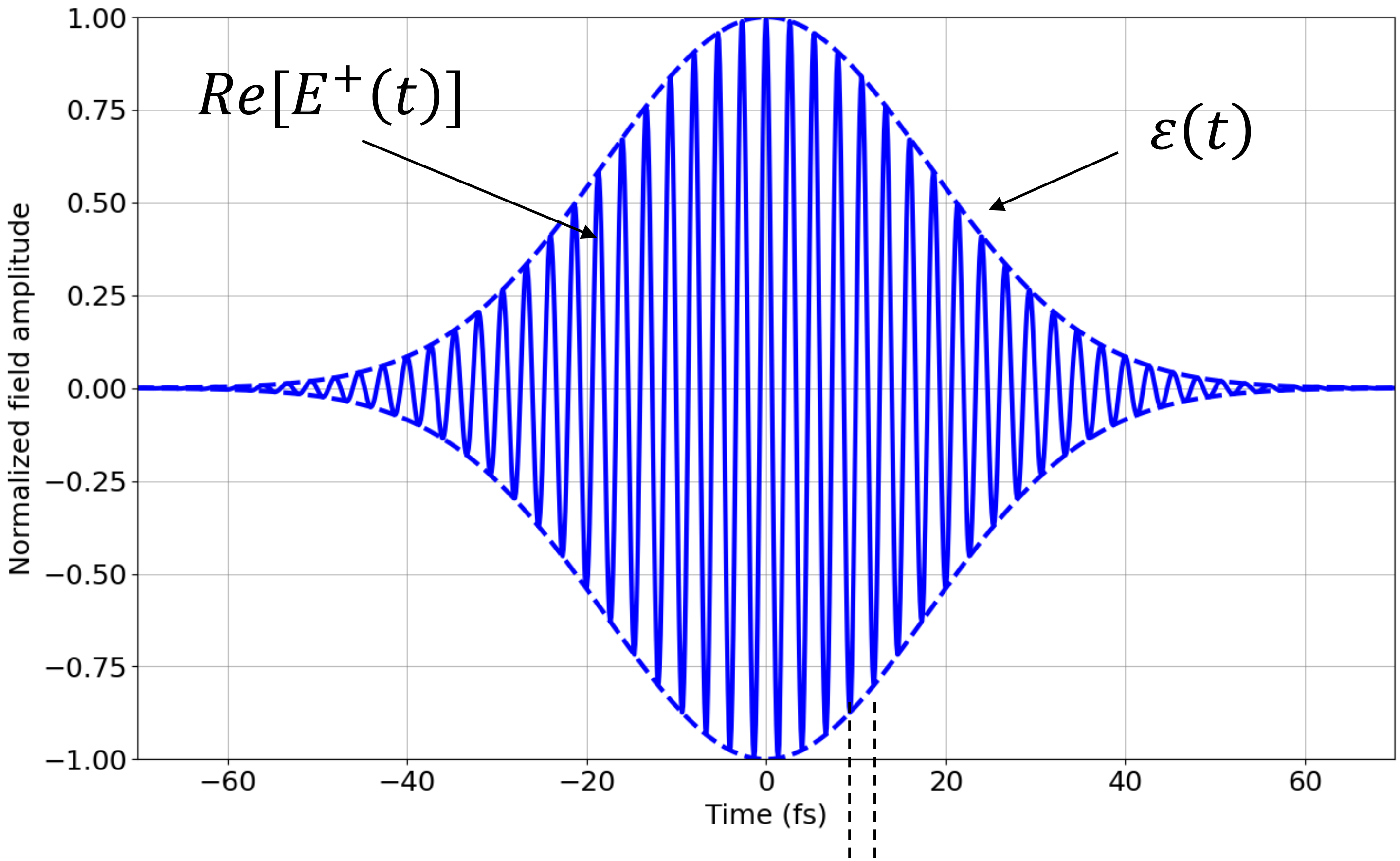


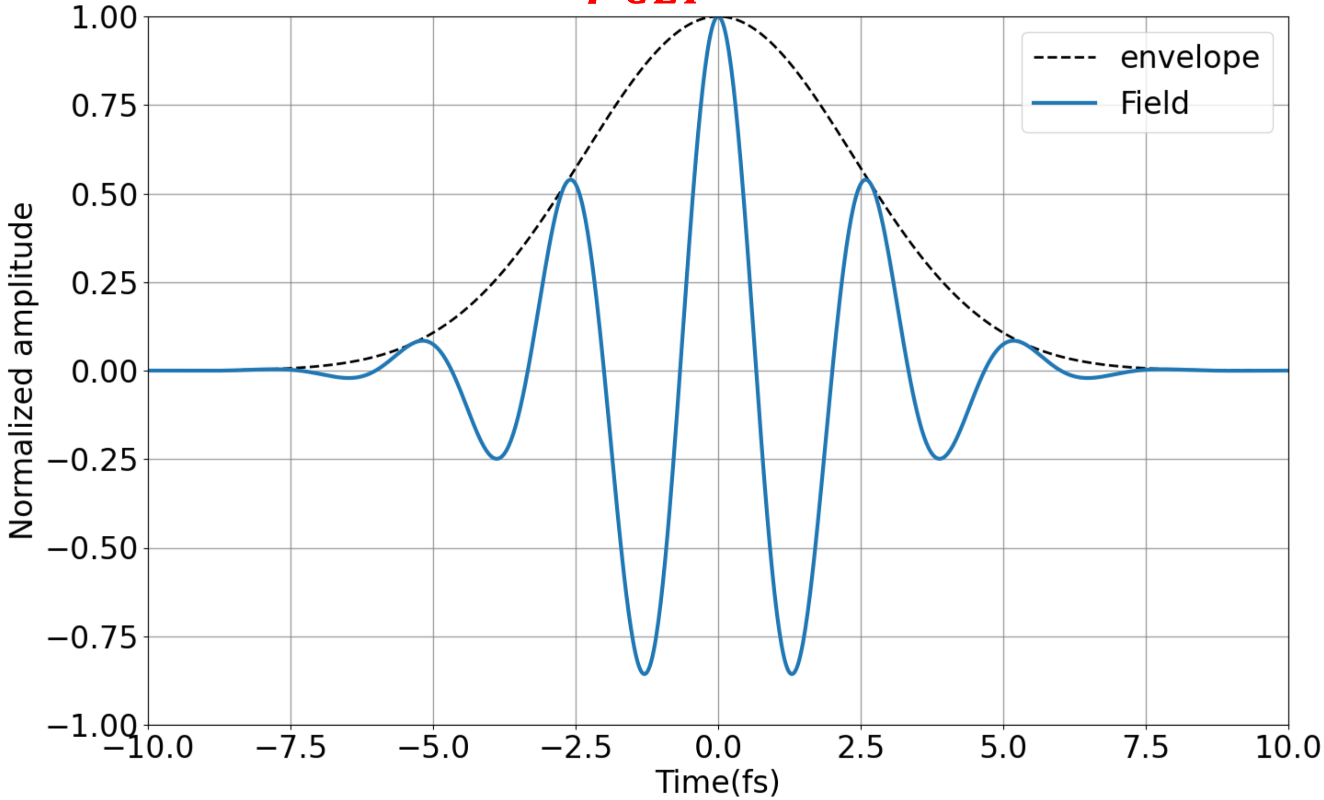
Optical cycle: $T = \frac{2\pi}{\omega_l} = \frac{2\pi\lambda}{c} \cong 2.67\ fs$

Carrier-envelope phase

$$E^{+}(t) = \frac{1}{2}\varepsilon(t)e^{i\varphi(t)}e^{i\varphi_{CEP}}e^{i\omega_l t}$$

Offset between max. of envelope and closest peak of carrier

$$\varphi_{CEP} = 0$$



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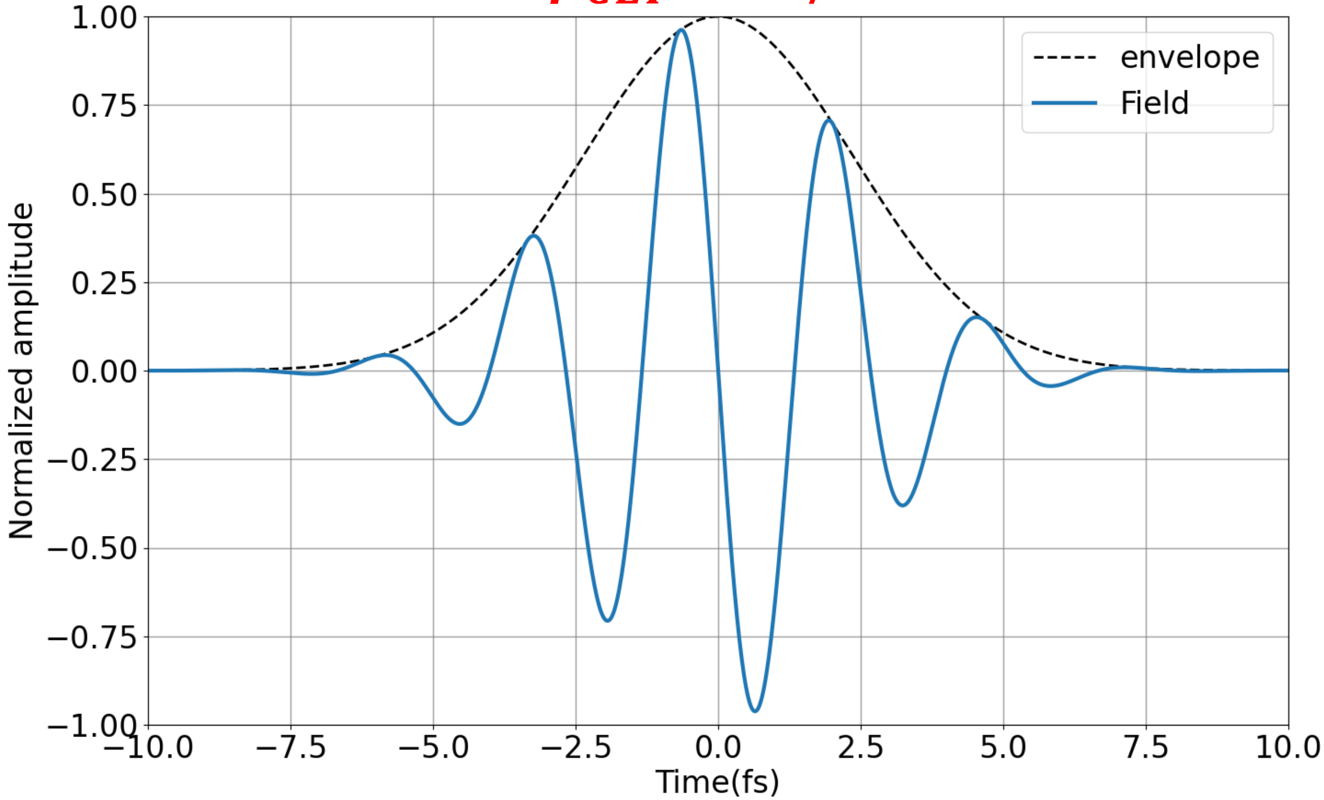
$$\boldsymbol{\varphi_{CEP} = \pi/2}$$



Pulse shape and duration

Pulse duration?

Laser Pulse Shape →$I(t) \propto |\varepsilon(t)|^2$ (intensity profile)

Pulse duration(τ_{FWHM}): Full Width at Half Maximum of $I(t)$

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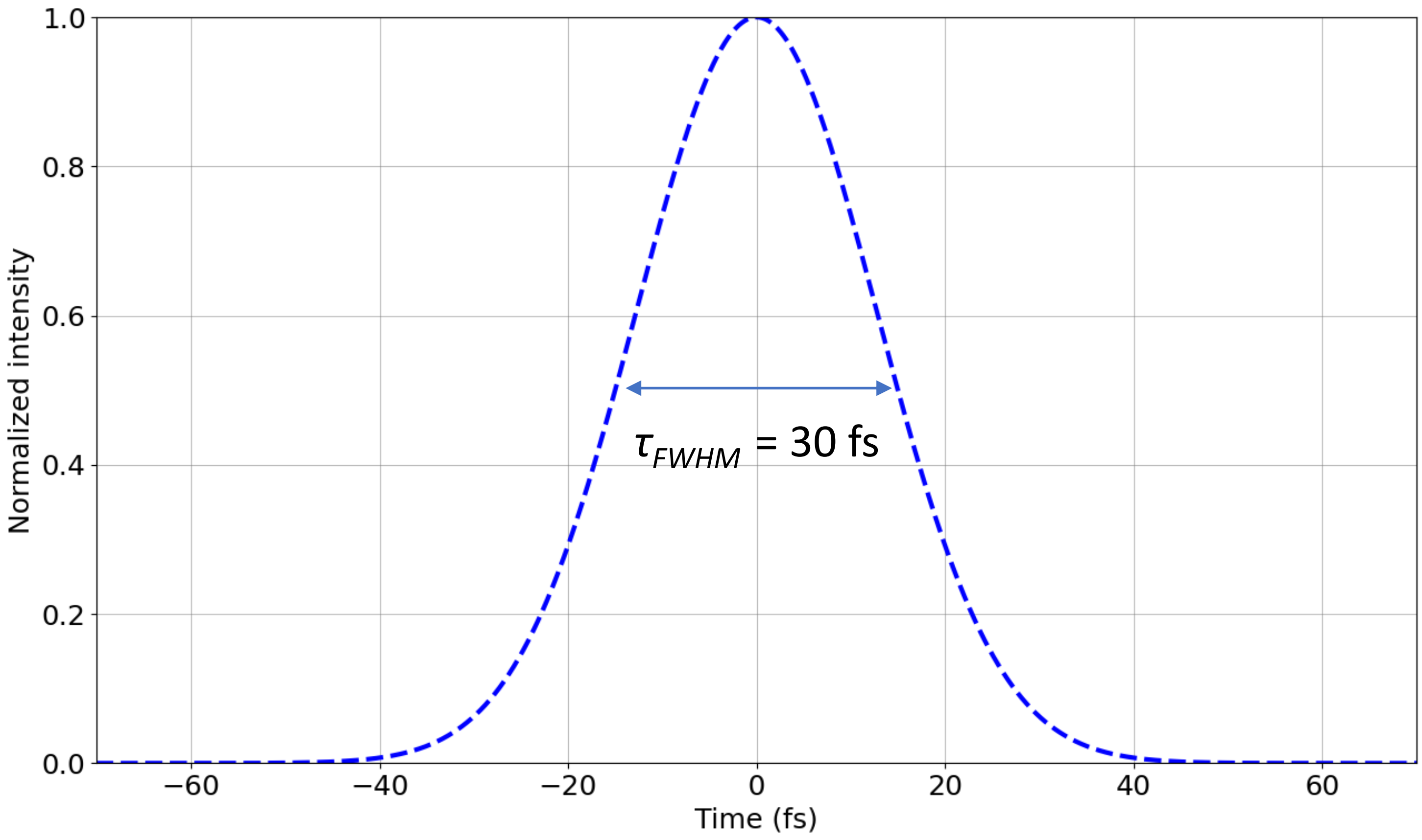
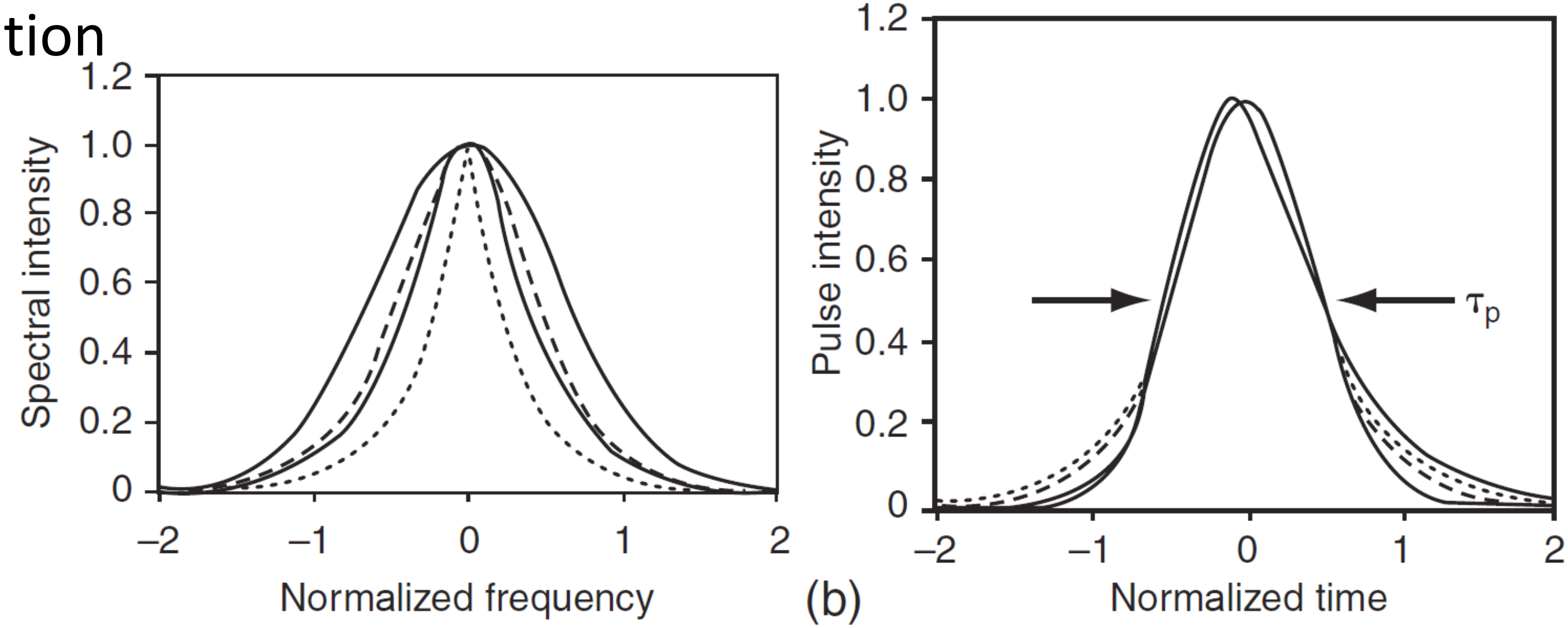


Bandwidth and pulse duration

The spectral width and shape determine the shortest laser pulse duration



————	Gaussian pulse	$\mathcal{E}(t) \propto \exp[-1.385(t/\tau_p)^2]$
– – – – – – – –	Sech pulse	$\mathcal{E}(t) \propto \text{sech}[1.763(t/\tau_p)]$
· · · · · · · · ·	Lorentzian pulse	$\mathcal{E}(t) \propto [1 + 1.656(t/\tau_p)^2]^{-1}$
———	Asymm. sech pulse	$\mathcal{E}(t) \propto [\exp(t/\tau_p) + \exp(-3t/\tau_p)]^{-1}$

$\tau_p = \tau_{FWHM}$

J.-C. Diels and W. Rudolph, *Ultrashort Laser Pulse Phenomena,* (Academic Press, 2006)

Bandwidth and pulse duration

Fourier-limited pulse durations

$$\Delta\omega_p \tau_p = 2\pi\Delta\nu_p \tau_p \geq 2\pi c_B$$

Examples of standard pulse profiles. The spectral values given are for unmodulated pulses. Note that the Gaussian is the shape with the minimum product of mean square deviation of the intensity and spectral intensity.

Shape	Intensity profile $I(t)$	τ_p FWHM	Spectral profile $S(\Omega)$	$\Delta\omega_p$ FWHM	c_B	$\langle\tau_p\rangle\langle\Delta\Omega_p\rangle$ MSQ
Gauss	$e^{-2(t/\tau_G)^2}$	$1.177\tau_G$	$e^{-\left(\frac{\Omega\tau_G}{2}\right)^2}$	$2.355/\tau_G$	0.441	0.5
Sech	$\text{sech}^2(t/\tau_s)$	$1.763\tau_s$	$\text{sech}^2 \frac{\pi\Omega\tau_s}{2}$	$1.122/\tau_s$	0.315	0.525
Lorentz	$[1+(t/\tau_L)^2]^{-2}$	$1.287\tau_L$	$e^{-2\|\Omega\|\tau_L}$	$0.693/\tau_L$	0.142	0.7
Asym. sech	$\left[e^{t/\tau_a}+e^{-3t/\tau_a}\right]^{-2}$	$1.043\tau_a$	$\text{sech} \frac{\pi\Omega\tau_a}{2}$	$1.677/\tau_a$	0.278	
Square	1 for $\|t/\tau_r\| \leq 1$, 0 elsewhere	τ_r	$\text{sinc}^2(\Omega\tau_r)$	$2.78/\tau_r$	0.443	3.27

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Some representative numbers

What is the minimum pulse duration that a 40 nm FWHM bandwidth Gaussian spectrum centered around 800 nm can support?

Step 1: calculate $\Delta\omega_p$

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$= 2.417 \text{ fs}^{-1}$

$\omega_{820} =$ $= 2.299 \text{ fs}^{-1}$

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$\tau_p = 0.2$ fs; $\Delta\omega_p \geq 13.85 \text{ fs}^{-1}$

i.e. $\omega_p \geq 13.85 \text{ fs}^{-1}$ ~ 136 nm, i.e. in the vacuum-ultraviolet

So attosecond science is VUV/XUV/X-ray science!

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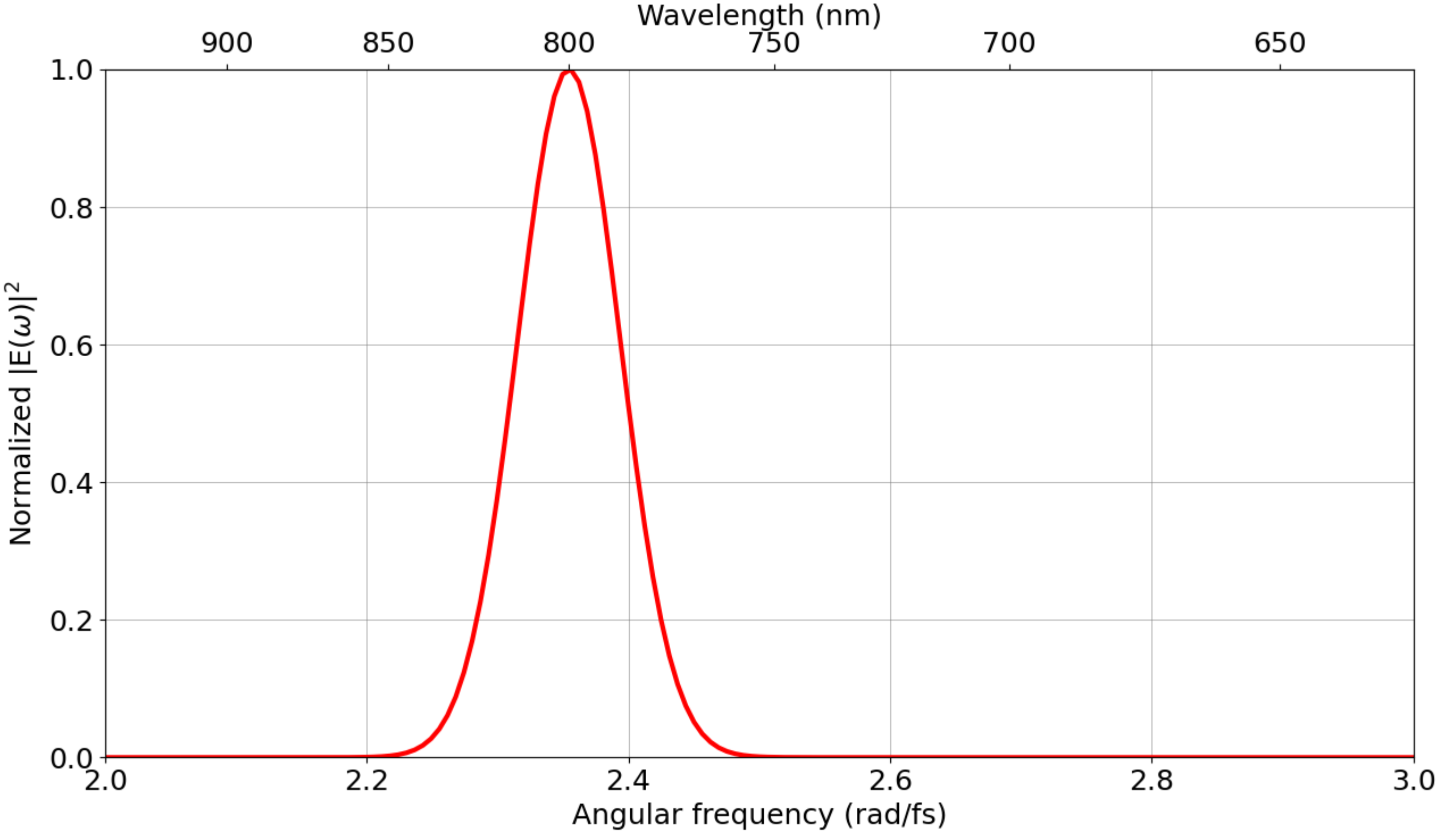
An alternative way to make an estimation of ω_p???

Bandwidth and pulse duration

From the following discussion… Does the spectrum determines the pulse shape and duration???

$$|\tilde{E}^{+}(\omega)|^2 = S(\omega) \quad \text{pulse spectrum}$$



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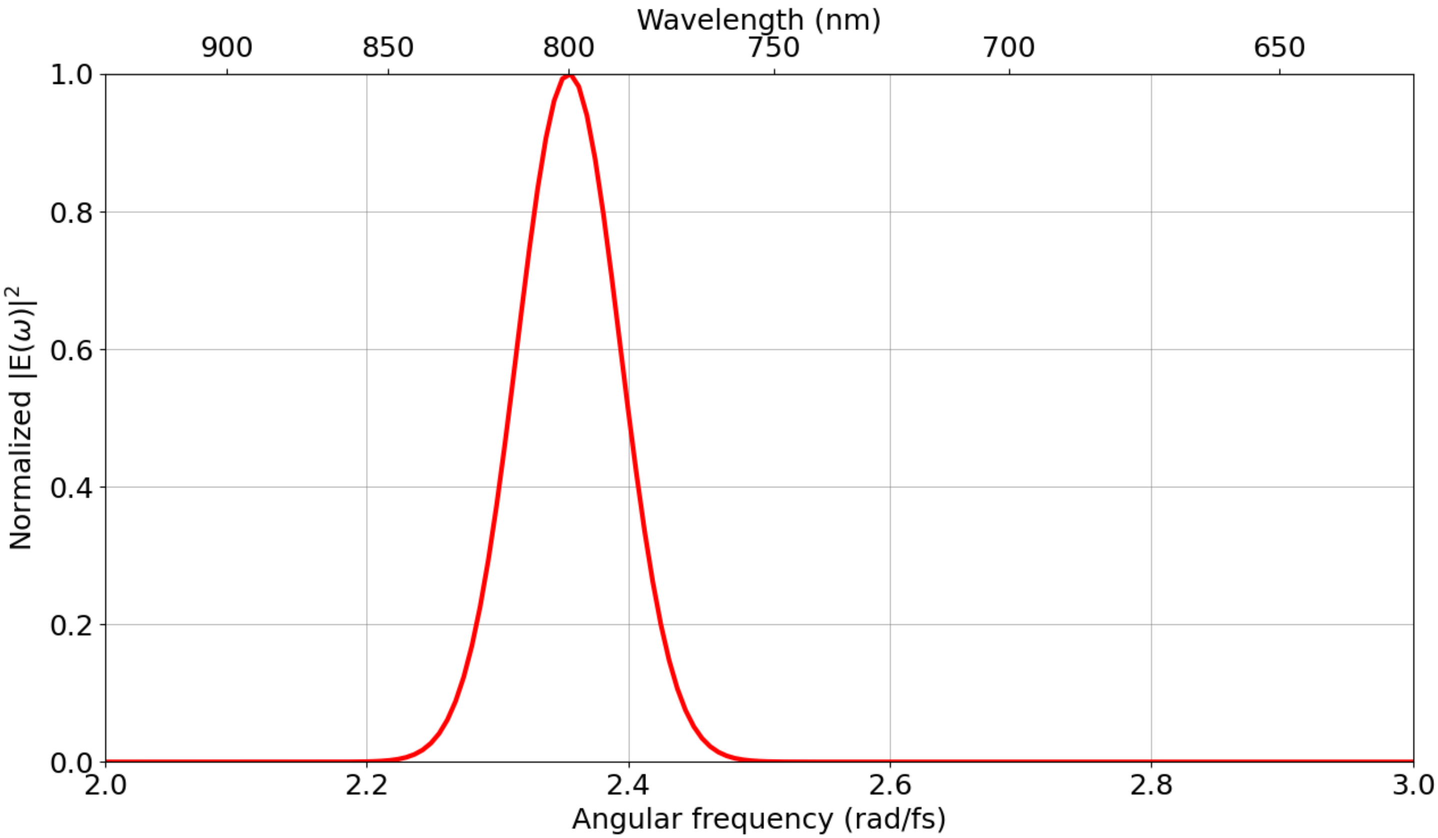

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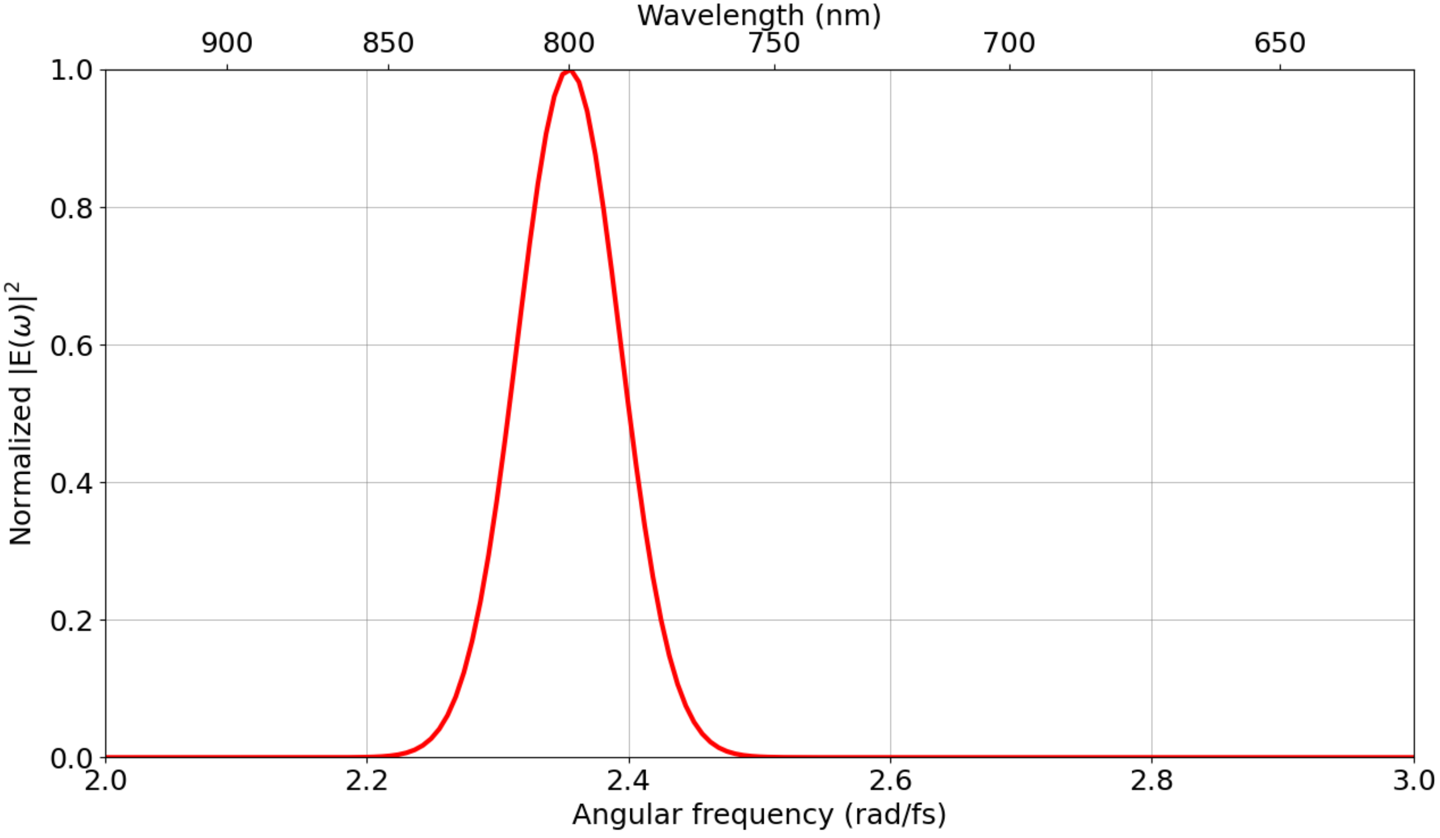

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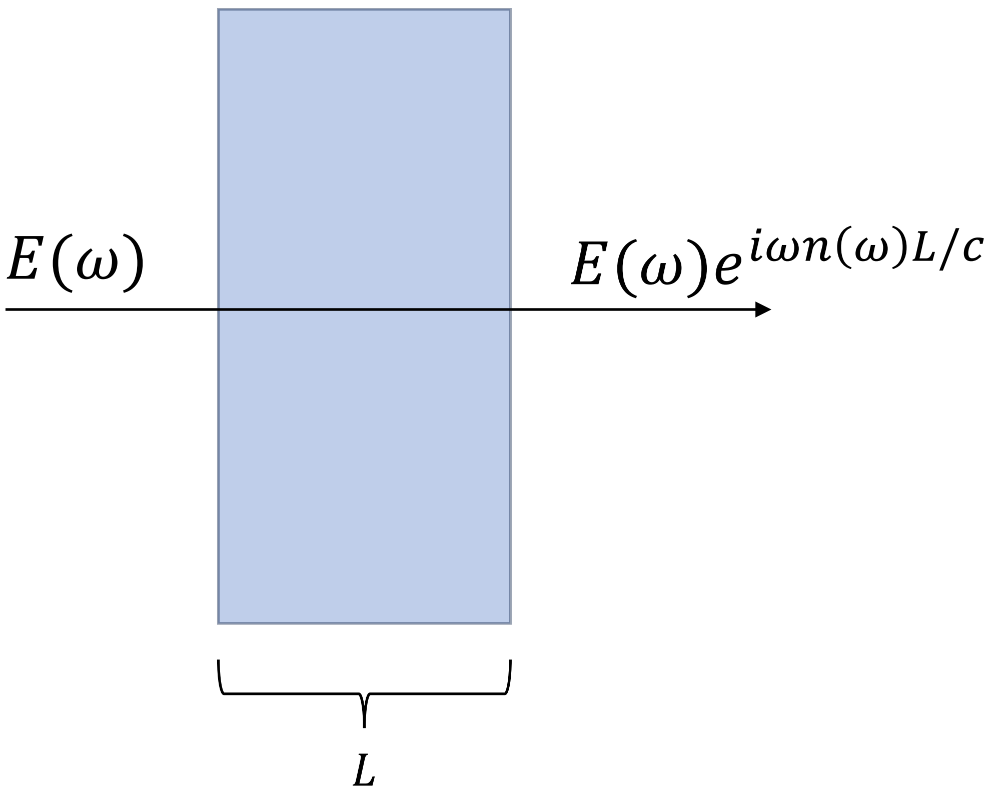
Linear propagation: the spectral phase

Why is the spectral phase important?

Laser pulse traveling through material with index of refraction $n(\omega)$



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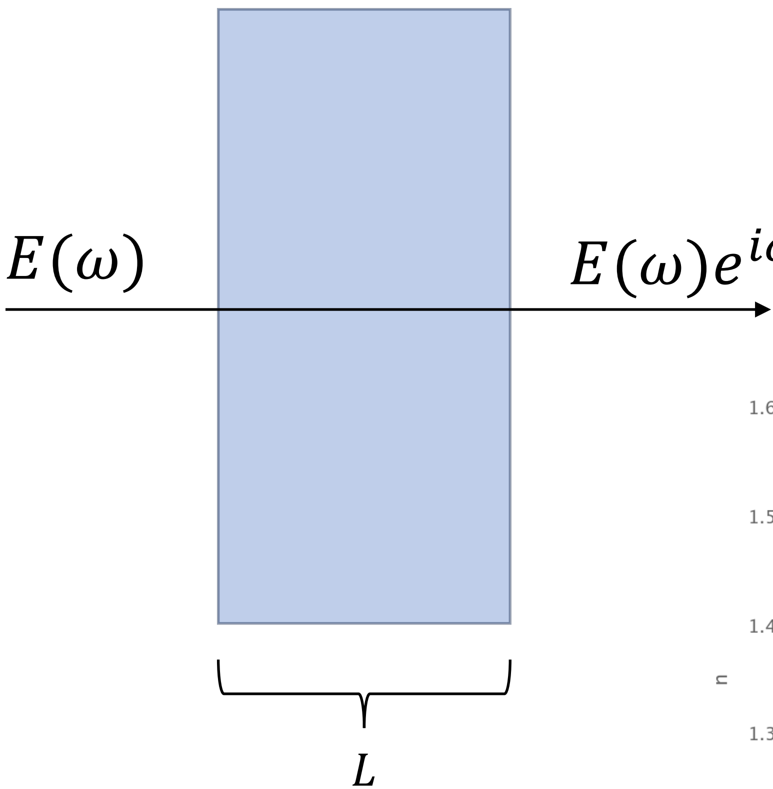
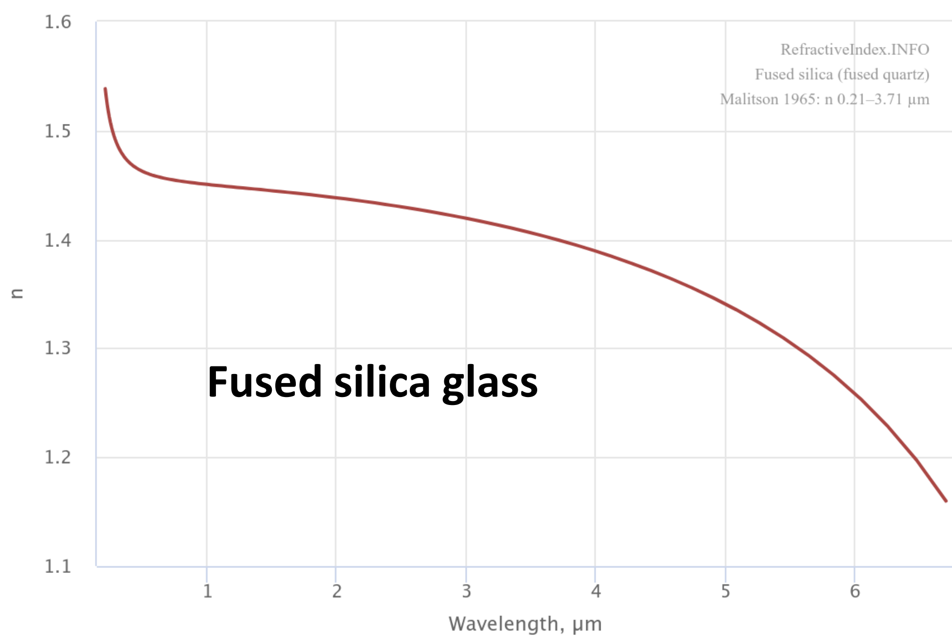


Phase is function of index of refraction, index of refraction is not constant over the frequency spectrum



https://refractiveindex.info/

Linear propagation: the spectral phase

The phase accumulated by passing through a medium of length L:

$$\varphi(\omega) = \omega n(\omega)L/c = k(\omega)L$$

$$= k(\omega_0)L + k'(\omega_0)[\omega - \omega_0]L + \frac{1}{2}k''(\omega_0)[\omega - \omega_0]^2 L + \cdots$$

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$$k(\omega_0) = \frac{\omega_0}{v_{phase}(\omega_0)}$$

$$k''(\omega_0) = \frac{d}{d\omega}\left(\frac{1}{v_{group}(\omega)}\right)_{\omega=\omega_0} = GVD$$

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$$v_{phase} = \frac{c}{n(\omega)} = \lambda\nu = \frac{\lambda\omega}{2\pi} = \frac{\omega}{k}$$

laser frequency

laser wavevector

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$$v_{phase} = \frac{c}{n(\omega)} = \lambda\nu = \frac{\lambda\omega}{2\pi} = \frac{\omega}{k}$$

laser frequency

laser wavevector

BUT, the pulse as a whole moves at the group velocity

$$v_{group} = \frac{\partial\omega}{\partial k}$$

Linear propagation: the spectral phase

The phase accumulated by passing through a medium of length L:

$$\varphi(\omega) = \omega n(\omega)L/c = k(\omega)L$$

$$= k(\omega_0)L + k'(\omega_0)[\omega - \omega_0]L + \frac{1}{2}k''(\omega_0)[\omega - \omega_0]^2 L + \cdots$$

$$k(\omega_0) = \frac{\omega_0}{v_{phase}(\omega_0)}$$

$$k''(\omega_0) = \frac{d}{d\omega}\left(\frac{1}{v_{group}(\omega)}\right)_{\omega=\omega_0} = GVD$$

$$k'(\omega_0) = \frac{1}{v_{group}(\omega_0)}$$

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laser frequency

laser wavevector

BUT, the pulse as a whole moves at the group velocity

$$v_{group} = \frac{\partial\omega}{\partial k}$$

Group velocity dispersion (GVD) implies that parts of the pulse move at different velocities, and changes the pulse duration

Linear propagation: the spectral phase

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e.g. GVD of fused silica (at 800 nm): 35 fs^2/mm

Linear propagation: the spectral phase

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$$\varphi(\omega) = \omega n(\omega)L/c = k(\omega)L$$

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group delay (GD) **group delay dispersion (GDD)**

$$\varphi(\omega) = \varphi_0 + \left(\frac{\partial\varphi(\omega)}{\partial\omega}\right)_{\omega=\omega_0}(\omega - \omega_0) + \frac{1}{2}\left(\frac{\partial^2\varphi(\omega)}{\partial\omega^2}\right)_{\omega=\omega_0}(\omega - \omega_0)^2 +$$

$$\frac{1}{6}\left(\frac{\partial^3\varphi(\omega)}{\partial\omega^3}\right)_{\omega=\omega_0}(\omega - \omega_0)^3 + \cdots$$

N.B. positive GDD; red before blue

Third Order Dispersion (TOD)

Phase in time and frequency domains

carrier envelope phase

group delay

group delay dispersion (GDD)

$$\varphi(\omega) = \varphi_0 + \left(\frac{\partial\varphi(\omega)}{\partial\omega}\right)_{\omega=\omega_0}(\omega-\omega_0) + \frac{1}{2}\left(\frac{\partial^2\varphi(\omega)}{\partial\omega^2}\right)_{\omega=\omega_0}(\omega-\omega_0)^2 + \frac{1}{6}\left(\frac{\partial^3\varphi(\omega)}{\partial\omega^3}\right)_{\omega=\omega_0}(\omega-\omega_0)^3 + \cdots$$

N.B. positive GDD; red before blue

Simulations of the effects of each term…

Phase in time and frequency domains

carrier envelope phase

group delay

group delay dispersion (GDD)

$$\varphi(\omega) = \varphi_0 + \left(\frac{\partial\varphi(\omega)}{\partial\omega}\right)_{\omega=\omega_0}(\omega-\omega_0) + \frac{1}{2}\left(\frac{\partial^2\varphi(\omega)}{\partial\omega^2}\right)_{\omega=\omega_0}(\omega-\omega_0)^2 + \frac{1}{6}\left(\frac{\partial^3\varphi(\omega)}{\partial\omega^3}\right)_{\omega=\omega_0}(\omega-\omega_0)^3 + \cdots$$

N.B. positive GDD; red before blue

Simulations of the effects of each term…

Wigner:

$$\boldsymbol{W(t,\omega) = \int_{-\infty}^{+\infty} E^{+}\left(t+\frac{s}{2}\right) E^{+*}\left(t-\frac{s}{2}\right) e^{-i\omega s} ds = \int_{-\infty}^{+\infty} \widetilde{E}^{+}\left(\omega+\frac{s}{2}\right) \widetilde{E}^{+*}\left(\omega-\frac{s}{2}\right) e^{its} ds =}$$

Temporal location of spectral components

Phase in time and frequency domains

carrier envelope phase

group delay

group delay dispersion (GDD)

$$\varphi(\omega) = \varphi_0 + \left(\frac{\partial \varphi(\omega)}{\partial \omega}\right)_{\omega=\omega_0} (\omega - \omega_0) + \frac{1}{2}\left(\frac{\partial^2 \varphi(\omega)}{\partial \omega^2}\right)_{\omega=\omega_0} (\omega - \omega_0)^2 + \frac{1}{6}\left(\frac{\partial^3 \varphi(\omega)}{\partial \omega^3}\right)_{\omega=\omega_0} (\omega - \omega_0)^3 + \cdots$$

N.B. positive GDD; red before blue

$$E^+(t) = \frac{1}{2}\varepsilon(t) e^{i\varphi(t)} e^{i\omega_0 t} = \frac{1}{2}\tilde{\varepsilon}(t) e^{i\omega_0 t}$$

carrier envelope phase

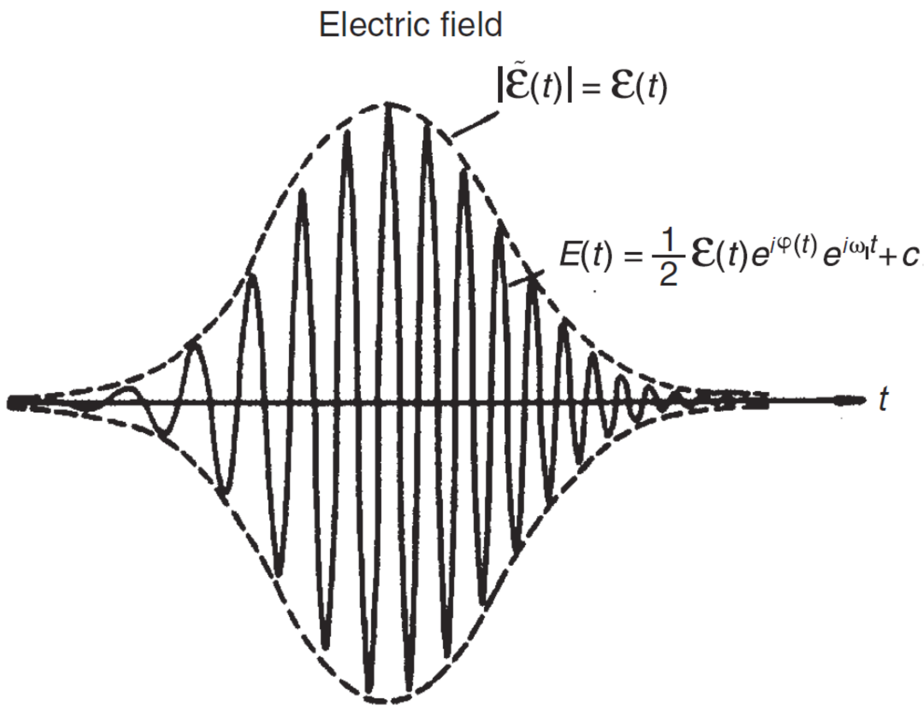
instantaneous frequency

linear chirp

N.B. positive chirp means $d\omega/dt > 0$

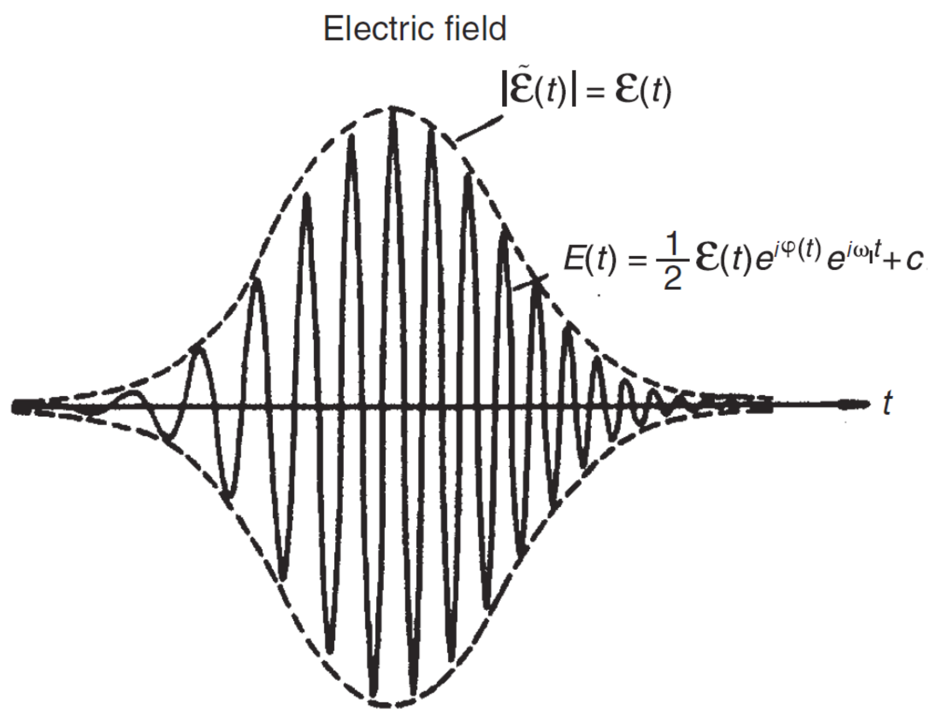
$$\varphi(t) = \varphi_0 + \left(\frac{\partial \varphi(t)}{\partial t}\right)_{t=t_0} (t - t_0) + \frac{1}{2}\left(\frac{\partial^2 \varphi(t)}{\partial t^2}\right)_{t=t_0} (t - t_0)^2 + \frac{1}{6}\left(\frac{\partial^3 \varphi(t)}{\partial t^3}\right)_{t=t_0} (t - t_0)^3 + \cdots$$

Pulse envelope and carrier: chirped pulse

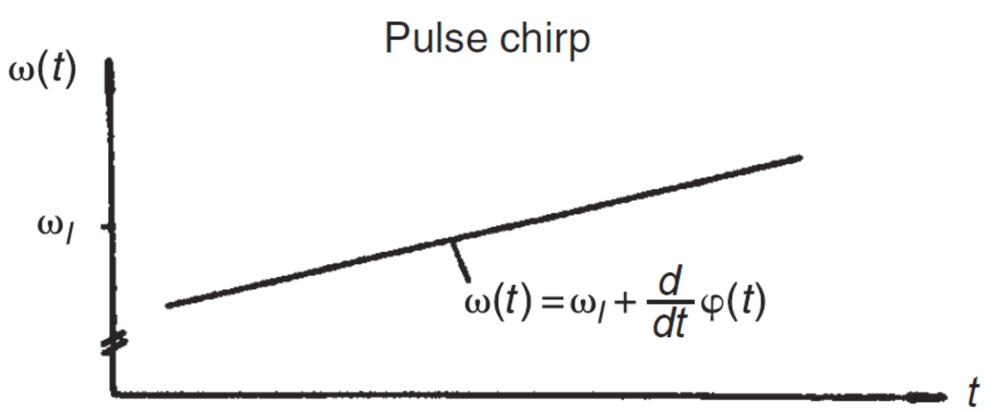


The laser electric field E(t) is described as the product of a real envelope function $\varepsilon(t)$ and an oscillatory term depending on the time-dependent phase φ(t) and the carrier frequency ω_l

Pulse envelope and carrier: chirped pulse



The laser electric field E(t) is described as the product of a real envelope function $\varepsilon(t)$ and an oscillatory term depending on the time-dependent phase φ(t) and the carrier frequency ω_l



The laser chirp is determined by the time-dependent phase φ(t)

Suggested literature

J.-C. Diels and W. Rudolph, *Ultrashort Laser Pulse Phenomena,* (Academic Press, 2006)

A. Siegman, *Lasers*, (University Science Books 1986)

U. Keller, *Ultrafast Lasers*, (Springer 2021)