Galaxy Bias

- One problem when interpreting galaxy surveys is that galaxies might be a "biased" realizatioon of the DM field.

- Typically one assumes

$$\delta_g = b\ \delta(\vec{x}) \quad \leftarrow \text{overdensities are proportional.}$$

$$\downarrow$$

$$P_g = b^2\ P(k)$$

- This might be valid on large Gaussian scales but NOT when non-gaussianities start to kick in.

- One would think that for any given cosmological model a hydrodynamical simulation could tell us the relation $g \leftrightarrow m$. But the process of galaxy formation is far from being understood so one needs to introduce phenomenological parameters → "bias parameters".

- 3 ways forward → hydro simulations (costly / unknown physics)

  → mapping of $\delta_m \longrightarrow \delta_g$ (bias parameters)

  → let dark-matter collapse into halos which is a gravitational problem; place galaxies into those halos w/ some assumptions to avoid complex gal. formation physics.

Clustering of tracers

- It's being known for a long time that the clustering of objects such as galaxies of different types, clusters, etc vary ⇒ [illegible] it depends on the object ⇒ clustering is not universal and it's not [illegible] directly the same as DM. ⇒ this is called "bias"

Simplest model is "local bias"

Let's assume we smooth both the DM and tracer field on some scale such that fluctuations are small ⇒ local bias assumes

$$\delta_g(\vec{x}) = f\{\delta(\vec{x})\} \qquad \leftarrow\!| \ \text{Eq}(*)$$

Since fluctuations are small we can Taylor expand

$$\delta_g = \sum_{k=0}^{\infty} \frac{b_k}{k!} \delta^k = b_1 \delta + \frac{b_2}{2} (\delta^2 - \sigma^2)$$

↑ such that $\langle \delta_g \rangle = 0$

We can then calculate

$$\langle \sigma_g^2 \rangle = b_1^2 \langle \sigma^2 \rangle$$

$$\langle \delta_g^3 \rangle = b_1^3 \langle \delta^3 \rangle + \frac{b_2}{2} 3b_1^2 \langle (\delta^2 - \sigma^2) \delta^2 \rangle$$

$$\downarrow$$

$$3\sigma^4 - \sigma^4 = 2\sigma^4$$

↑ all possible combinations of $\langle \delta\delta\delta\delta \rangle$

$$\langle \delta_g^3 \rangle = b_1^3 \langle \delta^3 \rangle + 3b_1^2 b_2 \sigma^4$$

$$S_{3g} = \frac{b_1^3 \langle \delta^3 \rangle + 3 b_2 b_1^2 \sigma^4}{b_1^4 \sigma^4} = \frac{1}{b_1} S_3 + 3 \frac{b_2}{b_1^2}$$

or in Fourier Space

$$P_g(k) \approx b_1^2 P(k)$$

$$B_g(k_1, k_2, k_3) = b_1^3 B(k_1, k_2, k_3) + \frac{b_2}{b_1^2} \{ P(k_1) P(k_2) + cyc \}$$

$$Q_g(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{1}{b_1} Q(k_1, k_2, k_3) + \frac{b_2}{b_1^2}$$

The last expression shows how ~~can~~ one can measure bias ⇒

Measure Q_g ; predict Q (matter) ⇒ this in independent of σ_8!

Measure b_1, b_2.

Otherwise note that $P_g(k) = b_1^2 P(k) \sim b_1^2 \sigma_8^2 P(k, z=0)$

$\downarrow$

but $P \sim D^2$ growth factor

or in other words σ_8

↦ Perfect degeneracy between b_1 and σ_8.

Eq(*) assumes <u>not</u> only that bias is <u>local</u> but also <u>deterministic</u>

In practice is likely that ~~bias~~ galaxy formation depends on ~~other~~ variables other than density (e.g. halo formation time)

Stochasticity

so

$$\hat{\delta}_g(\vec{x}) = \mathcal{F}\{\delta(\vec{x})\} + \epsilon_\delta(\vec{x})$$

such that $\langle \delta \epsilon \rangle = 0$

In general one measures the cross-correlation coefficient

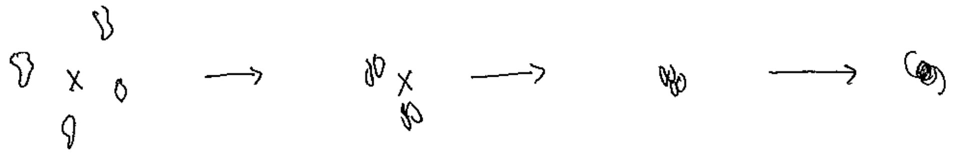
$$\delta_g = b_1 \delta + \epsilon \quad \text{and} \quad \delta$$

$$r = \frac{\langle \delta_g \delta_m \rangle}{\sqrt{\langle \delta_g \delta_g \rangle \langle \delta_m \delta_m \rangle}} = \frac{b_1 \delta^2}{\sqrt{(b_1^2 \delta^2 + \epsilon^2)\, \delta^2}} = \frac{1}{\sqrt{1 + P\epsilon / P_{gg}}}$$

On large scale $r \to 1$ typically ~~but~~ ($r < 1$ in general). For the combination of WL and Clustering we need $r = 1$.

Limits of local bias picture: Lets assume that at time t_* galaxies do follow local bias ⇒ what happens in the future?

In order to solve this let's think on the mass constituent of a galaxy and trace it's center of mass so number of objects in conserved.



Follow x ⇒

$$\frac{\partial \delta_g}{\partial \tau} + \vec{\nabla} \cdot \{(1 + \delta_g)\, \vec{v}_g\} = 0$$

Let's further assume that

→ $v_g = v_m$ since by Galilean Invariance this has to be the case at least for large scales.

→ Under these conditions one can show

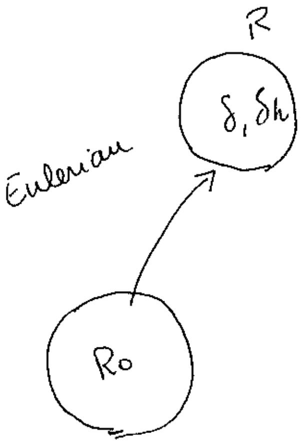
$$b_1 = 1 + \frac{b_1^* - 1}{D/D_*}$$

So → local bias is preserved on linear scales. At 2nd order this is NOT true anymore. Gravity generates non-local terms

$$\delta_g(x) = \int \underset{\substack{\downarrow \\ \text{non-linear}}}{\mathcal{F}\{\delta(x')\}} \; \underset{\substack{\downarrow \\ \text{non-local}}}{K(\vec{x}-\vec{x}')} \, d^3x$$

Halo bias

We are interested in the following problem



Region of comoving size R with some enclosed δ overdensity (nonlinear). This originated from some fluctuation at original comoving volume R_0 (recall from spherical collapse $R = R_0 (1+\delta)^{-1}$ the comoving size shrinks for $\delta > 0$).

This region will have some overdensity of halos δ_h that we want to relate to δ.

$$\frac{M}{V_\delta} = \bar{\rho}(1+\delta)$$

We want $\mathcal{N}(m|\delta)\,dm$: number of halos given the region is overdense. Recall the "standard" Press-Schechter formalism at $\delta = 0$

$$F(>M) = 2\,\mathcal{P}(\delta_m > \delta_c)$$: fraction of mass in halos more massive that m.

$$\frac{\partial F}{\partial m}(>m)\,dm$$: fraction of mass in halos $\{m, m+\delta m\}$.

$$\left(\begin{array}{c}\text{number of}\\ \text{halos with} > m\end{array}\right) \times \frac{m}{M} = \frac{\partial F}{\partial m}(>m)\,dm$$

← halo mass

↖ total enclosed mass

$W(m)\,dm$ = # of halos in the general case

$$W(m)\,dm = \frac{M}{m}\,\frac{\partial F(>m)}{\partial \sigma^2}\,\frac{\partial \sigma^2}{\partial m}\,dm$$

If we divide by the enclosed volume we get the standard mass function (i.e. # of halos per unit volume and mass)

$$n(m)\,dm = \frac{\bar{\rho}}{m}\,\frac{\partial F(>m)}{\partial \sigma}\,\frac{\partial \sigma}{\partial m}\,dm.$$

where since we are doing the standard case $\frac{M}{V} \equiv \bar{\rho}$

$$\frac{\partial F(>m)}{\partial \sigma} = 2\frac{\partial}{\partial \sigma}\mathcal{P}(\delta_m > \delta_c) = \frac{2}{2}\frac{\partial}{\partial \sigma}\,\mathrm{Erfc}\left(\frac{\delta_c}{\sqrt{2}\,\sigma}\right)$$

$$= \frac{-2}{\sqrt{\pi}}\,e^{-\delta_c^2/2\sigma^2}\left(\frac{-\delta_c}{\sqrt{2}\,\sigma^2}\right) = \sqrt{\frac{2}{\pi}}\,e^{-\delta_c^2/2\sigma^2}\,\frac{\delta_c}{\sigma^2} \checkmark$$

$$n(m)\,dm = \frac{\bar{\rho}}{m}\sqrt{\frac{2}{\pi}}\,e^{-\delta_c^2/2\sigma^2}\,\frac{\delta_c}{\sigma^2}\left|\frac{\partial \sigma}{\partial m}\right| dm$$

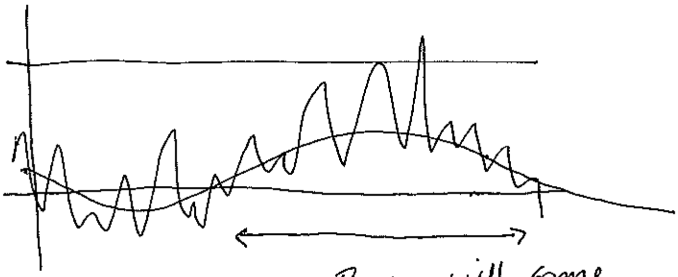
$$= \frac{\bar{\rho}}{m}\underbrace{\sqrt{\frac{1}{\pi 2}}\,e^{-\delta_c^2/2\sigma^2}\,\frac{\delta_c}{\sigma^3}}_{f(\nu)}\left|\frac{\partial \sigma^2}{\partial m}\right| dm$$

The extension to the conditional mass fraction can be done easily either using

$$f(\sigma, \delta_c \mid \sigma_0, \delta_0) = \frac{1}{\sqrt{2\pi}} \frac{\delta_c - \delta_0}{\{\sigma^2 - \sigma_0^2\}^{3/2}} \exp\left\{ -\frac{(\delta_c - \delta_0)^2}{2(\sigma^2 - \sigma_0^2)} \right\}.$$

Eq(551)
PT review

Here δ_0 is the linear extrapolation of the initial overdensity the in reality grew to become δ ; σ_0 it's variance. $\sigma \equiv \sigma(m)$ on halo-mass scale as before. Physically its easy to interpret using peak-background theory



Region will some small large scale over-density → the distance to the ~~bari~~ barrier for collapse is now less $\delta_c - \delta_0$. Region over large-scale overdensities collapse sooner => more halos

In our case we assume $\sigma_0 << \sigma(m)$ $\left\{ \sigma_0 \text{ variance over size } R = \frac{R_0}{1+\delta} \right\}$

We can write

$$n(m)\,dm = \frac{\bar{\rho}}{m}\, f(\delta_c,\sigma)\,\left|\frac{\partial\sigma^2}{\partial m}\right| dm$$

while now

$$n(m|\delta)\,dm = \frac{\bar{\rho}(1+\delta)}{m}\, f(\delta_c,\sigma|\delta_0)\,\left|\frac{\partial\sigma^2}{\partial m}\right| dm$$

the volume is smaller ; or at least a factor $(1+\delta)$ w.r.t the other case

Recall in this case $\frac{M}{V} = \bar{\rho}(1+\delta)$

$$n(m|\delta) = \frac{\bar{\rho}}{m}\,(1+\delta)\, f(\delta_c,\sigma|\delta_0)\,\left|\frac{\partial\sigma^2}{\partial m}\right| dm$$

Note : In PT review $(1+\delta)$ enters through the volume over $n(m)$ is averaged => same answer

one should expand $\delta_0 = \delta_0(\delta)$ using spherical collapse. To first order $\delta_0 = \delta$

$$\frac{n(m|\delta)}{n(m)} = (1+\delta)\,\frac{f(\delta=0) + \partial f/\partial\delta_0\,\delta}{f(\delta=0)}$$

$$1+\delta_h = (1+\delta)\left(1 - \frac{\partial \ln f}{\partial\delta_c}\,\delta\right)$$

$\delta_c - \delta_0$ so there is a change of sign

$$f = \frac{\delta}{\sigma^3} e^{-\delta^2/2\sigma^2}$$ Taking the derivative of the mass function..

$$\frac{\partial f}{\partial \delta_c} = \cancel{\frac{\delta}{\delta}} \frac{1}{\sigma^3} e^{-\delta^2/2\sigma^2} - \frac{\delta}{\sigma^3} e^{-\delta^2/2\sigma^2} \frac{\cancel{2}\delta}{\cancel{2}\sigma^2}$$

$$= \left(\frac{1}{\cancel{\delta}} - \frac{\delta^2}{\sigma^2} \right) \frac{1}{\sigma^3} e^{-\delta^2/2\sigma^2} = (1 - \nu^2) \frac{1}{\delta_c} f$$

$$1 + \delta_h = (1 + \delta) \left(1 - \left\{ \frac{1 - \nu^2}{\delta_c} \right\} \delta \right)$$

$$\cancel{1+} \delta_h = \cancel{1} - \left(\frac{1 - \nu^2}{\delta_c} \right) \delta + \delta - O(\delta^2)$$

$$\delta_h = \left\{ \underbrace{1 + \left(\frac{\nu^2 - 1}{\delta_c} \right)}_{b_h(m).} \right\} \delta$$

On large scales

$$\delta_h(m) = b(m)\, \delta$$

$$\boxed{b(m) = 1 + \frac{\nu^2 - 1}{\delta_c}}$$

$$\nu = \frac{\delta_c}{\sigma(m)}$$

↗ linearly extrapolated to

$$\nu(z) = \frac{\delta_c}{D(z)\, \sigma(m \,|\, z{=}0)}$$

Expanding $\delta = \delta(\delta_0)$ to 2nd, 3rd order etc one gets

$b_1(m)$ [DONE] $b_2(m)$ $b_3(m)$ etc.

If we use the ellipsoidal collapse multiplicity function

$$f_{ST}(\nu) = A_p \left\{ 1 + (q\nu^2)^{-p} \right\} \sqrt{\frac{q}{2\pi}}\ \nu\ e^{-q\nu^2/2}$$

Sheth and Tormen 1999

$p = 0$ $A_p = \frac{1}{2}$ $q = 1$ gives us back PS result.

$$b_n^{ST}(m) = 1 + \frac{q\nu^2 - 1}{\delta_c} + \frac{2p/\delta_c}{1 + (q\nu)^p}$$

see for instance Manera Scoccimarro and Sheth (2009).

Eq (22.)

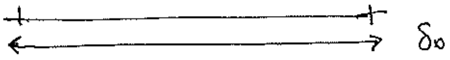
Ellipsoidal Collapse

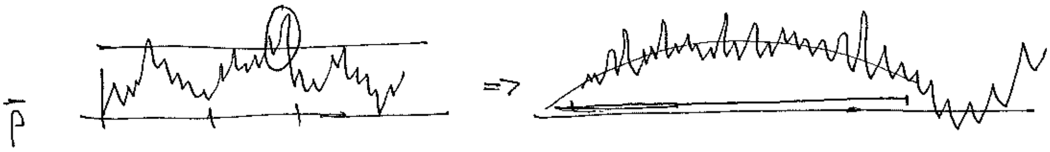
$$n(m)\,dm = \sqrt{\frac{2}{2\pi}}\;\bar{\rho}\quad e^{-\nu^2_2}\quad \left\{\frac{\nu}{\sigma^3}\right\}\qquad \underline{2\cancel{\sigma}}\;\;\partial\sigma$$

Conditional Mass Function

$$7 \qquad f(\sigma, \delta_c \mid \sigma_0, \delta_0) = \frac{1}{\sqrt{2\pi}}\;\frac{\delta_c - \delta_0}{(\sigma^2 - \sigma_0^2)^{3/2}}\;\exp\left\{-\frac{(\delta_c - \delta_0)}{2(\sigma^2 - \sigma_0^2)}\right\}$$

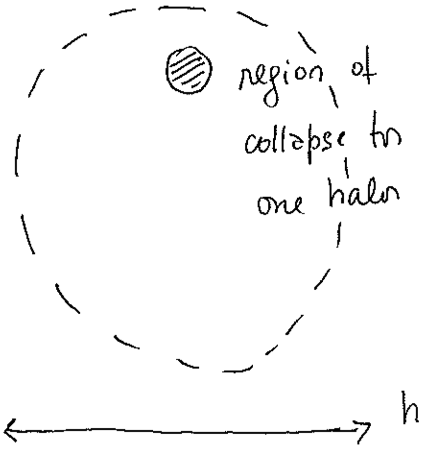
δ_0 = linearly evolves





$\bar{\rho}(1+\delta_0)$

{# of halos on some larger size}



=> we find the condition for collapse

here we assumed $\rho = \bar{\rho}$ before

Now we do the same but assuming here $\tilde{\rho} = \bar{\rho}(1 + \delta_0)$

δ_h = as function of $\bar{\rho}(1+\delta_0)$

↑ long wavelength.

$\textcircled{\bar{\rho}}$

$$n(m)\, dm \, \frac{m}{M'} = \frac{\partial F}{\partial m}\Big|_{\text{conditional}} dm$$

$\uparrow$

we can now
think that M' is a
bit more than M
or that volume is
smaller but
$\rho|_{\text{cond.}} = \bar{\rho}(1+\delta_0)$

$$dm \; n(m|\delta_0) = \cancel{M}' \rho(1+\delta) \; \frac{\partial F}{\partial \sigma} \left| \frac{\partial \sigma}{\partial m} \right| dm$$

Conditional mass fraction :

if we think of moving the barrier $\delta_c - \delta_0$ then

$$\delta = \delta_\ell + \delta_s$$

$$\delta^2 = \langle \delta_\ell^2 \rangle + \langle \delta_s^2 \rangle + \underset{=0}{\overset{\delta_s \delta_\ell}{\langle \cancel{\;x\;} \rangle}}$$

$\swarrow$ δ_ℓ does not change on scales δ_s

$$\sigma_s^2 = \sigma^2 - \sigma_0^2$$

$$P(\delta > \delta_c) = \int_{\delta_c - \delta_0} \frac{1}{\sqrt{2\pi}\,\sigma_s} e^{-\frac{\delta_s^2}{2\sigma_s^2}} =$$

$\uparrow$

all the same as before
but $\delta_c \longrightarrow \delta_c - \delta_0$
$\sigma \longrightarrow \sigma - \sigma_0^2$

Halo Model

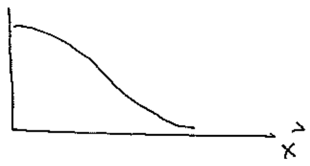
all we need to describe DM is:
- mass function
- halo bias
- halo profile

Assumptions

1- All dark matter is collapsed into halos of different masses

$$\rho(\vec{x}) = \sum_i m_i\, u_{m_i}(\vec{x}-\vec{x}_i)$$

↓ halo profile



$\int u_m(x-x)\,dx = 1$

2. Assume all halos have the same profile (!) Good to some extent see NFW profile.

$$\bar{\rho} = \langle \sum m_i\, u_{m_i}(\vec{x}-\vec{x}_i)\rangle = \langle \rho(\vec{x})\rangle$$

$$\langle \sum m_i \rangle \longrightarrow \int dm\; n(m)\; m$$

↑ mass function.

$\bar{\rho}^{-2}\xi$ = pairs of mass elements

$$\bar{\rho}^{-2}\xi(|x-x'|) = \langle \sum m_i\, u_{m_i}(\vec{x}-\vec{x}_i) \sum m_j\, u_{m_j}(\vec{x}'-x_j)\rangle$$

$i=j$ → one-halo term

$i\neq j$ → two-halo regime

$$\bar{\rho}^{-2}\xi(|\vec{x}-\vec{x}'|) = \int\int m^2\, n(m)\, dm\; u(x-y)\, u(x'-y)\, dy^3 +$$

↓ $u(y)\,u(y+r)$

$$+ \int m_1\, n(m_1)\, dm_1 \int m_2\, n(m_2)\, dm_2 \int dx_1\, dx_2\, \xi(|x_1-x_2|)\, u_{m_1} u_{m_2}$$

$$\rho = \bar{\rho}(1+\delta)$$

$$\langle \rho\rho \rangle = \bar{\rho}^2 (1+\xi)$$

$\downarrow$

disconnected piece

$$\langle \sum m_i u_i \sum_j m_j u_j \rangle = \underset{\downarrow}{\langle \sum m_i u_i \rangle \langle \sum m_j u_j \rangle} + \langle \sum_i m_i^2 \rangle + \langle \sum_{i \neq j} m_i m_j u_i u_j \rangle$$

$$\bar{\rho}^2(\cancel{1}+\xi) = \cancel{\bar{\rho}^2} + \underset{\xi_{1h}}{\text{one halo}} + \underset{\xi_{2h}}{\text{two-halo}}$$

$$\boxed{\xi = \xi_{1h} + \xi_{2h}}$$

one halo

$$\xi_{1h} = \frac{1}{\bar{\rho}^2} \int dm \, m^2 \, n(m) \int d^3x \; u_m(\vec{x}) \, u_m(\vec{x}+\vec{r})$$

two halo (LSS)

$$\xi_{2h} = \frac{1}{\bar{\rho}^2} \int dm_1 \, n(m_1) \, m_1 \int dm_2 \, n(m_2) \, m_2 \int d^3x_1 \int d^3x_2 \; u_{m_1}(x_1) \, u_{m_2}(x_2) \; \xi_{hh}(\vec{x}_1 - \vec{x}_2 + \vec{r} \,|\, m_1 m_2)$$

$$\xi_{hh}(r \,|\, m_1, m_2) = b_1(m_1) \, b(m_2) \, \xi(r).$$ ← in the limit of large separations

$\xi_{2h} \longrightarrow \xi_{lin}$ because $\int dm \, b(m) \, n(m) \left(\frac{m}{\bar{\rho}}\right) = 1.$

Halo Model for Galaxies

Assume galaxies live inside DM halo w/ some profile

HOD → halo occupation distribution $P(N_{gal}/M)$ prob. of finding N_{gal} in a halo of mass M.

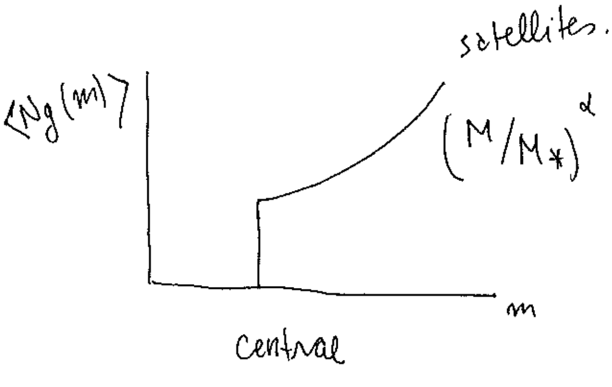
$$\xi^{2h}_{gg}(r) = \bar{n}_g^{-2} \int dm \int dm' \, n(m) \, n(m') \langle N_{gal}(m) \rangle \langle N_{gal}(m') \rangle$$

$$\int dx_1 \int dx_2 \; \underset{\uparrow \text{ galaxy profile}}{u_1(x_1|m_1)} \; u(x_2|m_2) \; \underset{\downarrow \; b(m_1)\, b(m_2)\, \xi(r)}{\xi(\vec{x}_1 - \vec{x}_2 - r \,|\, m_1 m_2)}$$

you end up w/

$$\boxed{b_{gal} = \frac{1}{\bar{n}_{gal}} \int dm \; n(m) \, b(m) \, \langle N_{gal}(m) \rangle}$$

Galaxy Bias.



Redshift Space Distortions

In galaxy redshift surveys we don't observe directly the distance to galaxies but rather we infer that from measuring the redshift. Redshifts are due to recesion speeds away from us. At low redshift one can schematically think of

$\frac{v}{c} \sim z$ → from Hubble flow $H_0 r = v \sim z$ → $r = \frac{z}{H_0}$

you get "r". In practice redshifts are due not only to Hubble flow but also to peculiar galaxy velocities and the later is unfortunately correlated w/ density which is what we want to get us.

$$\vec{x} = \frac{\vec{r}}{a} = \frac{\vec{v}}{aH} = \frac{\vec{v}}{\cancel{a}\mathcal{H}}$$

if now $\vec{v} \longrightarrow \underset{\substack{\uparrow \\ \text{due to} \\ \text{hubble} \\ \text{flow}}}{\vec{v}} + \underset{\substack{\uparrow \\ \text{peculiar} \\ \text{motion}}}{\vec{v}_{pec}}$

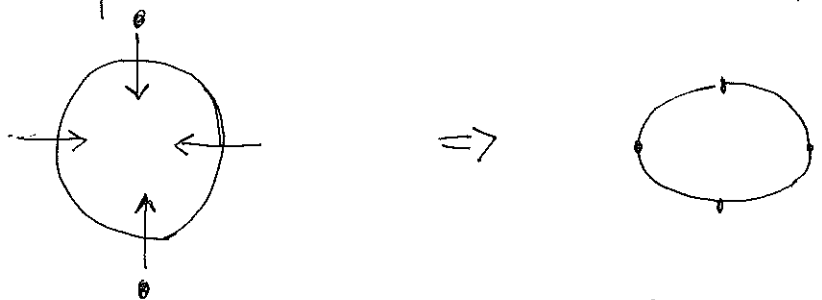
$$\vec{s} = \vec{x} + \frac{\vec{v}_p \cdot \hat{r}}{\mathcal{H}}$$

we only care along line of sight (i.e away from us).

If some moves on top of Hubble flow → we will think is farther aways (has more speed than just hubble) and otherwise

Pictorically

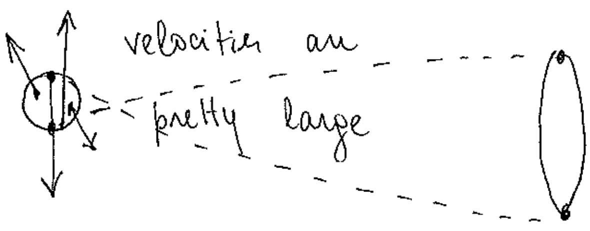
Kaiser effect.



↔ motion is ~ Mpc/h
velocities are few
hundred km/s.

on large scales you
generate an angle
dependent distortion ⇒ a quadrupole

On small scales however



This is called
Finger's of God.

virial velocities are $\sim 10^3$ km/s

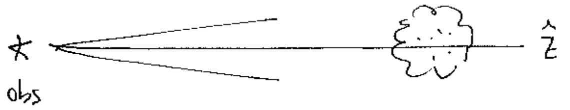
typical size
of the mapping
$$\frac{10^3 \text{ km/s}}{H_0 = 100 \text{ km/s } h/\text{Mpc}} \sim 10 \text{ Mpc}/h$$
while a
halo size is ~ few
Mpc/h !!

Recall that the relevant units for velocity were $-\theta/_{\mathcal{H}f} = \delta$ at large scales

$$-\frac{\vec{v}_p}{\mathcal{H}f} \equiv \vec{u} \quad \rightarrow \quad \vec{\nabla}\vec{u} = \delta$$

so z-space mapping is now

$$\vec{s} = \underset{\substack{\uparrow \\ \text{true comoving} \\ \text{coordinate.}}}{\vec{x}} - f u_z \hat{z}$$



distant observer approx ⇒ assume l.o.s is always at $\hat{z}$.

~~Conservation of number of~~

The mapping cannot change the number of galaxies in a particular region (!)

$$n_s(x_s)\, d^3s = n(x)\, d^3x$$

$$\{1 + \delta_s(\vec{s})\}\, d^3s = \{1 + \delta(\vec{x})\}\, d^3x$$

Let's integrate this

$$\int e^{-i\vec{k}\cdot\vec{s}} \{1 + \delta_s(\vec{s})\}\, d^3s/_{(2\pi)^3} = \int e^{-i\vec{k}(\vec{x} - f u_z \hat{z})} (1 + \delta(x))\, d^3x/_{(2\pi)^3}$$

$$\delta_D(\vec{k}) + \underset{\substack{\uparrow \\ \text{Fourier mode} \\ \text{in z-space}}}{\delta_s(\vec{k})} = \int \frac{d^3x}{(2\pi)^3}\, e^{-i\vec{k}\cdot\vec{x}} \{1 + \delta(x)\}\, e^{+i f k_z u_z}$$

$$\delta_D(\vec{k}) + \delta_s(\vec{k}) = \int \frac{d^3x}{(2\pi)^3}\, e^{-i\vec{k}\cdot\vec{x}} \{1 + \delta(x)\}\, e^{+i f k_z u_z(\vec{x})}$$

There is a rather non-linear mapping between real and redshift space!

Now we can expand assuming $f k_z u_z < 1$

$$\delta_D(\vec{k}) + \delta_S(\vec{k}) = \int \frac{d^3x}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \{1+\delta(\vec{x})\}\{1 + i f k_z u_z + \dots\}$$

Eq(**)

$$= \delta_D(\vec{k}) + \delta(\vec{k}) + \int \frac{d^3x}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}}\, i f k_z u_z(x)$$

$\downarrow$

$i f k_z . \hat{u}_z(k)$

Fourier transform of $\vec{u}$ field

Recall that on large scales

$$\vec{\nabla}\cdot\vec{u} = \delta$$

$\downarrow$ from $u(x) = \int d^3k\, e^{i\vec{k}\cdot\vec{u}}\, \hat{u}(k)$

$$i\vec{k}\cdot\vec{u} = \delta$$

$$\vec{u} = -i\frac{\vec{k}}{k^2}\delta$$

Back to Eq(**) $\delta_S(\vec{k}) = \delta(\vec{k}) + i f k_z \overbrace{\left(-i\frac{k_z}{k^2}\right)}^{u_z}\delta$

$$\delta_S(\vec{k}) = \delta(\vec{k})\left\{1 + f\frac{k_z^2}{k^2}\right\}$$

or defining μ cosine angle w/ l.o.s ⇒ $k_z = \mu k$

$$\delta_S(\vec{k}) = (1 + f\mu^2)\,\delta(\vec{k})$$

For a biased tracer with unbiased velocity field you get

$$\boxed{\delta_s(k) = (b + f\mu^2)\ \delta(k)}$$

Since $f\mu^2 > 1$ $\Rightarrow$ the apparent overdensity in z-space is larger than in real space $\Rightarrow$ makes sense.

<u>Power spectrum</u>

$$\underbrace{\langle \delta_s(\vec{k})\, \delta_s(\vec{k}') \rangle}_{P_s(\vec{k})\ \delta_D(\vec{k}+\vec{k}')} = (1+f\mu^2)(1+f\mu'^2)\ \langle \underbrace{\delta(\vec{k})\, \delta(\vec{k}')}_{P(\vec{k})\ \delta_D(\vec{k}+\vec{k}')} \rangle$$

$\downarrow$

this implies $-\mu = \mu'$ $(\mu'^2 = \mu^2)$.

$$\boxed{P_s(\vec{k}) = (b + f\mu^2)^2\ P(\vec{k})}$$

$\downarrow$ this is not isotropic anymore!

$\downarrow$ this one depends only on k

Typically one use multipole decomposition

$$P_s(k,\mu) = \sum_\ell \mathcal{L}_\ell(\mu)\ .\ P_s^{(\ell)}(k)$$

split of k, μ

$$P_s^{(\ell)}(k) = (2\ell+1) \int_{-1}^{1} P_s(k,\mu)\ \mathcal{L}_\ell(\mu)\ d\mu$$

For Kaiser model only $\ell = 0, 2, 4$ survive

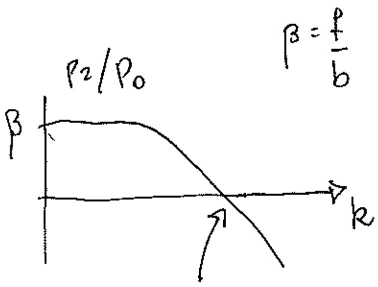
$$P_s^{(0)}(k) = \left(b^2 + \frac{2}{3} f b + \frac{f^2}{5}\right) P(k)$$

$$P_s^{(2)}(k) = \left(\frac{4}{3} b f + \frac{4}{7} f^2\right) P(k)$$

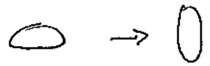
$$P_s^{(4)}(k) = \frac{8}{35} f^2 P(k)$$ ← does NOT depend on bias! but S/N is very low.

Another option is quadrupole to monopole ratio to measure f/b

$$\frac{P^{(2)}}{P^{(0)}}$$ → on large scales $P(k)$ cancels out and you get f/b and $f = \Omega_m^{0.6}$ measures Ω_m! you need to get rid of bias.



quadrupole changes sign when



The effect of virial velocities is highly non-trivial

Phenomenologically one uses a damping factor

$$P^{gal}(k) = P_* \, (b + f\mu^2)^2 \, \frac{1}{1 + k^2\mu^2\sigma^2/2}$$

or $e^{-k^2\mu^2\sigma^2/2}$

σ is a free fitting parameter.