The need for a statistical view

1. We do not have direct observational ~~evidence~~ access to primordial fluctuation that would provide the IC. Therefore; although evolution is deterministic we can't put IC

2. Time scale of evolution is very much longer than our timescale => imposible to follow a single system.

Random Fields

We can think of the observable Universe as being a particular realization of the statistical ensemble of possibilities. Recall that initial fluctuation (e.g. from inflation) are random by nature.

$$\phi(\vec{x}\, t) = \langle \phi(\vec{x}, t) + \delta\phi \rangle \equiv \phi(t) + \delta\phi\, (\vec{x}; t).$$

probability

Cosmology assumes Ergodic fields.

$$\int \Phi P(\phi)\, d\phi = \langle \phi(\vec{x}\, t) \rangle = \frac{1}{V} \int_V d^3x\ \phi(\vec{x} t) = \bar{\Phi}(t)$$

This is essential to compare theory to obs. $\Rightarrow$ measure statistical properties by spatial averaging!

Characterizing fluctuations:

$\langle \delta\phi \rangle \equiv 0$ by definition.

$\frac{\delta\phi}{\phi} \ll 1$ small fluctuations

$\rightarrow$ correlation functions:

$\langle \delta\phi_1 \ldots \delta\phi_N \rangle$: N point CF.

$$= \langle \delta\phi(x_1) \ldots \delta\phi(x_N) \rangle$$

Unless the field varies independently from point-to-point these quantities will be non-zero in gral.

They characterize completely the PDF

$$< \delta\phi(x_1) \ldots \delta\phi(x_N) > = \int P(\delta\phi(x_1) \ldots \delta\phi(x_N))\, \delta\phi \ldots d\,\delta\phi$$

$\boxed{\text{Gaussian Field}}$ → the only non-trivial correlation is the 2-point.

Poisson Field → all points are independent!

In general we assume $\boxed{\text{Homogeneity + Isotropy}}$ (in the statistical sense).

$$< \delta\phi(\vec{x}_1) \ldots \delta\phi(x_N) > = f(x_{21}, x_{31}, \ldots x_{N1})$$

only depends on relative separations

f is translational invariant

but also independent of overall orientation

$$< \delta(\vec{x}_1)\, \delta(\vec{x}_2) > = \xi(\vec{x}_2 - \vec{x}_1) = \xi(r_{12})$$

$\hookrightarrow r_{12} = |\vec{x}_2 - \vec{x}_1|$

homogeneity

isotropy.

$$< \delta(\vec{x}_1)\, \delta(\vec{x}_2)\, \delta(\vec{x}_3) > = f(x_{12}, x_{23}, x_{13})$$

9 variable

3 variable $\left\{ \begin{array}{l} \text{two sides} \\ 1 \text{ angle} \end{array} \right.$

In general one has connected and disconnected pieces

$$\langle A_1 A_2 \rangle = \langle A_1 \rangle\langle A_2 \rangle + \langle A_1 A_2 \rangle_c$$

things can have correlation due to intrinsic lower order "disconnected" pieces

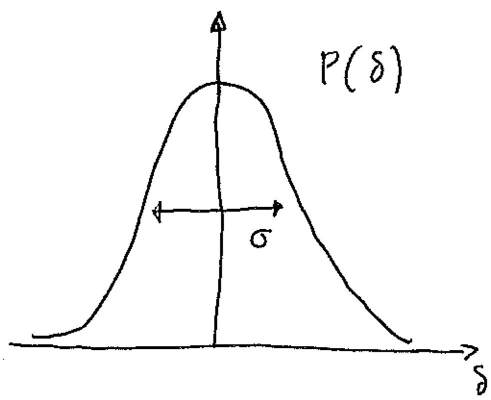
If we work with fluctuations $\Rightarrow$ $\langle A \rangle = 0$! But at 4 points

$$\langle A_1 A_2 A_3 A_4 \rangle = \langle A_1 A_2 \rangle\langle A_3 A_4 \rangle + \text{perm.} + \langle \quad \rangle_c$$

This is the NEW piece of information!

Let's talk now about ~~single~~ ~~point~~ <u>one-point correlations,</u>

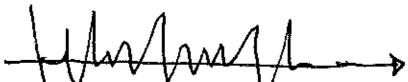
~~[illegible]~~ <u>1 point PDF</u>: Gaussian Fluctuations.



$$P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\delta^2}{2\sigma^2}\right].$$

where $\sigma^2 \equiv \langle \delta^2(x) \rangle = \int P_G(\delta)\, \delta^2 d\delta^2.$

There is equal probability of positive / negative fluctuation!



$\boxed{\text{Gaussian Field}}$ $\langle \delta^3 \rangle = 0$ $\quad \langle \delta^4 \rangle = 3 \langle \delta^2 \rangle^2$

All connected are zero if $N > 2$!

two - point correlations

in configuration space is simply defined as

$$\boxed{\xi(r) \equiv \langle \delta(\vec{x})\, \delta(\vec{x}+\vec{r}) \rangle}$$

→ Mention physical interpretation in page 20

if we now transform to Fourier space

$$\delta(\vec{x}) = \int d^3k \;\; \delta(\vec{k})\; e^{i\vec{k}.\vec{x}}$$

Notice that since $\delta(\vec{x})$ is real → $\delta(\vec{k}) = \delta^*(-\vec{k})$.

$$\langle \delta(\vec{k})\, \delta(\vec{k}') \rangle = \int \frac{d^3x}{(2\pi)^3} \frac{d^3x'}{(2\pi)^3}\; e^{-i\vec{k}.\vec{x}}\; e^{-i\vec{k}'.\vec{x}'} \langle \delta(\vec{x})\, \delta(\vec{x}') \rangle$$

$$\Downarrow$$

$$\vec{x}' = \vec{x} + \vec{r}$$

$$= \int \frac{d^3x}{(2\pi)^3} \frac{d^3x'}{(2\pi)^3}\; e^{-i\vec{k}.\vec{x} - i\vec{k}'(\vec{x}+\vec{r})} \;\; \xi(r)$$

$$= \int \frac{d^3r}{(2\pi)^3} \;\underbrace{\left\{ \int d^3x\; e^{-i(\vec{k}+\vec{k}').\vec{x}} \right\}}_{\delta(\vec{k}+\vec{k}')}\; \xi(r)\; e^{-i\vec{k}.\vec{r}}$$

$$\boxed{\langle \delta(\vec{k})\, \delta(\vec{k}') \rangle \equiv P(k)\;\delta(\vec{k}+\vec{k}')}$$

due to translation invariance $\vec{r} \to \vec{r} + \vec{\lambda}$

$\delta_k \to \delta_k\, e^{-i\vec{k}\vec{\lambda}}$

If we now go back to PDF and consider the 2-point PDF

$$P_G(\delta_1, \delta_2) = \frac{1}{\sqrt{(2\pi)^2(\sigma^2 - \xi^2)}} \exp\left[-\frac{1}{2} \quad \vec{\delta}\, C^{-1}\, \vec{\delta}\right]$$

$$\vec{\delta} = (\delta(x_1), \delta(x_2)) \quad \text{and} \quad \mathbb{C}_x = \begin{pmatrix} \sigma^2 & \xi \\ \xi & \sigma^2 \end{pmatrix}$$

We can now of course extend this to N-point

$$P_G(\delta_1 \text{---} \delta_N) = \frac{1}{(2\pi)^{N/2}} \frac{1}{(\det \mathbb{C})^{1/2}} \exp\left[-\frac{1}{2}\ \vec{\delta}\, C^{-1}\, \vec{\delta}\right].$$

For <u>Gaussian</u> field $\mathbb{C}_{ij}$ only depends on ξ!

In $\boxed{\text{Fourier Space}}$ the picture is much more simple b/c Fourier modes are independent $\rightarrow$ $\boxed{\text{Wick Theorem}}$

$$\begin{cases} \langle \delta(k_1)\,\delta(k_2) \rangle = \delta_D(k_{12})\, P(k_1) \\ \langle \delta(k_1)\,\delta(k_2)\,\delta(k_3) \rangle = 0 \\ \langle \delta(k_1)\,\delta(k_2)\,\delta(k_3)\,\delta(k_4) \rangle = \{P(k_1)\,P(k_2) + \text{perm.}\}\ \delta_D(k_{1-4}) \end{cases}$$

$$P_G(\delta(\vec{k}) \dots) = \prod_{\vec{k}} \frac{1}{\sqrt{2\pi P(k)}} \exp\left\{\frac{-|\delta|^2}{2P(k)}\right\}!$$

In particular $\sigma^2 = \langle \delta^2 \rangle = \int e^{i\vec{k}_1\cdot\vec{x}}\, e^{i\vec{k}_2\cdot\vec{x}}\, d\vec{k}_1\, d\vec{k}_2\, P(k)\, \delta_D(k_{12})$

$$\boxed{\sigma^2 = \int d^3k\, P(k) = \int d\ln k\, \Delta(k) \quad \text{and of course} \quad \sigma^2 \equiv \xi(0)}$$

In the first classes we found that linear evolution is <u>local</u> on top of linear

$$\delta(\vec{x}\ \tau) = D^{n}(\tau)\ \delta(\vec{x}\ \tau=0) \quad (*)$$

actually a_{DEC}

~~and this linear & local~~

instead of $\delta(\vec{x}\ \tau) = D(\tau) \int \delta(x-x')\ \delta(x')\ d^3x'\ F(x, x')$

And this linear / local evolution is valid well after decoupling; e.g. up to $z \sim 10$ (from $z \sim 1000$!).

<u>Linear Evolution preserves Gaussianity</u> ✓

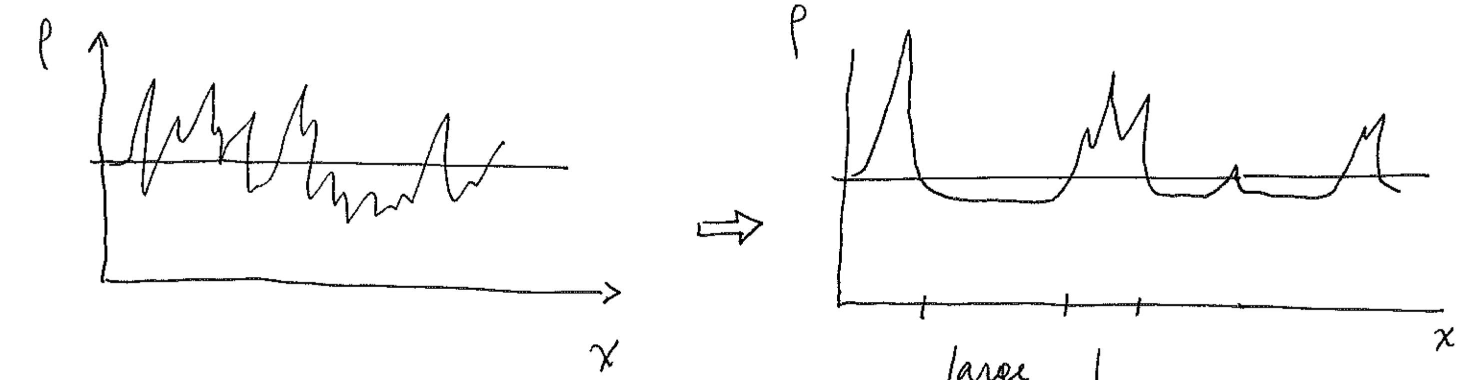
Recall that

$$\rho = \bar{\rho}\ (1+\delta)$$

so $\delta \geq -1$ by definition. In the early universe fluctuations are very small $\sigma \sim 10^{-5}$ → so $P(\delta < -1) \sim \exp\left(-\frac{1}{10^{-10}}\right) \sim \exp(-10^{10}) \sim 0$ the probability of having zero ~~negative~~ densities is zero in Gaussian fluctuation ✓

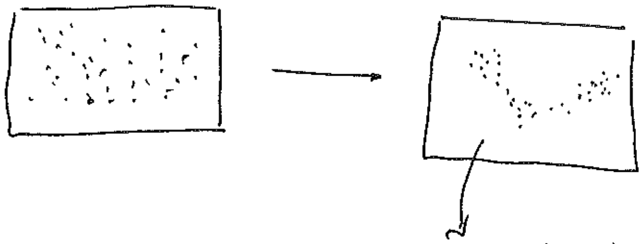
However gravitational evolution distorts this picture! As δ grows $\sigma \longrightarrow 1$ and Prob $(\delta \sim -1)$ is non-negligible → Problems with Gaussian stat.

Following grav collapse and by just looking at the Universe we see large asymmetries between $\delta > 0$ and $\delta < 0$

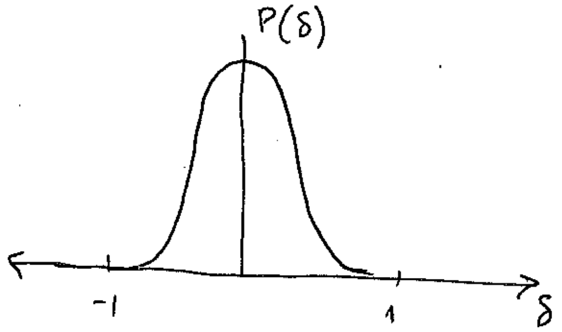


large voids

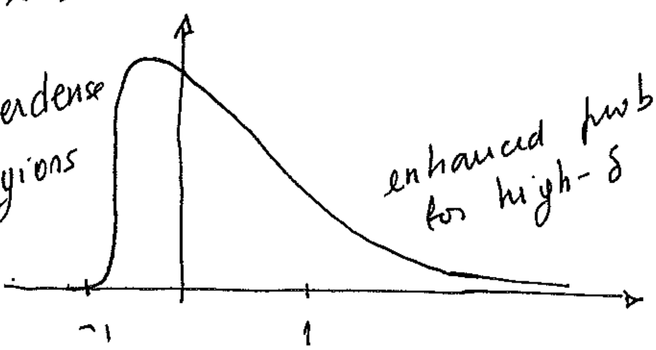
large overdensities in localized regions.



most of the Universe is empty



MAX shift to underdense regions



enhanced prob for high-δ

σ is also larger!

Deviations from Gaussianity can be understood expanding the PDF about a Gaussian + derivatives

$$P(\delta)\, d\delta = \phi(\nu)\, d\nu \qquad \nu \equiv \delta/\sigma$$

change of variables.

$$\phi_0(\nu) = \frac{1}{\sqrt{2\pi}}\, e^{-\nu^2/2}.$$

$$\phi(\nu) = \sum \frac{c_n}{n!} \frac{d^n \phi_G}{d\nu^n}$$

one can use the following property $\frac{d^n \phi_G}{d\nu^n} = (-1)^n H_n(\nu)\, \underbrace{e^{-\nu^2/2}}_{\phi_G(\nu)}$.

H_n are Hermite polynomials ; so

$$\phi(\nu) = \sum \frac{c_n}{n!} (-1)^n H_n(\nu)\, \phi_G(\nu)$$

$$c_n = (-1)^n \int \phi(\nu) H_n(\nu)\, d\nu = (-1)^n \langle H_n(\nu) \rangle$$

$H_0 = 1 \qquad H_2 = \nu^2 - 1$

$H_1 = \nu \qquad H_3 = \nu^3 - 3\nu$

$c_0 = 1$

$c_2 = \langle \nu^2 \rangle - 1 = \frac{1}{\sigma^2} \langle \delta^2 \rangle - 1 = 1 - 1 = 0$

$c_3 = -\langle \nu^3 - 3\nu \rangle = -\frac{1}{\sigma^3} \langle \delta^3 \rangle + \frac{3}{\sigma^3} \underbrace{\langle \delta \rangle}_{=0} = -\frac{\langle \delta^3 \rangle}{\sigma^3}$.

$c_5 = -\frac{\langle \delta^5 \rangle}{\sigma^5}$

$c_6 = \frac{\langle \delta^6 \rangle_c}{\sigma^6} + 10 \left[\frac{\langle \delta^3 \rangle}{\sigma^3} \right]^2$ $\qquad \langle \delta^N \rangle$ cumulants.

For n odd $\quad \langle \delta^n \rangle = \langle \delta^n \rangle_c$

We define

$$\boxed{S_p \equiv \frac{\langle \delta^n \rangle_c}{(\sigma^2)^{n-1}}}$$

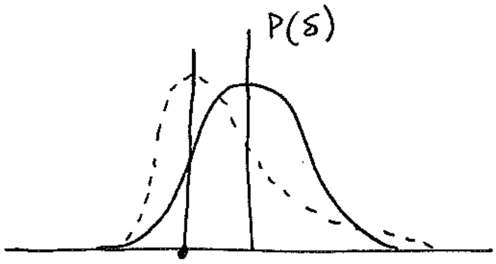
$\langle \delta^2 \rangle$

so for Gaussian $S_2 = 1$ and $S_n = 0$ $n>2$.

And you get to $\phi(\nu) \longrightarrow P(\delta)$.

$$P(\delta) = P_G(\delta)\left[1 + \frac{S_3\sigma}{3!}H_3(\nu) + \sigma^2\left[\frac{S_4}{4!}H_4(\nu) + \frac{10}{6!}S_3^2 H_6(\nu)\right] + \dots\right].$$

"Edgeworth expansion".

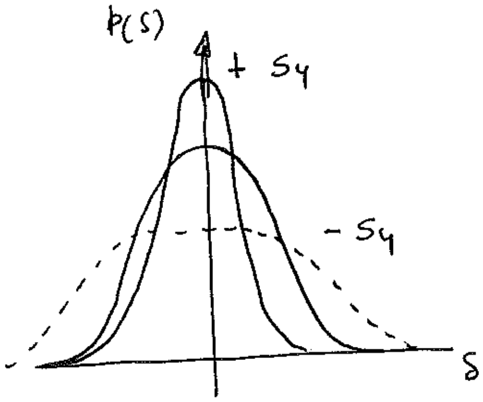


+ skewed.

$\boxed{S_3 \text{ is the skewness}}$

it measures shifts of the peak. $\delta_{max} = -\frac{S_3\sigma^2}{2}$

neglecting terms of δ^2



$\boxed{S_4 \text{ is the kurtosis}}$

it measures ~~the dependence~~ how peaked is the distribution → width

$$S_4 = \frac{\langle \delta^4 \rangle_c}{\langle \delta^2 \rangle^3}$$

Skewness and Kurtosis generated by gravity

$$\langle \delta^3 \rangle = \langle (\delta_1 + \delta_2 + \dots)^3 \rangle$$

$$= \langle (\delta_1)^3 \rangle + 3 \langle \delta_2 \delta_1 \delta_1 \rangle$$

this was Gaussian so $\langle \delta^3 \rangle = 0$

$$\langle \delta_1^3 (x) \rangle = \langle \delta_1(x)\, \delta_1(x)\, \delta(x) \rangle$$

$$= \int d^3k_1\, d^3k_2\, d^3k_3 \langle \delta(k_1)\, \delta(k_2)\, \delta(k_3) \rangle e^{i(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\cdot \vec{x}}$$

if we take all 3 as linear $\Rightarrow \langle \delta^3 \rangle = 0$

$$\delta_1(\vec{k}) = D\, \delta_0(k) \quad \text{for } k_1, k_2$$

$$\delta(k_3) = D^2 \int F_2(\vec{q}_1, \vec{q}_2)\, \delta_0(q_1)\, \delta_0(q_2)\, d^3q\, d^3q\, \delta_D(k-q)$$

$$\langle \delta^3 \rangle = 3 D^4 \int d^3k_1\, d^3k_2\, d^3q_1\, d^3q_2\, e^{i(k_1 + k_2 + q_1 + q_2)}\, F_2(q_1, q_2)$$

$$\langle \delta(k_1)\, \delta(k_2)\, \delta(q_1)\, \delta(q_2) \rangle$$

this pairing leads to $P(k_1)\, \delta_D(\vec{k}_1 + \vec{q}_1) \times P(k_2)\, \delta_D(\vec{k}_2 + \vec{q}_2)$

this is zero $\langle \delta_2 \rangle$

$k_1 \to q_1$
$k_1 \to q_2$

$$\langle \delta^3 \rangle = 6 D^4 \int F_2(\vec{q}_1, \vec{q}_2)\, P(q_1)\, P(q_2)\, d^3q_1\, d^3q_2$$

And the $\vec{x}$-dependence went away

$$\langle \delta^3 \rangle = 6\, D^4 \int P(q_1)\, q_1^2\, dq_1^3 \int P(q_2)\, q_2^2\, dq_2^3 \, .$$

$$\int F_2(q_1, q_2)\, d\Omega_1\, d\Omega_2 \, .$$

$$F_2 = \frac{5}{7} + x\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right) + \frac{2}{7}\, x^2$$

So ; let's take $\vec{q}_1$ in $\hat{z}$ direccion $\vec{q}_1 \cdot \vec{q}_2 = q_1 q_2 x$

$$d\Omega_2 \rightarrow (4\pi) \qquad d\Omega_1 \longrightarrow 2\pi\, dx = 4\pi\, \frac{dx}{2}$$

$$(4\pi)^2 \int_{-1}^{1} \frac{dx}{2}\, F_2(x) = (4\pi)^2 \left[\frac{5}{7} + \frac{2}{7}\cdot\frac{1}{3}\right] = (4\pi)^2\, \frac{17}{21}$$

We get

$$\langle \delta^3 \rangle = 6 \cdot \frac{17}{21} \Big[\underbrace{D^2 \int P(k)\, 4\pi k^2 dk}_{= \sigma^2}\Big]^2 = \frac{34}{7} \langle \delta^2 \rangle^2$$

$$\boxed{S_3 \equiv \frac{\langle \delta^3 \rangle}{\langle \delta^2 \rangle^2} = \frac{34}{7} > 0 \, !}$$

Similarly

$$S_4 = \frac{\langle \delta^4 \rangle_c}{\langle \delta^2 \rangle^3} = \frac{60712}{1323} > 0 \, !$$

~~Will I talk of ξ Cn or Sn? Sp & hierarchical ratios.~~

~~→ Me quede hablar de NL corrections to P(k); bispectrum; the Gaussian dist;~~

Non-linear evolution ⟶ non Gaussian features

All N-point correlators are needed!

Gravitational clustering from Gaussian IC. is special.

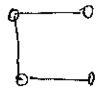
$$\boxed{\begin{array}{l}\xi_N \propto \xi_2^{N-1}\\ \text{at large scales}\end{array}}$$

$\boxed{\text{b/c nonlinearities are quadratic}}$ ~~2nd order~~. → leads to hierarchical clustering
S_p are numbers.

Fry (1984).

$$Q_N \equiv \frac{\xi_N}{\Sigma \prod \xi_2(r_{ij})}$$

hierarchical amplitudes.



Q_N is independent of the amplitude of the 2-point CF.

e.g.

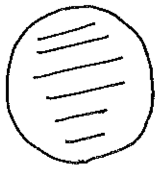
$$Q_4 = \frac{\xi_4}{\xi_2^3} = \frac{\xi_4}{\xi_2(r_{12})\,\xi_2(r_{13})\,\xi_2(r_{32})}$$

⇨ Q_N and their one-point cousins S_p parameters are the most natural set of statistics to describe the non-Gaussianity that results from grav. clustering!

The $\boxed{\text{hierarchical scaling}}$ is expected to work also in the strongly non-linear regime.

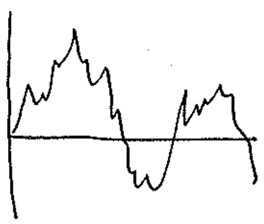
The case of smoothed density field.

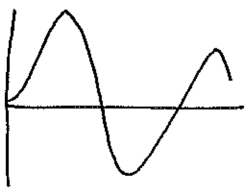
In reality one works w/ smoothed density fields b/c fluctuations would diverge



$$\delta_R(\vec{x}) \equiv \int \delta(\vec{x}')\, W_R(\vec{x}-\vec{x}')\, d^3x'$$

top-hat in real space. $\longrightarrow$ $W_R(\vec{r}) \begin{cases} 1 & \text{if } r \lesssim R \\ 0 & \text{otherwise} \end{cases}$





all high frequency mode are washed-out

$kR > 1 \qquad k \gtrsim 1/R$

Eq (*) in Fourier is just $\delta_R(\vec{k}) = W(kR)\, \delta(\vec{k})$

$$W(x) = \frac{3}{x^3}(\sin x - x\cos x)$$ Fourier Transf. of W_R

so

$$P_R(k) = W^2(kR)\, P(k)$$

$$\sigma^2(R) = \int d^3k\, P_R(k) = \int d^3k\, P(k)\, W^2(kR)$$

$\rightarrow$ it's often used to normalize the amplitude of P ($\sigma_8 \sim 0.8$) $\xleftarrow{}$ today

For smoothed fields

$$\boxed{S_3 = \frac{34}{7} + \frac{d\ln\sigma^2(R)}{d\ln R}}$$

Back to power spectrum and bispectrum

- Recall the physical interpretation of the CF

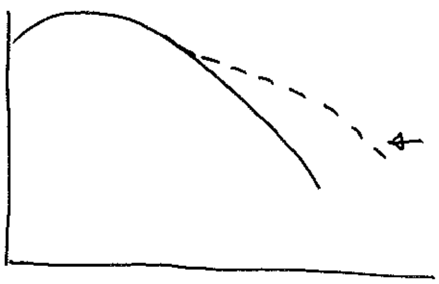
$\xi(r)$ measures the excess "over random" probability that two particles at volumes dV_1 and dV_2 are separated by distance x_{12}

$$dP_{12} = \langle n_1 dV_1 \, n_2 dV_2 \rangle = \langle \bar{n}(1+\delta_1) \, \bar{n}(1+\delta_2) \rangle \, dV_1 dV_2$$

$$= \bar{n}^2 \left(1 + \langle \delta_1 \delta_2 \rangle\right) dV_1 dV_2$$

$$\boxed{dP_{12} = \bar{n}^2 \{ 1 + \xi(r_{12}) \} \, dV_1 dV_2}$$

Measuring ξ :
$$\hat{\xi} = \int \frac{d\Omega_r}{4\pi} \int d^3r' \; \delta(r'+r) \, \delta(r)$$



← means that at high-k (small scales) things are more clustered.

$$dP(2|1) = n \left(1 + \xi(r_{12})\right) dV_2$$

the enhancement comes from NL gravity!

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)}$$

$$< \delta(\vec{k}) \delta(\vec{k}') > = \underbrace{< \delta^{(1)} \delta^{(1)} >}_{= P_L(k)} + \underbrace{\langle \delta^{(1)} \delta^{(2)} \rangle}_{=0} + < \delta^{(1)} \delta^{(3)} > 2 + \langle \delta^{(2)} \delta^{(2)} \rangle$$

$$= P_L(k) + \{ P_{13} + P_{22} \}$$

$$P_{22} = 2 \int [F_2(\vec{k}, -\vec{k} + \vec{q})]^2 \, P(q) \, P(|k-q|) \, d^3q$$

$$P_{13} = 6 \int F_3(\vec{k}, \vec{q}, -\vec{q}) \, P(k) \, P(q) \, d^3q$$

→ $\boxed{\text{show \underline{slide}}}$

Maybe more interesting is the 3 point function (in Fourier space is called bispectrum)

$$< \delta(\vec{k}_1) \, \delta(\vec{k}_2) \, \delta(\vec{k}_3) > \equiv B(\vec{k}_1, \vec{k}_2, \vec{k}_3) \, \delta_D(k_{123})$$

At linear order is zero; at first NL one

$$\delta_1 \rightarrow \delta^{(1)}(k_1) \qquad \delta_2 - \delta^{(1)}(k_2) \qquad \delta_3 \rightarrow \delta^{(2)}(k_3)$$

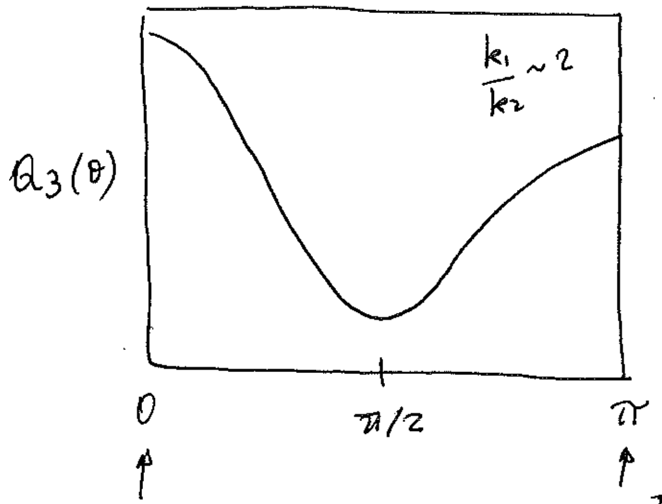
$$\delta^{(2)} = \int F_2(\vec{q}_1, \vec{q}_2) \, \delta_0(q_1) \, \delta_0(q_2) \, \delta_D(\vec{k}_3 - \vec{q}_{12})$$

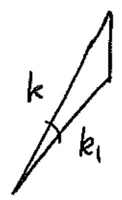
$$B(\vec{k}_1, \vec{k}_2, \vec{k}_3) = 2 F_2(\vec{k}_1, \vec{k}_2) \, P(k_1) P(k_2) + \text{cyclic}$$

Defining the reduced bispectrum

$$Q(k_1, k_2, k_3) = \frac{B(\vec{k}_1, \vec{k}_2, \vec{k}_3)}{P(k_1) P(k_2) + \ldots \text{cyclic}}.$$

one finds tipically





~~the~~ elongated configurations are more "probable"!

The configuration dependence of the bispectrum (which enhance correlations for colinear wave-vectors) reflects the anisotropies of structures and flows generated by gravity $(\vec{v} \cdot \vec{\nabla}) \delta$ $(\vec{v} \cdot \vec{\nabla}) \vec{v}$

Gravity generates density and velocity perturbations mostly parallel to the flow $\Rightarrow$ filaments

$$F_2(\vec{k}_1, \vec{k}_2) = \frac{5}{7} + \frac{\vec{k}_1 \cdot \vec{k}_2}{k_1 k_2} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) + \frac{2}{7} \chi^2$$

BAO :

coupled plasma (proton + e⁻) ionized H.

Initially baryons and photons are tightly coupled through Compton scattering;

high pressure drives the gas + photon outwards at speeds close to c. → this expansion continues for over 10^5 years ; after that time universe have cooled enough that protons capture electrons to form neutral H. This decouples the photon from the baryons; and γ stream away.

The baryons remain overdense $c_s t$. Now gravity takes over and the central DM potential well drives baryons back. Perturbations grow by $O(1000)$.

Baryons and DM reach equilibrium in $\frac{\Omega_b}{\Omega_m} \sim \frac{0.044}{0.25}$

(baryons are only 10% of DM) ⇒ this effect will be tiny. The radious of the outer shell is called <u>sound horizon</u>

$\boxed{\text{Show slide}} \Rightarrow \boxed{\text{BAO detection by Eisenstein}}$

$$c_s = \left(\frac{\delta p}{\delta \rho}\right)_s \qquad \lambda_p = a\,\lambda_{com}$$

$$\underset{\substack{\uparrow\\ \text{comoving}}}{r_s} = \int_0^{t_{rec}} c_s \underset{\nwarrow\ \text{comoving}}{\frac{dt}{a}} = \int_{z_{rec}}^{\infty} \frac{c_s(z)}{H(z)}\,dz$$

$$a = \frac{1}{1+z} \qquad dt\,\overbrace{\frac{da}{dt}}^{Ha} = -\frac{1}{(1+z)^2}\,dz = -$$

~~c_s~~ ~~etc~~

$$aH\,dt = -\frac{1}{(1+z)^2}\,dz$$

$$H(z) = H_0\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3} \qquad \cancel{a}H\,dt = -a^{\cancel{2}}\,dz \longrightarrow \frac{dt}{a} = \frac{dz}{H}$$

Now

$$c_s^2 = \frac{\delta P_{rad} + \cancel{\delta P_b}^{\approx 0}}{\delta \rho_{rad} + \delta \rho_b} = \frac{\delta P_{rad}/\delta\rho_{rad}}{1 + (\delta\rho_b/\delta\rho_{rad})}$$

$$P_{rad} = \frac{c^2}{3}\rho_{rad} \qquad \boxed{c_s^2 = \frac{c^2/3}{1 + \delta\rho_b/\delta\rho_{rad}}}$$

Now $\rho_b \sim \frac{1}{a^3}$ $\quad \rho_{rad} \sim \frac{1}{a^4}$ $\quad \frac{\delta\rho_b}{\rho_b} = -3\frac{\delta a}{a}$ $\quad \frac{\delta\rho}{\rho} = -4\frac{\delta a}{a}$

$$\frac{\delta\rho_b}{\delta\rho_{rad}} = \frac{3}{4}\frac{\rho_b}{\rho_{rad}} = \frac{3}{4}\frac{\Omega_b}{\Omega_{rad}}\frac{1}{1+z} \checkmark$$

$$\boxed{c_s = \frac{c}{\sqrt{3(1+R)}} \qquad R = \frac{3}{4}\frac{\rho_b}{\rho_\gamma}}$$

$$c_s(z) = \frac{c}{\sqrt{3}\left[1 + \dfrac{3\Omega_b/4\Omega_{rad}}{1+z}\right]^{1/2}}$$

Replacing this into Eq (*) one obtains

$$\boxed{r_s = \frac{c}{\sqrt{3}} \frac{1}{H_0 \Omega_m^{1/2}} \, 0.0313 \approx 108.6 \text{ Mpc}/h.}$$

using $z_{rec} \approx 1100$ $\Omega_b = 0.046$ $\Omega_m = 0.25$ $h = 0.7$

$\Omega_{rad} \approx 4.2 \times 10^{-5}$

We should find an excess correlation at this scale!

$\boxed{r_s \text{ only depends on } \Omega_m h^2 \text{ and } \Omega_b h^2}$

$\Omega_m h^2$ → set the matt rad equality

$\Omega_b h^2$ → baryon ~~the~~ to photon ratio

$$r_s = \frac{c}{\sqrt{3}} \frac{1}{\Omega_m^{1/2} H_0} \cdot \frac{2}{\sqrt{z_{eq} R_{eq}}} \left\{ \ln \frac{\sqrt{1+R_{rec}} + \sqrt{R_{rec}+R_{eq}}}{1+\sqrt{R_{eq}}} \right\}$$

But the most important think is that this scale can be very well calibrated from CMB (it's the same physics).

Galaxies form in matter (DM + baryon) overdensities. Most of the galaxies will be at the original perturbation position but there is a 1% enhancement of galaxies at the acoustic scale ⟹ it should ~~pop~~ show up in the CF!

BAO in Fourier Space

Evolution of baryon fluid with non-zero pressure

$$\frac{\partial^2 \delta_k}{\partial \tau^2} + \mathcal{H}\frac{\partial \delta_k}{\partial \tau} = \frac{3}{2}\Omega_m \mathcal{H}^2 \delta_k - k^2 \frac{p_k}{\rho}$$

Let's look at a simplified picture ⇒ full calculation should include photon perturbations and the coupled γ-baryon fluid.

$$-k^2\frac{p_k}{\rho} \longrightarrow \frac{\nabla^2 \delta p}{\rho} = c_s^2 \frac{\nabla \delta \rho}{\rho} \longrightarrow -k^2 c_s^2 \delta_k$$

where we assumed that pressure is a function of ρ alone.

$$\frac{\partial^2 \delta_k}{\partial \tau} + \mathcal{H}\frac{\partial \delta_k}{\partial \tau} = \left(\frac{3}{2}\Omega_m \mathcal{H}^2 - k^2 c_s^2\right)\delta_k$$

so we define Jeans wavenumber such that

$$k_J^2 c_s^2 = \frac{3}{2}\Omega_m \mathcal{H}^2$$

Jeans scale $\lambda_J = \frac{2\pi}{k_J}$ one can show that $\frac{\lambda_J}{c_s} \sim H^{-1}$.

scales larger than Jeans scale grow by gravity as usual

scales smaller than Jean scale are suppresed due to pression.

Simple case; in the absence of expansion

$$\ddot{\delta}_k = -c_s^2 k^2 \delta_k \longleftarrow \text{harmonic oscillator}$$

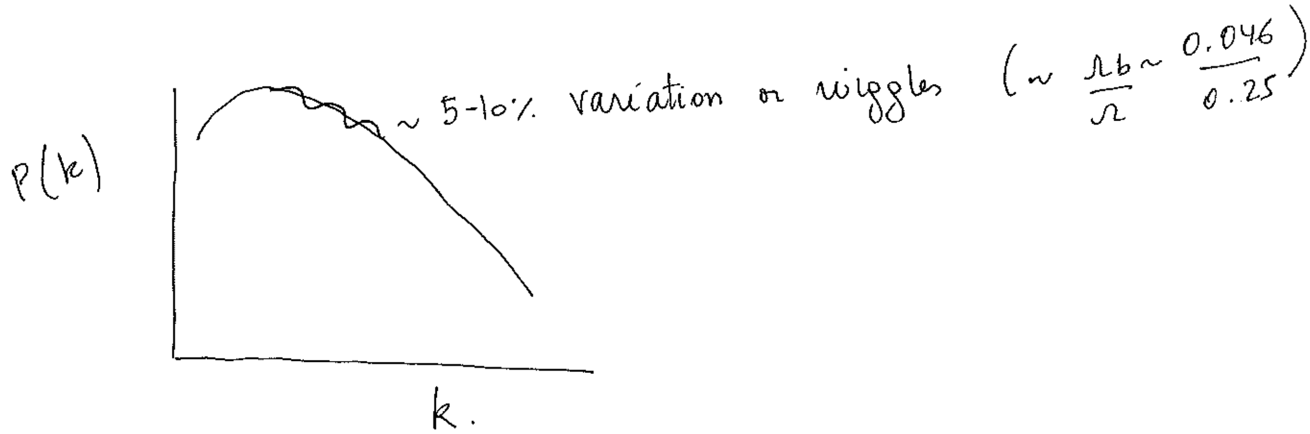
leads to acoustic or standing wave oscillations in δ_k !

Jean scale for baryon - photon fluid is close to horizon size at that time (b/c $c_s \propto c$) $\rightarrow$ this will show up at about the same scale at k_{break} ! } !!

$$T(k) = \frac{\Omega_b}{\Omega_0} T_b(k) + \frac{\Omega_{CDM}}{\Omega_0} T_{CDM}(k)$$

$$T_b(k) \longrightarrow \alpha_b \frac{\sin(ks)}{k \cdot s}$$

s : sound horizon ! $\sim \int c_s \, dt \, (1+z)$
(comoving)



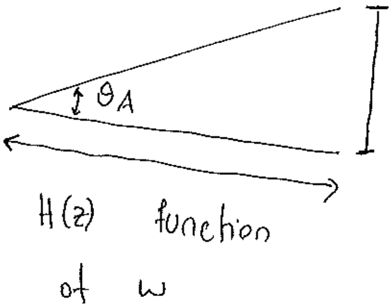
Best Observational Probes

- DE changes distances (e.g. background cosmology) but also growth w/ perturbations

1) Weak Lensing (Geom + Growth)
2) BAO (Geom)
3) Supernova (Geom)
4) Clusters (Growth + Geom).

Degeneracy w/ CMB and Dark Energy

Problem is that DE only happens at low - z so it only affects distances. CMB is 2D, just an angle on sky



$$r_s = r_s(\Omega_m h^2, \Omega_b h^2)$$

I can change $\Omega_m h^2$ and $\Omega_b h^2$ to increase r_s but then more w to dilute $H(z)$ and get the same θ_A

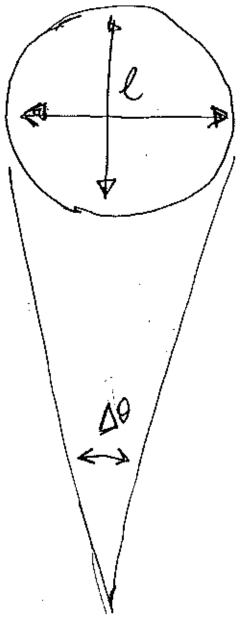
Probing dark energy

Through $H(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_{rad} (1+z)^4 + \Omega_k (1+z)^2 + \Omega_{DE} (1+z)^{3(1+w)} \right]^{1/2}$

$\frac{p}{\rho} \equiv w$

Probes
- Geometric : through exp rate. (supernova ; BAO ; clusters)
- Growth of structure : Lensing ; RSD. → Pablo

Standard ruler test



You know the size of the "ruler" $\Rightarrow$ you can get the angular diameter distance d_A

$$\Delta\theta = \frac{\ell}{d_A (1+z)}$$

$$d_A = \frac{r(z)}{1+z} \qquad r(z) = \int_0^z \frac{dz'}{H(z')}$$

w!

But it could potentially be used also along the line of sight

$$\ell = \frac{c \, \Delta z}{H(z)}$$ to yield $H(z)$ directly

The problem is that the ruler is very large! you need huge volumes to get an accurate measure