Evaluation for the course in Cosmology

Martin Crocce, Program of visiting professors, Phys. Dep., UBA

November 16, 2018

The following set of exercises are a mixture of calculations, numerical evaluations and plotting. Use the plotting software you like best, but plots should be clear neat and include labels and units. You can code in whatever language you prefer.

Please produce the write-up in Latex with plots merged in (I can send an example file for you to edit if you need one). Calculations should also be a bit elaborated until the final expression (that's why we use Latex). And answers explained with logical arguments.

Referenced papers can be found at https://arxiv.org/search/astro-ph

Useful constants and definitions

- (Newton's constant) $G = 6.673 \times 10^{-8}\,\mathrm{cm}^3\,\mathrm{gr}^{-1}\,\mathrm{sec}^{-2}$
- (Hubble expansion rate) $H = 100\,h\,\mathrm{km/sec\,Mpc}^{-1}$
- (Critical density) $\rho_{\rm crit}(t) = 3H(t)^2/(8\pi G) = 1.879\,h^2\,10^{-29}\,\mathrm{gr\,cm}^{-3}$
- (Total relative density) $\Omega_0 = \rho(t)/\rho_{\rm crit}$
- (Energy density in radiation) $\Omega_{\rm rad} = 2.47 \times h^{-2}(T/T_0)^4$
- (CMB temperature today) $T_0 = 2.726$ Kelvin ($\sim$ -270 Celsius)
- (MICE Cosmology): Throughout the exercises we will compare against simulations that were run with the following cosmological model: $\Omega_m = 0.25$, $\Omega_b = 0.044$, $\Omega_\nu = 0$, $n_s = 0.95$, $\sigma_8 = 0.75$, $h = 0.7$ and a flat Universe.

1.- The background Universe

Age of the Universe. Starting from the Friedmann equation

$$H^2(t) = \frac{8\pi G}{3}\rho(r) - \frac{k}{a^2} \tag{1}$$

where $\rho(t) = \rho_m(t) + \rho_\Lambda + \rho_{\rm rad}(t)$ show that the age of the Universe t_0 is given by,

a) In a matter dominated Universe with zero cosmological constant as

$$t_0 H_0 = \int_0^1 \frac{dx}{\sqrt{\Omega_0/x + 1 - \Omega_0}} \tag{2}$$

b) In a radiation dominated Universe with zero cosmological constant as

$$t_0 H_0 = \int_0^1 \frac{dx}{\sqrt{\Omega_0/x^2 + 1 - \Omega_0}} \tag{3}$$

c) In a flat Universe with cosmological constant (i.e. $k = 0$ and $\Omega_0 = 1 = \Omega_m + \Omega_\Lambda$) as

$$t_0 H_0 = \frac{2}{3\sqrt{\Lambda}} \ln\left[\frac{1+\sqrt{\Lambda}}{\sqrt{\Omega_m}}\right] \tag{4}$$

For a fixed H_0 value, compare the age of the Universe for two cases ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$) vs ($\Omega_m = 1$, $\Omega_\Lambda = 0$), which one is older and why?

d) *How old is the Universe.* From the Planck 2018 results (see Table 1 in e-print arXiv 1807.06209), take the value of H_0, $\Omega_b h^2$ and $\Omega_c h^2$ from the table and estimate the age of the Universe today (in Gyrs) using what you believe is the most appropriate formula from those derived above. Compare your result with theirs (10th raw in Table 1 of the paper). Think of reasons why they are not "exactly" the same.

e) *The earliest time we can see.* Choose what you think is the appropriate scenario above and estimate how old was the Universe at the Last Scattering Surface (take $z_{\rm rec} = 1100$).

f) *The age of the accelerating Universe.* Using the values from the Planck table derive the redshift at which the matter and vacuum energy densities are the same (i.e. when the Universe started to be dominated by dark energy and hence expanding with acceleration) and find the age of the Universe the age of the Universe there.

2.- Linear growth and linear growth rate of structure

The linear evolution of (cold dark matter) density perturbations $\delta(\tau)$ in the matter dominated era is given by the growth factor $D(\tau)$ which, as we saw in class, obeys

$$\frac{d^2D}{d\tau^2} + \mathcal{H}\frac{dD}{d\tau} = \frac{3}{2}\Omega_m \mathcal{H}^2 D, \tag{5}$$

where $\mathcal{H}(\tau)$ is the conformal expansion rate $\mathcal{H} \equiv (1/a)da/d\tau = aH(a)$, τ is conformal time and $\Omega_m(\tau)$ is the ratio of matter to critical density. In the general case where there is only matter and vacuum energy Eq. 5 has the following growing mode solution,

$$D^{(+)}(a) = \frac{\mathcal{H}(a)}{a}\frac{5\Omega_m^0}{2}\int_0^a \frac{da'}{\mathcal{H}(a')^3}. \tag{6}$$

In turn, in linear perturbation theory, the velocity field divergence $\theta = \nabla \mathbf{v}$ is related to the density contrast δ, by $\theta = -\mathcal{H}f\delta$, where $f \equiv d\ln D/d\ln a$ is directly related to the growth rate of structure ($dD/d\tau$). The logarithmic growth rate f measures how fast velocity perturbations grow with respect to density ones.

a) Integrate Eq. (6) numerically for the MICE Cosmology (flat Universe) to find the linear growth factor at $z = 0.5$ (i.e. $D^{(+)}(0.5)/D^{(+)}(0)$, growth normalized to 1 at $z = 0$).

b) We will now find a widespread approximation for f. From the Friedmann equation for arbitrary curvature and matter domination find an expression for $d\mathcal{H}/d\tau$ and then $d\Omega_m/d\tau$.

c) Use Eq. 5 to find a differential equation for f as a function of Ω_m (i.e. change to Ω_m as a time variable). Show that this differential equation has the expected solution for $\Omega_m = 1$ that we found in class. By expanding around $\Omega_m = 1$, show that $f(\Omega_m) = \Omega_m^p$ is a solution, and find the value of p (note that here Ω_m is a function of redshift). The usual fitting formula is $f \sim \Omega_m(z)^{0.6}$ (valid for $0.1 < \Omega_m < 1$). How does the p-value compare?.

d) Plot the relative error of your approximate expression as a function of Ω_m^0, w.r.t. the "exact" value obtained by numerically differentiating Eq. (6) [1]. For $0.2 < \Omega_m^0 < 1$ and $0 < z < 2$ (e.g. do four curves at fix values, $z = 0, 0.5, 1, 2$), and assuming flat Universe, show that your result from c) is better than 2% (and certainly better than $f = \Omega_m^{0.6}$).

Note: The growth rate of structure and its redshift dependence is a very important quantity in cosmology. Measuring f allow us to test, not only Ω_m in general relativity, but also different theories of gravity in extended models (see Weinberg et al, e-print arXiv 1201.2434).

[1] This can be obtained as $f(a) = \frac{D^{(+)}(a)}{a} \lim_{h\to 0} \frac{D^{(+)}(a+h) - D^{(+)}(a-h)}{2h}$

3.- The linear and nonlinear matter power spectrum

Download CAMB from `http://camb.info/`.

CAMB is a numerical code that solves exactly the Boltzmann equations governing the evolution of linear modes after they leave inflation, evolving outside the horizon with general relativity, and throughout the matter and radiation dominated Universe. Exactly the same idea to what we did in class but taking into account all species, baryons, dark matter, radiation, neutrinos, etc, and the exact equations in GR (to linear order).

CAMB is the code used and developed by the Planck Collaboration to derive the cosmological parameters as we know them now, out of CMB power spectrum measurements.

Install it and try to run the demo, we will use it for some of the exercises below.

Exercises

a) Derive $z_{\rm eq}$ (the redshift of matter radiation equality) and $k_{\rm eq}$ (the inverse of the comoving size of the Horizon at matter-radiation equality $H^{-1}(z_{\rm eq})$) as a function of $\Omega_m h^2$ and $\Omega_{\rm rad} h^2$.

b) Assuming the MICE cosmology given in the first page run CAMB to output a linear power spectrum and a linear transfer function at $z = 0$. Are they related in the way we discussed in class ? does $P(k)$ follow the expected low-and-high-k asymptotes ?
[*Note*: this is the cosmology of the simulations that we will use later on, so keep this output]

c) For the cosmological model in b) and assuming the value of $\Omega_{rad} h^2$ given at the beginning, derive the value of $k_{\rm eq}$ in units of h/Mpc. This should be similar to which feature in the linear power spectrum ? (explain why).

d) A popular measure of the amplitude of the density perturbations is the RMS overdensity in a sphere of radius R, defined as

$$\sigma_R^2 = \langle \delta_R^2(x) \rangle \tag{7}$$

with

$$\delta_R(\mathbf{x}) = \int d^3\mathbf{x}' \delta(\mathbf{x}) W_R(\mathbf{x} - \mathbf{x}') \tag{8}$$

where $W_R(\mathbf{x})$ is equal to 1 for $|x| < R$ and vanishes otherwise (spherical top-hat filter). Show that

$$\sigma_R^2 = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P(k) W^2(kR) \tag{9}$$

where $W(x) = (3/x^3)(\sin x - x\cos x)$. It is standard to quote fluctuations at $R = 8\,\mathrm{Mpc}\,h^{-1}$, which are denoted by σ_8.

Implement a code in your preferred language that evaluates the above integral for a given input power spectrum.

e) Assume the following Planck 2018 best fit values from Table 1 in e-print arXiv 1807.06209: $\Omega_b h^2 = 0.0224$, $\Omega_c h^2 = 0.1201$, $n_s = 0.966$ (scalar spectral index), $\ln(10^{10} A_s) = 3.0448$ (where A_s is the scalar amplitude of fluctuations) and $h = 0.6732$. Compute σ_8 and Ω_m at this cosmology. Decrease the cold dark matter density Ω_c by 15%, re-run CAMB, and recompute σ_8 and Ω_m. Explain the result (why σ_8 decreases/increases).

Locate these two results in the contour plot for (Ω_m, σ_8) given by the cosmological analysis of the first year of DES data, Fig 5 in e-print arXiv 1708.01530. Which one is ruled out or disfavoured by DES ?

f) Run CAMB but output instead the nonlinear power spectrum (look for the variable `do_nonlinear`) at $z = 0$ and $z = 0.5$. At each of these redshifts, find the scale at which the linear and nonlinear power spectra start to differ (at each redshift). This is the scale in which nonlinear gravitational collapse starts to be important (basically when the amplitude of linear fluctuations $k^3 P(k)/(2\pi^2) \sim 1$). Does this scale increase of decrease with time ? Explain whether this makes sense for you (or not).

Make a (nice) plot of these 4 spectra (two linear, two nonlinear) showing clearly the transition to the nonlinear regime.

4.- The halo mass function

For this exercise you will need these halo catalogs from the MICE simulation,

$https://www.dropbox.com/s/a56sj2tfqpynthd/halo\_MICE\_N4096\_L3072\_z0\_b0p2\_np40.tar?dl = 0$
$https://www.dropbox.com/s/tkhfk9x9vwmxwxb/halo\_MICE\_N4096\_L3072\_z0p5_b0p2\_np40.tar?dl = 0$

The above are formatted ascii files with columns $(n_p, x, y, z, v_x, v_y, v_z)$, where n_P is the number of particles in the given halo (i.e. $n_p m_p$ is halo mass, with the dark-matter particle mass m_p given below), (x,y,z) are the coordinates of the halo center of mass in $\mathrm{Mpc}\, h^{-1}$ and (v_x, v_y, v_z) its velocity in km/sec. Filenames are halo_$i_$j with $i, j = 1$ to 10.

The MICE Grand Challenge N-body simulation is described in detail in e-print arXiv 1312.1707 and 1312.2013. An N-body run refers to a simulation that evolves only particles interacting gravitationally, from initial conditions at some high redshift were fluctuations are small (typically $z \sim 100$). These simulations track the nonlinear collapse of structure.

MICE evolves gravitationally 6.8×10^{10} cold dark-matter particles (4096^3) from their initial conditions at $z = 50$, within a periodic comoving box of $L = 3072\,\mathrm{Mpc}\,h^{-1}$. The cosmology is given in the 1st page. The dark-matter particle mass is $m_p = 2.93 \times 10^{10}\, M_\odot h^{-1}$. Bound structure (i.e. Halos) in the simulation were found using a Friend-of-Friends algorithm with linking length $b = 0.2$. Such algorithm finds all the neighbours to a given particle whose distance is less than a fixed fraction of the mean interparticle distance (in this case 0.2), and repeats the process iteratively on all the linked neighbours or "friends" until no new "friend" is found. These effectively defined halos as iso-density contours (which are typicall not spherical). Other algorithms define "spherical over-densities" instead, or find halos in phase-space.

For what follows consider the expression for the halo mass function $n(m, z)$ representing the number density of halos of mass m and redshift z in terms of the multiplicity function $f(\nu)$ with $\nu = \delta_c/\sigma(m, z)$ ($\delta_c \equiv 1.686$)

$$n(m) = \frac{\rho_m}{m^2} f(\nu) \frac{d \ln \sigma^{-1}}{d \ln M}, \tag{10}$$

where for Press and Schechter (1974, ApJ **187**, p. 425),

$$f_{\mathrm{PS}}(\nu) = \sqrt{\frac{2}{\pi}}\, \nu\, \exp[-\nu^2/2] \tag{11}$$

while the more accurate expression from Sheth and Tormen (e-print arXiv 0105113) is,

$$f_{\rm ST}(\nu) = A\sqrt{\frac{2q}{\pi}}\nu\left[1+\left(q\nu^2\right)^{-p}\right]\exp[-q\nu^2/2] \tag{12}$$

with $A = 0.3222$, $q = 0.707$ and $p = 0.3$. In $\nu = \delta_c/\sigma(m, z)$ the variance is computed as,

$$\sigma^2(m, z) = \frac{D^2(z)}{2\pi^2}\int_0^\infty dk k^2 P(k) W^2(kR) \tag{13}$$

where $W(x) = (3/x^3)(\sin x - x\cos x)$, $m = \rho_m 4\pi R^3/3$, and $D(z)$ is the linear growth factor normalised to 1 at $z = 0$ (use $\rho_m = \Omega_m\rho_c$ and $\rho_c = 2.775\times 10^{11}\,h^{-1}\,{\rm M}_\odot/(\,h^{-1}\,{\rm Mpc})^3$).

Exercices

a) Modify the code that computed $\sigma(R)$ to compute the mass variance $\sigma(m, 0)$ from roughly $m = 10^8\,h^{-1}\,{\rm M}_\odot$ to $10^{16}\,h^{-1}\,{\rm M}_\odot$ and plot it in logarithmic scale. Explain the result.

b) Plot $n(m)$ vs. m (in log-log scale) for the MICE cosmology but assuming $\sigma_8 = 0.7, 0.8$ and 0.9. Explain the dependence of the halo abundance on this cosmological parameter. A similar dependence is expected for increasing Ω_m. The cluster (halo) mass function is an excellent probe of growth of structure.

c) For $M_\star = 10^{13}\,h^{-1}\,{\rm M}_\odot$ integrate the mass function to find all halos above this threshold : $n_h(> M_\star) = \int_{M_\star}^\infty n(m)dm$. Plot this as a function of redshift $(0 < z < 1)$. The dependence with redshift is also an excellent tracer of growth.

d) Measure the "cumulative" halo mass function $n(> m)$ vs. m in the MICE halo catalog[2] at $z = 0$ and compare it with the ST and the PS predictions. For this you will need to write a little code to read and count halos above a certain mass limit. Note that the predictions are for number densities. Which prediction works better? Do you find the trends that we discussed in class (at low and high mass) ?

e) Measure the "cumulative" mass function at $z = 0.5$ and compare it with the one at $z = 0$. Below the cut-off mass scale, the abundance of halos should be roughly constant with redshift. What is the value of this halo mass ?

`Bib: Overall, e-print arXiv 1312.2013 could be of help (e.g. Fig 2)`

[2] Suggestion: use a log binning in mass, from $\sim 1.2\times 10^{12}\ h^{-1}\,{\rm M}_\odot$ to $\sim 7.5\times 10^{15}\ h^{-1}\,{\rm M}_\odot$, and ~ 30 bins

5.- Clustering of tracers: halo bias and bias evolution

For this exercise you will need `PowerI4` from https://github.com/sefusatti/PowerI4 (it requires FFTW libraries to be installed, see the README for instructions).

Note: It is faster to read the halo catalogs as unformatted (fortran binary)
$https://www.dropbox.com/s/s1jlf1orloojhd3/halo.uf\_MICE\_N4096\_L3072\_z0\_b0p2\_np40.tar?dl=0$
$https://www.dropbox.com/s/v2khtan43s5tvqa/halo.uf\_MICE\_N4096\_L3072\_z0p5\_b0p2\_np40.tar?dl=0$

In class we show that halo bias can be obtained by taking derivatives of the conditional mass function with respect to the barrier for halo colapse:

$$b_1 \equiv 1 - \frac{\partial \ln n(m,z)}{\partial \delta_c} \tag{14}$$

For the Sheth and Thormen mass function given in Eq. (12) this leads to

$$b_1^{\rm ST}(m,z) = 1 + \frac{q\nu^2 - 1}{\delta_c} + \frac{2p/\delta_c}{1+(q\nu^2)^p} \tag{15}$$

The bias of "halo samples" defined by mass ranges is then computed as,

$$\begin{aligned} b_h &= \frac{1}{n_h}\int_{m_1}^{m_2} b_{\rm ST}(m,z)\, n_{\rm ST}(m,z)\, dm \qquad (16)\\ n_h &= \int_{m_1}^{m_2} n_{\rm ST}(m,z)\, dm. \end{aligned}$$

Press Schechter predictions are trivially obtained by setting $q=1$ and $p=0$.

Exercises

a) *Evolution of bias.* Suppose there is a component in the universe (e.g. galaxies) that forms at some characteristic time $t = t_\star$, with density contrast satisfying the local linear bias relation $\delta_g = b_0\delta$ at $t = t_\star$, where δ is the dark matter density. Assume that the number of these objects is conserved (which is a strong assumption for halos because of halo merger and accretion), and that their velocities equal those of dark matter particles. Show that at linear order the bias factor $b = \delta_g/\delta$ evolves towards unity at late times and that the relation stays local.

b) Consider 6 different halo samples at $z=0$ defined by the following mass thresholds,

$M_h/\,h^{-1}\,\mathrm{M}_\odot > 1.465 \times 10^{12}, 5.86 \times 10^{12}, 1.465 \times 10^{13}, 2.93 \times 10^{13}, 8.79 \times 10^{13}, 1.465 \times 10^{14},$

corresponding to number of particles per halo > 50, 200, 500, 1000, 3000, 5000. Select those samples from the catalogs given above and measure their power spectrum P_{hh}. Then estimate halo bias as $b_h = \sqrt{P_{hh}/P_{linear}}$ where P_{linear} is the linear power spectrum you obtained by running CAMB for the MICE cosmology. You can for example average the ratio for the range of scales where bias seems linear.

Plot $b(> M_h)$ vs. M_h in log-linear scales. On large scales you should find the regime of "linear bias" that we discussed in class where b_h is scale independent. At what scale does linear bias brakes down? How does it compare with the scale at which matter power spectrum becomes nonlinear, found in Exercise 3f ?

Comment: Power spectrum measurements need to be corrected by shot-noise from the discrete nature of the number of objects used to measure it. This is more important for the most massive halos that have the lowest number density. If one assumes shot-noise is "Poisson" then the correction is simply $P = P - (2\pi)^{-3}\bar{n}^{-1}$, where $\bar{n}$ is the number density of objects. `PowerI4` outputs in its first line the number of objects and the shot-noise correction [3].

Comment: You will need to modify the routine "input_catalog.f90" in `PowerI4` to read in halo catalogs. I strongly recommend to read binary files which is much faster. My routine for doing this is here

https://www.dropbox.com/s/z4et436ofvbmd2q/input_catalog.f90?dl=0

you should be able to use it right away. For params.ini I use

```
# Input file type = 2
# FFT grid sizes = 256 256 256
# Interpolation order (integer 0 to 4) = 4
# Interlacing (true/false) = true
# Box sides = 3072. 3072. 3072.
# bin size in units of the fundamental frequency (linear binning) 3
# center of the first bin in units of the fundamental frequency 3
# measure multiples for anisotropic clustering (0,1,2,3, see below) 0
# output density file (see below) 0
```

[3] The factor $(2\pi)^3$ is coming from the convention used in the FFT transform

c) Predict $b(> M_h)$ from the Sheth and Tormen expression in Eq. (16) setting m_2 very large (e.g. 10^{16} or so), and plot them together with the measurements made above Discuss the results (e.g. in terms of the accuracy of the prediction)

d) Select a sample of halos in the range $50 \leq n_p \leq 200$ both at $z = 0$ and $z = 0.5$. From the mass function exercise we see that number density of this objects is roughly constant with time. Estimate the linear bias in both cases (recall that at $z = 0.5$ bias must be defined w.r.t. the linear power spectrum at $z = 0.5$, which is $D(z = 0.5)^2 P_{CAMB}(z = 0)$). Do these two biases relate in the way you found in point a) ? In reality halos are being created constantly and the evolution of bias is more consistent with a constant clustering amplitude, i.e. $b(z) = b_\star D(z_\star)/D(z)$. Do you find this ?

`Bib: e-print arXiv 0906.1314 or 1405.5521 could be of help in this exercise`

6.- Redshift Space Distortions

When studying the large-scale structure of the universe, the redshift of galaxies is used as a distance indicator. This would be exact in an homogeneous universe where velocities are only due to the Hubble flow; however, because of density and velocity perturbations, the comoving distance thus obtained is affected by peculiar velocities,

$$\mathbf{s} = \mathbf{x} + \hat{z}(\mathbf{v}_p \cdot \hat{z})/\mathcal{H} \tag{17}$$

where $\mathbf{s}$ is the so-called redshift-space (comoving) position that corresponds to real-space (comoving) position $\mathbf{x}$, $\mathbf{v}_p$ is the peculiar velocity, $\mathcal{H}$ is the conformal Hubble factor ($\mathcal{H} \equiv da/d\tau = aH(a)$) and we assumed that the line of sight is along a fixed direction denoted by $\hat{z}$.

In class we showed that, in linear theory ($\delta \ll 1$), this mapping translates into this relation between the Fourier components of the redshift-space and real-space density contrasts,

$$\delta_s(\mathbf{k}) = (b + f\mu_{\mathbf{k}}^2)\delta(k) \tag{18}$$

where f is the growth rate of structure derived above, b the linear bias of the tracer under consideration (galaxies, halo, etc) and μ the cosine angle of the Fourier mode $\mathbf{k}$ with the line of sight ($\mu_{\mathbf{k}} = \mathbf{k} \cdot \hat{z}/k$). Equation 18 implies that the power spectrum of tracers, in redshift space, is now anisotropic and given by

$$P_s(k, \mu) = (b + f\mu_{\mathbf{k}}^2)^2 P(k). \tag{19}$$

Exercises

a) Eq. 19 says that modes parallel to the line of sight ($\mu_{\mathbf{k}} = 1$) have enhanced amplitude compared to those perpendicular to it ($\mu_{\mathbf{k}} = 0$); which, as expected, are unaffected by the mapping). Explain physically where this enhancement is coming from.

b) Use a multipole decomposition of the anisotropic power spectrum,

$$P_s(k, \mu_{\mathbf{k}}) = \sum_{\ell} P_\ell(\mu_{\mathbf{k}}) P_s^{(\ell)}(k) \tag{20}$$

where

$$P_s^{(\ell)}(k) = (2\ell + 1) \int_{-1}^{1} \frac{d\mu}{2} P_\ell(\mu) P_s(k, \mu) \tag{21}$$

and P_ℓ are the Legendre polynomials of order ℓ to show that, in linear theory, only $\ell = 0, 2, 4$ are non-zero and given by

$$P_s^{(0)}(k) = (b^2 + \frac{2}{3}b\,f + \frac{1}{5}f^2)\,P(k) \quad (22)$$

$$P_s^{(2)}(k) = (\frac{4}{3}b\,f + \frac{4}{7}f^2)\,P(k) \quad (23)$$

$$P_s^{(4)}(k) = \frac{8}{35}f^2\,P(k) \quad (24)$$

b) Using PowerI4 measure the power spectrum multipoles for halos in the second sample of the previous exercise (i.e. with $M_h > 5.86 \times 10^{12}\,h^{-1}\,\mathrm{M}_\odot$, corresponding to 200 or more particles) at $z = 0$. Change he params.ini to "# measure multiples for anisotropic clustering (0,1,2,3, see below) → 1,2 or 3". This will move the particles to z-space along, x, y or z [4]. I suggest doing the 3 cases (which are roughly independent) and average the results.

Do a log-linear plot of the monopole to quadrupole ratio $P^{\ell=0}(k)/P^{\ell=2}(k)$ in the range $(0.01 - 0.10)\,h\,\mathrm{Mpc}^{-1}$. Assuming the bias from real space measurements found in exercise 5b and the approximation for f in exercise 2c plot the large-scale predictions for the monopole-to-quadrupole ratio for $\Omega_m = 0.2, 0.25, 0.3$. Is the better fit the one corresponding to the value of matter density used in the simulation ?

Note : the quadrupole to monopole ratio "on large scales" has two important properties, a) it cancels the statistical noise (known as cosmic variance) b) it does not depend of the power spectrum itself (!). For a long time this was considered a very good way of measuring f/b. By now we have precise enough measurements to fit both quadrupole and monopole independently and hence derive both $b\sigma_8$ and $f\sigma_8$.

c) Plot the ratio of $P^{(\ell=0)}$ (redshift space monopole), $P^{(\ell=2)}$ (redshift space quadrupole) to their respective linear models given by the expressions in Eq. (22). At what scale does the model acquires a strong scale dependence ? how does it compare with the scale for nonlinear matter clustering and nonlinear halo bias ?

One particular complication in interpreting large scale structure surveys measuring galaxy clustering is that all the non-linear effects (matter clustering, bias, redshift-space distortions) appear at roughly the same scale. The modeling becomes quickly complicated !.

`Bib: e-print arXiv 1206.4070 Fig 5 could be of reference`

---

[4]In a simulation we can place the observer in many places!

General Bibliography

1-Modern Cosmology, Scott Dodelson (Academic Press ISBN 0-12-219141-2)
2-Cosmological Inflation and Large Scale Structure, Liddle and Lyth

By topics or a bit more specialized

- "*Basic concepts*": The cosmology course by Anthony Lewis is a nice and clear introduction: https://cosmologist.info/teaching/Cosmology/. Download the Course Notes.
- "*Cosmic Acceleration Review*": Weinberg et al, e-print arXiv 1201.2434. A very good and complete overview of observational probes of cosmic acceleration, from basic concepts to a discussion of systematics of different observations.
- "*Perturbation Theory*": By now a standard source of reference is the "Review on perturbation theory" at e-print arXiv 0112551. It also has discuss bias and statistics.
- "*Halo Model*": Check the review "Halo Models of Large Scale Structure" by Cooray and Sheth at e-print arXiv 0206508, for mass functions and halo bias.
- "*Weak Lensing*": There are many reviews and articles on this subject, I like Hoekstra e-print arXiv 1312.5981 and Kilbinger e-print arXiv 1411.0115.