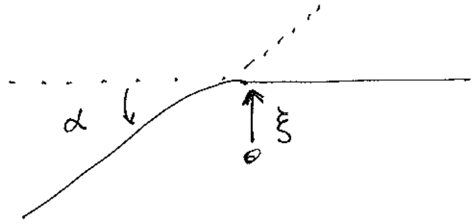
General Overview for Weak Lensing

1) light bending by a point-like mass object in GR.

starting from Schwarzschild metric.



$$\alpha = \frac{2 r_g}{r_m} = \frac{4GM}{r_m c^2} \to \frac{4GM}{\xi c^2}$$

r_m : impact parameter.

2) Extended mass : thin lens approximation

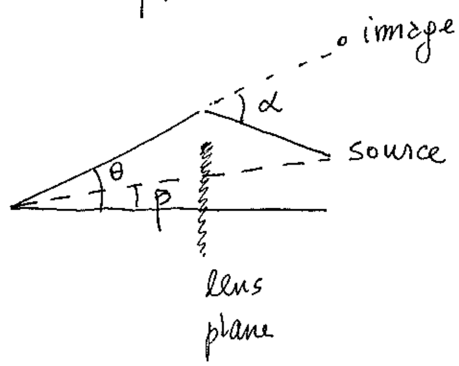
$$\Sigma(\vec{\xi}) \equiv \int dz\, \rho(\vec{\xi}, z)$$

position on lens plane.

$$\vec{\alpha}(\vec{\xi}) = \frac{2}{c^2} \vec{\nabla}_\xi \psi(\vec{\xi}).$$

3) Lens equation : converting the above to angles and β, θ

$$\vec{\beta} = \vec{\theta} - \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \vec{\nabla}_{\vec{\theta}} \psi(\vec{\theta})$$



4) Amplification matrix: convergence, shear

$$A_{ij} \equiv \frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \frac{\partial^2 \psi_i}{\partial \theta_i \partial \theta_j}$$

$$\frac{\partial \vec{\beta}\ (\text{source})}{\partial \vec{\theta}\ (\text{observed})}$$

- definition of shear, etc
- how to get γ from κ and viceversa.

5) Cosmic shear

Star from the geodesic equation in a perturbed FRW metric; write the geodesic equation for the evolution of light ray $\vec{\theta}$ and get to

$$\underset{\text{source}}{\theta_a(\chi)} = \underset{\text{observed}}{\theta_a^{(0)}} - \frac{2}{c^2} \int_0^{\chi} d\chi' \, \varphi_{,a} [\vec{\theta} \chi', \chi'] \, \frac{\chi - \chi'}{\chi \chi'}$$

Check Dodelson's book.

Compute A

projected potential.

$$\Psi \equiv \frac{2}{c^2} \frac{1}{\chi} \int_0^{\chi} d\chi' \, \varphi [\vec{\theta} \chi', \chi'] \, \frac{\chi - \chi'}{\chi'}$$

6) Power Spectrum of convergence

$$K(\theta) = \frac{1}{2} \nabla_\theta^2 \Psi(\vec{\theta})$$

$k(\theta)$ in terms of $\nabla_\perp^2 \varphi \to \delta$ using $\nabla^2 \phi$

→ Compute

$$\langle K(\vec{\ell})\, K(\vec{\ell}')\rangle = \delta_D(\vec{\ell}+\vec{\ell}')\, P_K(\ell)$$

small angle and flat sky → $P_K(\ell) = 2\pi \int d\chi \, \frac{g^2(\chi)}{\chi^2} P_\delta(\ell/\chi)$

7) Power Spectrum of γ_1, γ_2 → E/B decomposition

from $\gamma_1 = \frac{\ell_1^2 - \ell_2^2}{\ell^2} K \qquad \gamma_2 = \frac{2\ell_1 \ell_2}{\ell^2} K$

$P_{\gamma_1} = \cos 2\phi \, P_K$

$P_{\gamma_2} = \sin 2\phi \, P_K$

→ E definition → $P_E = P_K$

B → $P_B = 0$

8) Kind of pattern induced → $(\gamma_t, \gamma_\times)$ → (ξ_+, ξ_-). ← dependence on $P(\delta)$ No bias

$\delta > 0 \quad \delta < 0$ → E mode $\quad -1$ → No lensing

9) How do you measure shear from ellipticity (Dodelson)

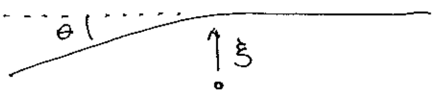
and $\epsilon = \epsilon_{intrinsic} + \epsilon_{lensing}$

→ plan for this.

Weak Lensing

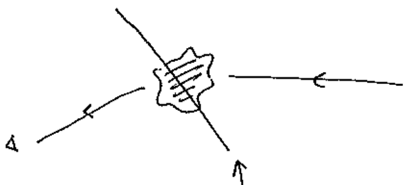
- Light bending ~~b~~ by point - source is one of main prediction by GR

A factor 2 larger the Newtonian for test particle → $\alpha = \frac{4GM}{c^2 r_m}$



> For the sun the deflection is $\sim 1.75''$ (seconds of arc) => major test of GR

For an extended mass distribution



$$\alpha = \sum_i^{\text{lenses}} \alpha_i$$

Assume deflection happens all in one plane => thin lens approximation deflection happens at center of mass for ρ

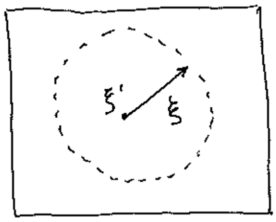
ξ : coordinate (2D vector in the lens plane)

$$\Sigma(\xi) = \int dz\, \rho(\vec{\xi}, z)$$ ↔ projected mass density

$$\vec{\alpha}(\xi) = \frac{4G}{c^2} \int \underbrace{d^2\xi' \, \Sigma(\xi')}_{\text{mass}} \frac{\vec{b}}{|b|^2} \qquad \vec{b} = \vec{\xi} - \vec{\xi}'$$

~~enclosed mass~~

↓ impact parameter w.r.t. ξ'



lens plane

If we instead work in terms of projected gravitational potential

$$\psi(\vec{\xi}) = \int dz\, \phi(\vec{\xi}, z)$$

→ by construction obeys the Poisson eq in 2D

$$\nabla^2_{\xi}\psi = 4\pi G\, \Sigma(\xi)$$

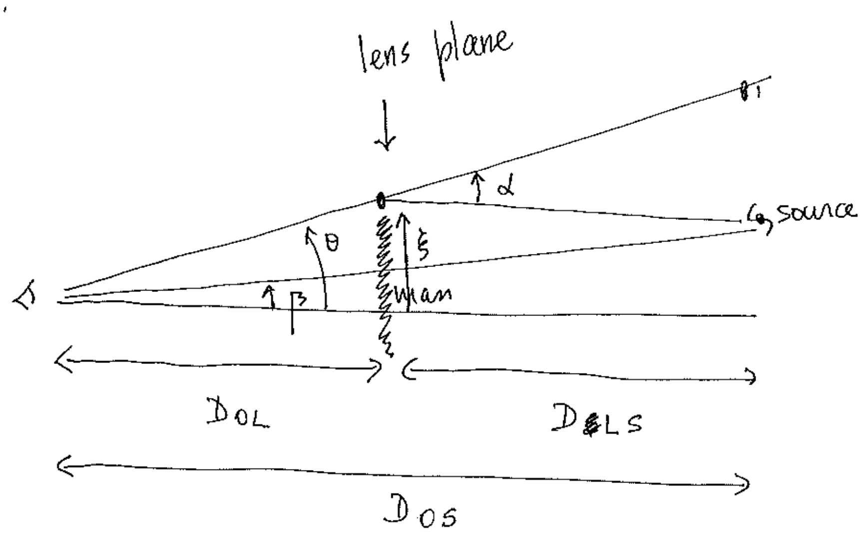
$$\psi(\xi) = 2G \int d^2\xi'\, \Sigma(\xi') \ln|\xi - \xi'|$$ solution in 2D

$$\vec{\alpha}(\vec{\xi}) = \frac{4G}{c^2}\int d^2\xi' \frac{\vec{\xi}-\vec{\xi}'}{|\vec{\xi}-\vec{\xi}'|^2}\Sigma(\xi')$$ from before

So

$$\boxed{\vec{\alpha} = \frac{2}{c^2}\vec{\nabla}_{\xi}\psi(\xi)}$$ gradient of the 2D potential

Let's try to relate this to emission from a source of know distance and lensing too



$$\theta \cdot D_{OS} = \beta\, D_{OS} + \alpha\, D_{LS}$$

$$\vec{\beta} = \vec{\theta} - \frac{D_{LS}}{D_{OS}} \vec{\alpha}(\vec{\xi}) \qquad \hookrightarrow \quad D_{OL}\vec{\theta} \approx \vec{\xi}$$

Now we can write everything in terms of $\vec{\theta}$

$$\vec{\beta} = \vec{\theta} - \frac{D_{LS}}{D_{OS}} \vec{\alpha}\left(D_{OL}\vec{\theta}\right) = \vec{\theta} - \vec{\alpha}'(\vec{\theta})$$

If we define $K(\vec{\theta}) \equiv \dfrac{\Sigma(\vec{\theta})}{\Sigma_{crit}}$ when we mean $\Sigma(D_{OL}\vec{\theta})$

and $\Sigma_{crit} =$

$$\vec{\alpha} = \frac{4G}{c^2} \int d^2\xi' \, \frac{\xi - \xi'}{|\vec{\xi} - \vec{\xi}'|^2} \, \Sigma(\xi')$$

→ Re-scaled angle

$$\alpha' = \frac{4G}{c^2} \frac{D_{OL} \cdot D_{LS}}{D_{OS}} \frac{\pi}{\pi} \int d^2\theta \, \frac{\theta - \theta'}{|\theta - \theta'|} \, \Sigma(\theta)$$

$$\Sigma_{crit} = \frac{c^2}{4\pi G} \frac{D_{OS}}{D_{OL} D_{LS}}$$

$$\vec{\alpha}' = \frac{1}{\pi} \int d^2\theta' \, \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2} \, K(\theta)$$

Here all distances mean angular diameter distance that relate angles to comoving distances transverse to the line of sight measured at the time of emission

$$D(z_e, z_o) = a(z_e)\, r[\chi(\cancel{z}_e, z_o)]$$

$$\chi = c \int_{t_e}^{t_o} \frac{dt}{a}$$

$$r(\chi) = \begin{cases} 1/\sqrt{k}\, \sin \sqrt{k}\,\chi \\ \chi \\ \frac{1}{\sqrt{-k}} \sinh \sqrt{k}\,\chi \end{cases}$$

In general $D_{OS} \neq D_{OL} + D_{LS}$.

$$\vec{\theta}_S = \theta_0 - \frac{D_{LS}}{D_{OS}} \frac{2}{c^2} \vec{\nabla}_{\vec{\xi}} \psi = \vec{\theta}_0 - \frac{D_{LS}}{D_{OS} D_{OL}} \frac{2}{c^2} \frac{\partial \psi}{\partial \theta_i}$$

$$\vec{\xi} = D_{OL} \cdot \vec{\theta}$$

$$\frac{\partial \theta_S}{\partial \theta_i} = \delta_{ij} - \frac{D_{LS}}{D_{OS} D_{OL}} \frac{2}{c^2} \frac{\partial \psi}{\partial \theta_i \partial \theta_j}$$

$$A_{ij} = \begin{pmatrix} 1 - K - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - k + \gamma_1 \end{pmatrix}$$

$$\mathrm{Tr}\{A_{ij}\} = 2 - K = 2 - \frac{D_{LS}}{D_{OS} D_{OL}} \frac{2}{c^2} \nabla_\theta^2 \psi \Rightarrow$$

Define the re-scale potential

$$\Psi = \frac{D_{LS}}{D_{OS} D_{OL}} \frac{2}{c^2} \psi$$

such that

$$\nabla_{\vec{\theta}}^2 \Psi = K(\theta) = \Sigma / \Sigma_{crit}$$

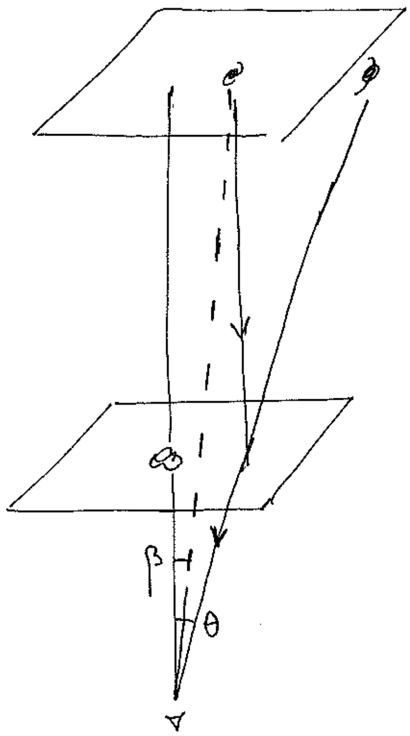
↑ if you go back to the relation of ψ and Σ

Then

$$\gamma_1 \equiv \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \frac{1}{2} \left(\frac{\partial^2 \psi}{\partial \theta_1} - \frac{\partial^2 \psi}{\partial \theta_2} \right)$$

$$\gamma_2 \equiv \frac{2}{c^2} \frac{D_{LS}}{D_{OS} D_{OL}} \frac{\partial^2 \psi}{\partial \theta_1 \partial \theta_2}$$

Distortions of faint galaxy images



$\vec{\theta}$: observed angle

$\vec{\beta}$: undistorted image

Galaxies are distorted in shape and size (also in magnitude / flux → brightness)

$$\beta - \beta_0 = A(\theta_0)\,(\theta - \theta_0)$$

$$\beta_0 = \beta(\theta_0)$$

observed surface brightness $\longrightarrow \; I(\theta) = I^{s}(\beta) = I^{(s)}\big(\beta_0 + A(\theta_0)\,(\theta - \theta_0)\big)$

$$A(\theta) = (1-\kappa)\begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix}$$

local Jacobi matrix

$$g(\theta) = \frac{\gamma(\theta)}{|1-\kappa(\theta)|}$$ reduced shear

$$\gamma = \gamma_1 + i\,\gamma_2 = |\gamma|\, e^{2i\varphi}$$

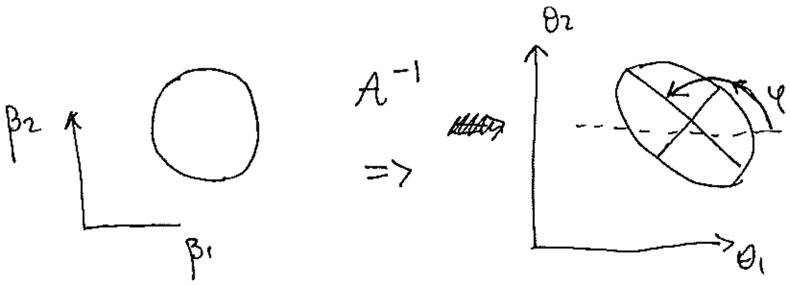
$$g = \frac{\gamma}{1-\kappa} = g_1 + i\, g_2$$

$\dfrac{R}{1-\kappa-|\gamma|}$: semi-major : $[(1-\kappa)(1-g)]^{-1}$

$\dfrac{R}{1-\kappa+|\gamma|}$: semi-minor $[(1-\kappa)(1+g)]^{-1}$

$$\frac{b}{a} = \frac{1-|g|}{1+|g|} \;\longrightarrow\; |g| = \frac{1-b/a}{1+b/a}$$

measuring ellipticities leads to reduced shear.



φ : angle to the semi-major axis

Measuring Shear

$$\mathcal{A} = \delta_{ij} - \frac{\partial^2 \psi}{\partial x_i \partial x_j} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

these are angular coordinates

$$\gamma_1 = \frac{1}{2}\left(\frac{\partial^2\psi}{\partial\theta_1^2} - \frac{\partial^2\psi}{\partial\theta_2}\right) \qquad \gamma_2 = \frac{\partial^2\psi}{\partial\theta_1\partial\theta_2}$$

describe isotropic distortion

$$\kappa = \frac{1}{2}\left(\frac{\partial^2\psi}{\partial\theta_1^2} + \frac{\partial^2\psi}{\partial\theta_2^2}\right) = \frac{1}{2}\nabla_\theta^2\psi$$

*) The effect is to transform a circular source into an ellipse with axis ratio $\frac{1-|g|}{1+|g|}$ and position angle

$\alpha = 1/2 \arctan(g_2/g_1)$

**) Source is magnified by

$$\mu = \frac{1}{\det\mathcal{A}} = \frac{1}{(1-\kappa)^2 - |\gamma|^2} = \frac{1}{(1-\kappa)^2(1-|g|^2)}$$

$\mu > 1$.

> Note that to first order μ only depends on κ!

in the weak lensing regime $g = \frac{\gamma}{1-\kappa} \simeq \gamma$

Distortion Matrix

Dodelson

$$A_{ij} \equiv \frac{\partial \theta_s^i(x)}{\partial \theta^j(0)}$$

White

$$\frac{\partial \theta_i(x)}{\partial \theta_j(0)} \equiv \delta_{ij} + A_{ij}$$

Roman

$$A^{-1}_{ij} = \frac{\partial \theta_s(x)}{\partial \theta^j(0)}$$

Henk

$$A_{ij} = \delta_{ij} - \frac{\partial^2 \Psi_6}{\partial x_i \partial x_j}$$

Dodelson (and same for all except Roman)

$$A_{ij} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

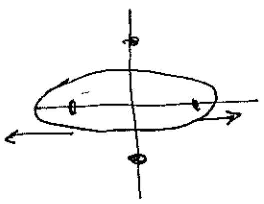
To see how this transforms a circular image or induces rotations we have to do A^{-1}_{ij}

$$A^{-1}_{ij} = \begin{pmatrix} 1+\kappa+\gamma_1 & +\gamma_2 \\ +\gamma_2 & 1+\kappa-\gamma_1 \end{pmatrix}$$

now $\theta^{(obs)} = A^{-1}\,\theta^{(s)}$

$\gamma_1 > 0$

$\gamma_1 < 0$

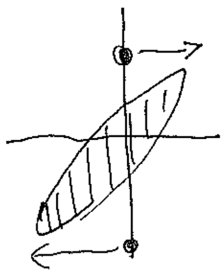


$$\theta_x = (1+\kappa+\gamma_1)\,\theta_x^{(s)}$$

$$\theta_y = (1+\kappa-\gamma_1)\,\theta_y^{(s)}$$

if $\theta_x^{(s)} > 0$ then $\gamma_1 > 0$ makes it grow ; and the opposite for θ_x negative

if $\theta_y^{(s)} > 0$ then θ_y is less positive



if we take $\theta_x^{(S)} = 0$ then $\theta_x = \gamma_2 \, \theta_y^{(S)}$ will be generated, and opposite if $\theta_y^{(S)} < 0$ (from $\theta_x^{(S)} = 0$)

$\gamma_2 > 0$ $\quad$ $\gamma_2 < 0$

The same convention for ellipticity (see Dodelson page 301)

Magnification of the size is obtained by

$$\det A = (1-\kappa)^2 - |\gamma|^2$$

← dominated by κ! that enters linearly

$$\mu = \frac{1}{\det A} = \frac{1}{(1-\kappa)^2 - \gamma^2}$$

$$\mu \approx 1 + 2\kappa$$

Cosmic Shear

We need the geodesic equation for the path of a photon in a perturbed FRW Universe in Newtonian gauge

$$ds^2 = a^2(\tau)\left\{\left(1+\frac{2\phi}{c^2}\right)d\tau^2 - \left(1-\frac{2\psi}{c^2}\right)\left[d\chi^2 + r^2(\chi)\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]\right\}$$

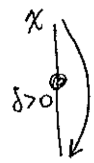
where χ is comoving distance. Let's take a fiducial direction along the light beam, say $\hat{z}$, and take angular coordinates $a=1,2$ corresponding to x, y so $d\theta_1^2 + d\theta_2^2 = d\theta_x^2 + d\theta_y^2 = d\theta^2 + \sin^2\theta\, d\phi^2$

In GR $\phi = \psi$ = "Newtonian potential". From spatial parts of Einsteins eqs

$$\frac{d^2}{d\chi^2}(\chi\theta^i) = -\frac{2}{c^2}\phi_{,i}$$

in this notation $\vec{x} = \chi\hat{\theta}$ and $_{,i} \equiv \frac{\partial}{\partial x_i}$

if we are at one overdensity $\Rightarrow \phi_{,i} \leq 0 \Rightarrow \frac{\partial^2}{\partial\chi^2} < 0$

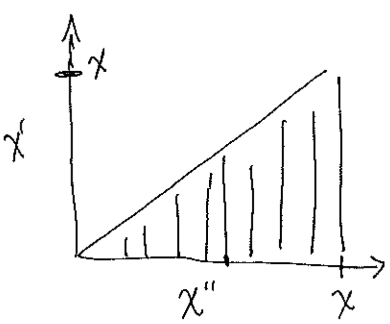


$$\frac{d}{d\chi}(\chi\theta_i) = -\frac{2}{c^2}\int_0^{\chi} d\chi'\, \phi_{,i}(\theta^{(0)}\chi', \chi') + \text{constant}$$

is bending towards overdensity

$$\theta_i^{s} = -\frac{1}{\chi}\frac{2}{c^2}\int_0^{\chi} d\chi'' \int_0^{\chi''} d\chi'\, \phi_{,i}(\theta^{(0)}\chi', \chi')$$

$r(\chi')\,\theta^{(0)}$ in the general geometry



$$\rightarrow \int_0^{\chi} d\chi'' \int_0^{\chi''} d\chi' = \int_0^{\chi} d\chi' \cdot \underbrace{\int_{\chi'}^{\chi} d\chi''}_{= \chi - \chi'}$$

$$\theta_s^i = \theta_0^i - \frac{2}{c^2} \int_0^{\chi} d\chi' \, \phi_{,i}(\theta\chi', \chi') \quad \frac{\chi - \chi'}{\chi}$$

now we can assume this is $\theta^{(0)}$ unperturbed $\theta\chi' = \theta^{(0)}\chi'$

And recall that derivatives are <u>NOT</u> w.r.t. angles but rather transverse coordinates $\vec{X}_\perp = \chi\theta$ or $r(\chi)\theta$.

Deformation matrix

$$A_{ij} \equiv \frac{\partial\theta_s^i}{\partial\theta_0^j} = \delta_{ij} - \frac{2}{c^2}\int_0^{\chi} d\chi' \, \phi_{,ij} \quad \frac{\chi'(\chi-\chi')}{\chi}$$

where we use $\frac{\partial}{\partial\theta^j} = \chi' \frac{\partial}{\partial x_j}$

transverse comoving coordinate at fixed χ'.

There is a difference in what potential is being used

The scaled version of the projected potential is given by

$$\Psi \equiv \frac{2}{c^2} \int_0^{\chi} d\chi' \, \phi \quad \frac{\chi'(\chi-\chi')}{\chi}$$

Recall that

$$A_{ij} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

means that

$$K(\theta) = \frac{1}{2}\nabla_\theta^2\psi = \frac{1}{c^2}\int_0^\chi d\chi' \; \nabla_\perp^2\phi(\theta^{(0)}\chi', \chi') \; \frac{\chi'}{\chi}(\chi-\chi').$$

Newtonian potential in 3D

$$\nabla_\perp = \frac{\partial^2}{\partial x_1} + \frac{\partial^2}{\partial x_2} = \frac{1}{\chi'^2}\left\{\frac{\partial}{\partial\theta_1^2} + \frac{\partial}{\partial\theta_2^2}\right\}.$$

The full 3D Laplacian is given by. $\nabla^2 = \nabla_\perp^2 + \nabla_\chi^2$ the radial part can be integrated by parts and only gives boundary terms (in some approximation).

$$\nabla^2\phi = \frac{3}{2}\frac{H_0\Omega_m}{a(\tau)}\delta(\vec{x},\tau)$$

Recall

$$\frac{1}{a^2}\nabla^2\phi = 4\pi G\rho_m\delta$$

$$\frac{1}{a^2}\nabla^2\phi = 4\pi G\frac{\rho_m^0}{a^3}\delta$$

$$\nabla^2\phi = 4\pi G\rho_{crit}\Omega_m^0\frac{\delta}{a}$$

$$= \frac{3}{2}H_0^2\Omega_m^0\frac{\delta}{a}$$

$$\frac{3H^2}{8\pi G} = \rho_{crit}$$

So

$$K(\theta) = \int_0^\chi d\chi' \; W(\chi,\chi')\,\delta(x_\perp,\chi')$$

$$W(\chi,\chi') = \frac{3}{2}\frac{H_0^2}{c^2}\Omega_m^0\,(\chi-\chi')\frac{\chi'}{\chi}\frac{1}{a(\chi)}$$

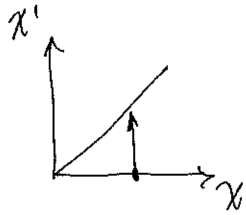
Distances depend on H_0 so to first order H_0 cancels out but it does depend on Ω_m^0 and cosmology through χ, χ'

$$K(\theta) = \frac{3}{2}\left(\frac{H_0}{c}\right)^2\Omega_m^0\int_0^\chi d\chi'\,(\chi-\chi')\frac{\chi'}{\chi}\frac{\delta_\perp(x_\perp,\chi')}{a(\chi')}$$

The kernel "peaks" at $x' \sim X/2$ half way to the source.

Extended source distribution

$$K(\theta) = \int_0^{\chi_\infty} dx \, n(\chi) \int_0^{\chi} dx' \, W(x, x') \, \delta_{3D}(x', x_\perp)$$



$$\int_0^{\chi_\infty} dx' \underbrace{\int_{x'}^{\chi_\infty} dx \; n(\chi) \, W(x, x')}_{F(\chi')} \; \delta(x', x_\perp)$$

$$K(\theta) = \int^{\chi_\infty} dx' \, F(x') \; \underset{3D}{\delta}(x', \underset{\downarrow \; x'\theta}{x_\perp})$$

F also called g ← peaks at $x' = x/2$. halfways.

Power Spectrum

$$K(\vec{l}) = \int \frac{d^2\theta}{(2\pi)^2} K(\theta) \, e^{-i \vec{\theta} \cdot \vec{l}}$$

← flat sky approximation ; we directly expand in 2D Fourier modes

$$\langle K(\vec{\theta}) K(\vec{\theta}') \rangle = \int d^2\vec{l} \; P_K(\vec{l}) \, e^{i \vec{l} \cdot (\vec{\theta} - \vec{\theta}')}$$

$$= \int dx \int dx' \, F(x) \, F(x') \, \langle \delta(\theta x, x) \, \delta(\theta' x', x') \rangle$$

$$= \int d\chi\, F(\chi) \int d\chi'\, F(\chi') \int d^3k\, e^{-i\vec{k}\cdot\vec{r}}\, P_\delta(\vec{k})$$ ← separation between $\vec{\theta}$ and $\vec{\theta}'$

$$= \int d\chi\, F(\chi) \int d\chi'\, F(\chi') \int d^3k\, P_\delta(k)\, e^{-i\{k_{\parallel}\cdot(\chi-\chi') + k_\perp(\chi\theta - \chi'\theta')\}}$$

The arguments of the exponential are

$$k_{\parallel}\, \Delta\chi$$

$$k_\perp\left(\chi\{\theta' + \Delta\theta\} - \chi'\theta'\right)$$

$$k_\perp \chi\, \Delta\theta - \cancel{k_\perp \theta' \Delta\chi}$$

→ The correlation can only depend on $\Delta\theta$ so we pick $\theta' = 0$

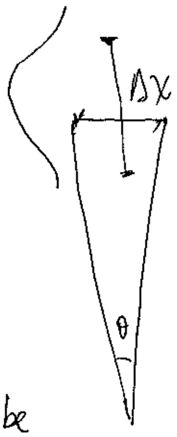
Two arguments

$k_{\parallel}\, \Delta\chi$ — $\Delta\chi$ radial extent of the source distribution

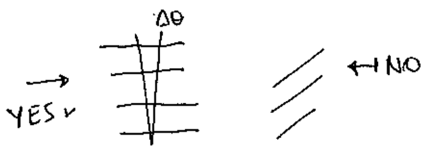
$k_\perp\, \chi\, \Delta\theta$ — $\chi\, \Delta\theta$ distances under consideration

In the small angle approximation

$$\Delta\theta\, \chi \ll \Delta\chi$$



Since both arguments must be $\lesssim 1$ for the $\exp\{i\}$ to survive (integral is dominated by argument of order 1 or less) then $k_\perp \gg k_{\parallel}$ ⇒ modes are all transverse



$$k = \sqrt{k_{\parallel}^2 + k_{\perp}^2} \sim k_{\perp}$$

$$= \int d\chi\, F(\chi) \int d\chi'\, F(\chi') \int dk_{\parallel}\, d^2k_{\perp}\, P(k_{\perp})\, \underbrace{e^{-i k_{\parallel}(\chi - \chi')}}_{\text{gives } 2\pi\, \delta_D(\chi - \chi')}\, e^{-i k_{\perp}(\theta - \theta')\chi}$$

$$\langle K(\theta)\, K(\theta') \rangle = \int d\chi\, F(\chi)^2 \int d^2k_{\perp}\, P(k_{\perp})\, e^{i k_{\perp}(\vec{\theta} - \vec{\theta}')\cdot\chi}$$

$$\ell = \chi\, k_{\perp}$$

$$= 2\pi \int d\chi\, \frac{F^2(\chi)}{\chi^2} \int d^2\ell\, P_\delta(\ell/\chi)\, e^{i\vec{\ell}(\vec{\theta} - \vec{\theta}')}$$

From here we get

$$\boxed{P_K(\ell) = 2\pi \int d\chi\, \frac{g^2(\chi)}{\chi^2}\, P_\delta(\ell/\chi)}$$

With the lensing kernel given by

Sensitive to Ω_m and σ_8

$$g(\chi) = \int_{\chi}^{\chi(\infty)} d\chi'\, n(\chi')\, \frac{3}{2}\left(\frac{H_0}{c}\right)^2 \Omega_m\, (\chi - \chi')\, \frac{\chi'}{\chi}\, \frac{1}{a(\chi)}$$

Since γ_1, γ_2 are derived from the same proyected potential we have

$$\gamma_1 = \frac{1}{2}\left(\frac{\partial^2 \psi}{\partial \theta_1} - \frac{\partial^2 \psi}{\partial \theta_2}\right) \qquad \frac{1}{2}\, \frac{\partial \psi}{\partial \theta_1} + \frac{\partial \psi}{\partial \theta_2} = K$$

$$\gamma_2 = \frac{\partial^2 \psi}{\partial \theta_1 \partial \theta_2}$$

In fourier space

$$\gamma_1(\vec{\ell}) = \frac{1}{2}(\ell_1^2 - \ell_2^2)\,\psi(\ell) \qquad \frac{1}{2}\underbrace{(\ell_1^2 + \ell_2^2)}_{=\ell^2}\,\psi(\ell) = \kappa(\ell)$$

$$\gamma_2(\vec{\ell}) = \ell_1 \ell_2\, \psi(\ell)$$

$$\boxed{\gamma_1 = \frac{\ell_1^2 - \ell_2^2}{\ell^2}\,\kappa(\ell)} \quad \& \quad \boxed{\gamma_2 = 2\,\frac{\ell_1 \ell_2}{\ell^2}\,\kappa}$$

The shear can be thought as spin 2-field

$$\gamma_{ab} = \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

$$= \gamma \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} \qquad \gamma^2 = \gamma_1^2 + \gamma_2^2$$

The rotation symmetry is only π → not 2π → stretching

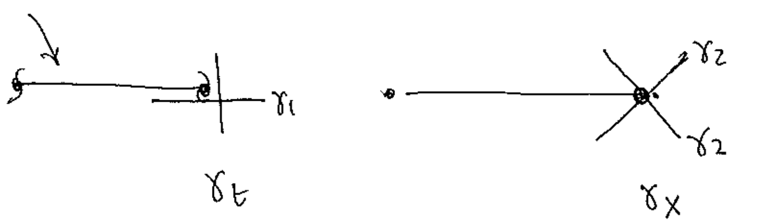
Images have no direction

$$\gamma_1 = \cos(2\alpha)\,\gamma$$

$$\gamma_2 = \sin(2\alpha)\,\gamma$$

$$\alpha \to \alpha + \pi$$
$$\gamma_1 \to \gamma_1$$
$$\gamma_2 \to \gamma_2$$

line connecting the galaxies



$$P_{\gamma_1} = \cos^2(2\alpha)\, P_\kappa$$

$$P_{\gamma_2} = \sin^2(2\alpha)\, P_\kappa.$$

From a spin-2 field you can construct E/B modes

$$\nabla^2 E = (\partial_1^2 - \gamma_2^2)\gamma_1 + 2\partial_1\partial_2\gamma_2 \rightarrow \cos(2\alpha)\gamma_1 + \sin(2\alpha)\gamma_2 = E$$

$$\nabla^2 B = (\partial_1^2 - \partial_2^2)\gamma_2 - 2\partial_1\partial_2\gamma_1 \rightarrow \sin(2\alpha)\gamma_1 - \cos(2\alpha)\gamma_2 = B$$

From here E mode $\rightarrow$ K

B mode $\rightarrow$ its zero!

overdensity underdensity

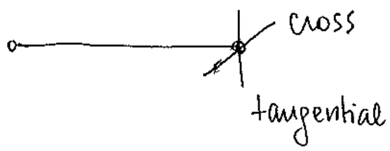
E- mode

B - mode

γ here is complex!

Tangential and cross shear:

$$\gamma_t = -\,\mathrm{Re}\{\gamma e^{-2i\phi}\} \qquad \gamma_\times = \mathrm{Im}(-\gamma e^{-2\phi i})$$

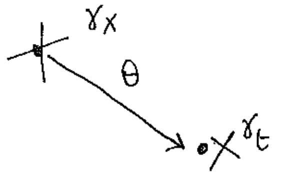


$$\xi_\pm = \langle \gamma_T(\theta_1)\,\gamma_T(\theta_2)\rangle \pm \langle \gamma_\times(\theta_1)\,\gamma_\times(\theta_2)\rangle$$

$$\xi_\pm = \frac{1}{2\pi}\int d\ell\, J_{0/4}(\ell\theta)\, P_K(\ell)$$

γ_t = tangential shear

ϕ : polar angle defined w.r.t. the separation vector between the galaxies.



Measuring γ_t is like computing γ_1 in the coordinate system aligned with the separation vector.

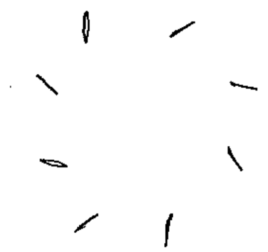
$$\xi_\pm(\theta) : \langle \gamma_t \gamma_t \rangle \pm \langle \gamma_\times \gamma_\times \rangle = \frac{1}{2\pi} \int d\ell \; P_\kappa(\ell)\, \ell \cdot J_{0,4}(\ell\theta)$$

$$\begin{aligned} \xi_+ &\longrightarrow J_0 \\ \xi_- &\longrightarrow J_4 \end{aligned} \quad \longleftrightarrow \text{ combination of EE and EB}$$

$$P_{E/B}(\ell) = \pi \int_0^\infty d\theta \; \theta \left[\xi_+(\theta)\, J_0(\ell\theta) \pm \xi_-(\theta)\, J_4(\ell\theta) \right] -$$

ξ_- receives more contributions from non-linear modes than ξ_+.

• Quiero mostrar E, B modes $\Rightarrow$ weak lensing <u>NO</u> genera B modes $\rightarrow$ source of systematics.



B-mode (same as E mode but rotated 45°)

$$\langle EE \rangle = P_K \quad (\rightarrow P_\delta)$$

$$\langle EB \rangle = 0$$

$$\langle BB \rangle = 0$$

In practice

$$(\gamma_1 + i\gamma_2) \longrightarrow (\gamma_1 + i\gamma_2)\, e^{-2i\phi}$$

counter-clockwise rotation by ϕ

1) : invariant if $\phi = \pi$

2) : $\phi = \pi/2$ chances $\gamma_2 \leftrightarrow \gamma_1$ ($\gamma_1 \rightarrow \gamma_2$ and $\gamma_2 \rightarrow -\gamma_1$)

$$\gamma_t = -\mathrm{Re}\left[(\gamma_1 + i\gamma_2)\, e^{-2i\phi}\right]$$

$$\gamma_\times = -\mathrm{Im}\left[(\gamma_1 + i\gamma_2)\, e^{-2i\phi}\right]$$

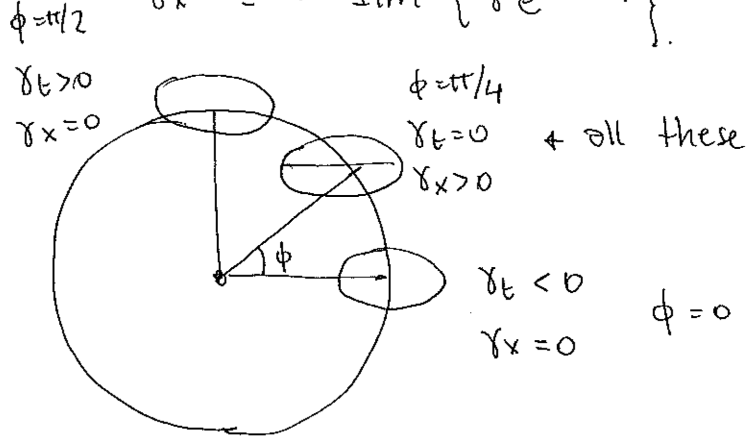
Tangential and cross component of shear

γ_1, γ_2 (shear components) are defined ~~xxxxxxxx~~ relative to a reference Cartesian coordinate frame. Note that shear is <u>NOT</u> a vector; because under a rotation by φ shear components are multiplied by $\cos 2\varphi$, $\sin 2\varphi$

Observationally it is useful to measure them w.r.t a rotated frame

$$\gamma_t = -\mathrm{Re}\{\gamma e^{-2i\phi}\}$$

$$\gamma_\times = -\mathrm{Im}\{\gamma e^{-2i\phi}\}.$$



& all these have $\gamma_1 > 0$ $\gamma_2 = 0$

$$\gamma_t = -\mathrm{Re}\{(\gamma_1 + i\gamma_2)e^{-2i\phi}\} = -\gamma_1 \cos 2\phi - \gamma_2 \sin 2\phi$$

$$\gamma_\times = -\mathrm{Im}\{(\gamma_1 + i\gamma_2)e^{-2i\phi}\} = \gamma_1 \sin 2\phi - \gamma_2 \cos 2\phi$$

↓ From here $\xi_{+/-}$

The effect of WL is to change the observed ellipticity of galaxies as

\- How to measure shear -

$$\epsilon_{orig} = \frac{a-b}{a+b}$$

$$\epsilon^{obs} = \frac{\epsilon^{orig} + g}{1 + g^* \epsilon^{orig}} \sim \epsilon^{orig} + \gamma$$

But $\epsilon^{orig} \sim 0.25$ for real galaxies. $\rightarrow$ we need to average many galaxies!

Angular Correlations : Power Spectrum.

$$\vec{x}(\chi,\theta) = \chi(\theta_1,\theta_2,1) \quad \text{where} \quad \chi = \chi(z)$$

Measuring all galaxies along the line-of-sight

$$\delta_{2D}(\vec{\theta}) = \int_0^{\chi_\infty} d\chi \; W(\chi) \; \delta(\vec{x}(\chi,\theta))$$

$\chi_\infty = (H_0/2)^{-1}$ in a flat matter dominated Universe

$\chi_\infty = \chi(z \to \infty)$.

$$\delta_{2D}(\vec{\ell}) = \int d^2\theta \; e^{-i\vec{\ell}\cdot\vec{\theta}} \; \delta_{2D}(\vec{\theta})$$

$$\langle \delta_{2D}(\vec{\ell}) \; \delta^*_{2D}(\vec{\ell}') \rangle = (2\pi)^2 \, \delta_D^2(\vec{\ell}-\vec{\ell}') \, P_{2D}(\ell)$$

$\hookrightarrow$ 2D power spectrum

Integrating this expression

$$P_{2D}(\ell) = \frac{1}{(2\pi)^2} \int d^2\ell' \, \langle \delta_{2D}(\vec{\ell}) \; \delta^*_{2D}(\vec{\ell}') \rangle$$

$$= \frac{1}{(2\pi)^2} \int d^2\ell' \, d^2\theta \, d^2\theta' \; e^{-i\vec{\ell}\cdot\vec{\theta}} \; e^{+i\vec{\ell}'\cdot\vec{\theta}'} \int_0^{\chi_\infty} d\chi \int_0^{\chi_\infty} d\chi'$$

$$W(\chi) \, W(\chi') \langle \delta(\vec{x}(\chi,\theta)) \; \delta(\vec{x}(\chi',\theta')) \rangle$$

The integral over $\vec{\ell}'$ gives $(2\pi)^2 \delta_D(\vec{\theta}')$

$$\xi(\vec{x} - \vec{x}') \equiv \langle \delta(\vec{x})\, \delta(\vec{x}') \rangle$$

$$= \int \frac{d^3k}{(2\pi)^3}\, P(k)\, e^{i \vec{k}.(\vec{x} - \vec{x}')}$$

Integrating out $\vec{\theta}'$

$$P_{2D}(\ell) = \int d^2\theta\, e^{-i\vec{\ell}.\vec{\theta}} \int d\chi\, W(\chi) \int d\chi' W(\chi') \int \frac{d^3k}{(2\pi)^3}\, P(k)\, e^{i\vec{k}.[\vec{x}(\chi,\theta) - \vec{x}(\chi',0]}$$

$\Downarrow$

This argument is

$$i\,[k_3(\chi - \chi') + i\,k_1\,\chi\theta_1 + i\,k_2\,\chi\,\theta_2\,]$$

$$i\,[k_3\,(\chi - \chi')] + i(k_1\,\theta_1 + k_2\theta_2)\,\chi$$

We can now do the integration over $d^2\theta$

$$\int d\theta_1\, e^{-i\ell_1\theta_1 + i\,k_1\theta_1\chi} = \int d\theta_1\, e^{-i\theta_1(\ell_1 - k_1\chi)} = \delta_D(\ell_1 - k_1\chi)$$

same for $d\theta_2$. Then we have

$$\int dk_1\, \delta_D(\ell_1 - k_1\chi) = \quad \text{using} \qquad \int \delta_D(f(x))\, dx = \frac{1}{f'(x)}\, \delta_D(x)$$

the derivative gives $1/\chi$ for each integral and

$$k = \sqrt{k_1^2 + k_2^2 + k_3^2} \longrightarrow \sqrt{\frac{\ell_1^2 + \ell_2^2}{\chi^2} + k_3^2} = \sqrt{\ell^2/\chi^2 + k_3^2}$$

So

$$P_2(\ell) = \int dx \frac{W(x)}{\chi^2} \int dx' W(x') \int_{-\infty}^{+\infty} \frac{dk_3}{(2\pi)} P(\sqrt{k_3^2 + \ell^2/\chi^2}) \, e^{ik_3(\chi-\chi')}$$

↑

if you expand this for $k_3 \sim 0$

$$P\left(\ell/\chi \left(1 + \frac{1}{2}\left[k_3 \frac{\chi}{\ell}\right]^2\right)\right)$$

$$\sim P(\ell/\chi) + \frac{1}{2} k_3^2 \frac{\chi^2}{\ell^2} P'(\ell/\chi)$$

In the small angle approximation $\ell/\chi \ll 1 \Rightarrow$ basically $\ell \sim \frac{1}{\theta}$

$$P_2(\ell) = \int dx \frac{W(x)}{\chi^2} \int dx' W(x') P(\ell/\chi) \, \delta_D(\chi-\chi') + O(1/\ell^2)$$

$$P_2(\ell) = \int_0^{\chi_\infty} dx \frac{W^2(x)}{\chi^2} P(\ell/\chi)$$

or in terms of k (doing $x \to k \equiv \ell/\chi$)

$$\boxed{P_2(\ell) = \frac{1}{\ell} \int dk \, P(k) \, W^2(\ell/k)}$$

In the small angle approx the relevant transverse modes $k_\perp = \ell/\chi$ are much larger than the longitudinal ones. k_χ.