Cosmic Acceleration – Age of the Universe

Acceleration of the Universe : Supernovae results

A lot of evidence has been gathered in the last few years that show that currently the universe is undergoing a period of accelerated expansion (somewhat analogous to ~~the~~ inflationary epoch). Here we will discuss the evidence from studying Supernovae type Ia as a function of redshift. Other lines of evidence will be discussed later in the course, as they require understanding of the growth of fluctuations.

The classic test for measuring the acceleration of the universe is to measure the luminosity distance $d_L = L/4\pi F$ as a function of redshift, as we already discussed this dependence gives information on $a(t)$ and its derivatives. For a flat universe,

$$d_L(z) = c\,(1+z) \int_0^z \frac{dz'}{H(z')}$$

and expanding near zero

$$\frac{a(t)}{a_0} \simeq 1 + H_0 (t-t_0) - \frac{1}{2} q_0 H_0^2 (t-t_0)^2 + \frac{1}{3} j_0 H_0^3 (t-t_0)^3 + \cdots$$

where as we discussed already $H_0 = \left.\frac{\dot a}{a}\right]_0$ $\quad q_0 = -\left.\frac{\ddot a}{a}\frac{1}{H^2}\right]_0$ $\quad j_0 = \left.\frac{\dddot a}{a}\frac{1}{H^3}\right]_0$

$$\Rightarrow d_L(z) = \frac{cz}{H_0}\left\{ 1 + \frac{1}{2}(1-q_0)\, z - \frac{1}{6}\left[1 - q_0 - 3q_0^2 + j_0 \right] z^2 + O(z^3) \right\}$$

Then acceleration gives a quadratic correction to Hubble's law, change in acceleration a cubic term. To study the change in acceleration it is useful to consider e.g. $q(z) = q_0 + z \left.\frac{dq}{dz}\right]_0 + \cdots$

then the transition redshift when one goes from deceleration to accelerati[on] is given by

$$z_t = \frac{-q_0}{(dq/dz)_0} = 0.46 \pm 0.13 \qquad (q(z_t) = 0)$$

Riess et al 2004

How are these observations done in practice? Let's discuss the basic idea. First, we need to switch from physicist's to astronomer's notation / conventions. Instead of ~~expressing~~ working with L and F astronomers work with magnitudes, which are logarithmic measures:

$$M : \text{absolute magnitude} \sim \log L$$
$$m : \text{apparent} \quad " \quad \sim \log F$$

In fact: $m = -2.5 \log F + \text{const.}$ (where the constant is fixed by units of F and passband over which m is measured)

What we are interested in is $m-M$ (called distance modulus) which is a logarithmic measure of d_L. M is defined as the apparent magnitude an object would have @ 10 pc, then

$$m = -2.5 \log \left[L \left(\frac{10\,pc}{d_L} \right)^2 \right] = -2.5 \log L - 5 \log \frac{10\,pc}{d_L}$$

$$\Rightarrow \mu \equiv m - M = 5 \log d_L(\text{Mpc}) + 25 \qquad \underline{\text{distance modulus}}$$

We measure m (or F) but we need a way to estimate M (or L) of sources at different redshifts to turn this into $d_L(z)$. A source with constant M (L) is said to be a standard candle. If we can find such objects in the universe, studying them as a function of z we can determine $d_L(z)$.

In the last decade or so, significant progress has been made using type Ia supernovae as "standarizable candles". Supernovae are rare (~ few / century / galaxy) but they are very bright ($M \sim -19.5$, typically comparable to the host galaxy) and therefore can be seen to high redshifts ($z \sim 1-2$), which probes cosmological evolution.

Why type Ia supernovae? to understand this, let's look at the classification of supernovae

Supernovae
- type I → deficiency of hydrogen in their spectra
  - type Ia → strong Si II } Carbon-Oxygen White Dwarf progenitors
  - type Ib → prominent He I }
  - type Ic → No Si II / He I } Massive star progenitors
- type 2 → Presence of hydrogen

type Ia: Occur in all types of galaxies, though most frequently in spiral galaxies

The progenitors are Carbon-Oxygen white dwarfs (WD) that accrete from a companion star and undergo thermonuclear runaway - Probably the WD reach the Chandrasekhar limit before exploding. The usefulness of type Ia SN rests on this property, with $M_{Ch} \simeq 1.4\, M_\odot$ being a nearly universal quantity.

Other types: Do not ocurr in elliptical galaxies, and typically are in or near spiral arms and HII regions.

The progenitors are massive stars ($M \gtrsim 8-10\, M_\odot$) that suffer core collapse (generally iron) and then rebound leaving a neutron star or a black hole.

In type Ib, Ic progenitors are thought to be stripped of their hydrogen (Ib) and helium (Ic) envelopes prior to exploding, either by mass transfer to companion stars or due to winds.

Although ideally one expects type Ia SN to be standard candles, resulting from $M_{Ch} c^2$ worth of energy in the explosion, in practice there is a ~40% scatter in peak brightness of nearby supernovae, which may be traced perhaps to difference in composition of the WD atmospheres.

However, the observed difference in peak luminosities are correlated with the shape of the light curves: dimmer SN decline faster, thus measuring the light curve one can empirically correct for this, decreasing the scatter from 40% to ~15%.

It appears that physically the main reason for this correlated behavior can be traced to the amount of ^{56}Ni produced in the SN explosion, more ^{56}Ni implies higher peak luminosity and thus higher temperature and opacity which leads to a slower decline of the light curve.

[Show SN Ia results]

Age of the Universe

Let's first assume there is a single component (either MAT or RAD). Then from

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

we have

i) MAT :
$$\dot a^2 = \frac{8\pi G}{3}\rho_M a^2 - k \overset{\rho_M \propto a^{-3}}{=} H_0^2 a_0^2 \left[\Omega_M^0 \frac{a_0}{a} - (\Omega_M^0 - 1)\right]$$

$$\int_0^{t_0} dt = \int_0^{a_0} \frac{da}{\dot a} \Rightarrow \text{age}: t_0 = \int_0^{a_0} \frac{da}{H_0 a_0 \sqrt{\Omega_M^0 \frac{a_0}{a} + 1 - \Omega_M^0}}$$

let $x \equiv \frac{a}{a_0}$ $\Rightarrow$

$$\boxed{H_0 t_0 = \int_0^1 \frac{dx}{\sqrt{\frac{\Omega_M^0}{x} + 1 - \Omega_M^0}}}$$

if $\Omega_M^0 (= \Omega_{TOT}^0) = 1$ $\Rightarrow$ $H_0 t_0 = \frac{2}{3}$

ii) RAD $\quad \dot{a}^2 = H_0^2 a_0^2 \left[\Omega_R^0 \frac{a_0^2}{a^2} + 1 - \Omega_R^0 \right]$

$$\Rightarrow \boxed{H_0 t_0 = \int_0^1 \frac{dx}{\sqrt{\frac{\Omega_0^R}{x^2} + 1 - \Omega_0^R}}} \qquad \text{if } \Omega_0^R \; (= \Omega_0^{TOT}) = 1 \;\Rightarrow\; H_0 t_0 = \frac{1}{2}$$

Notice that both expressions (for MAT and RAD) are decreasing functions of Ω_0:

$$\frac{\partial}{\partial \Omega_0} \left[\frac{1}{\sqrt{\frac{\Omega_0}{x^p} + 1 - \Omega_0}} \right] = \underbrace{-\frac{1}{2}}_{<0} \underbrace{\left(\frac{\Omega_0}{x^p} + 1 - \Omega_0 \right)^{-3/2}}_{>0} \underbrace{\left(\frac{1}{x^p} - 1 \right)}_{>0 \text{ since } x \leq 1} < 0 \qquad \left(\begin{array}{c} p = 1, 2 \text{ for} \\ \text{MAT, RAD} \end{array} \right)$$

Therefore, the age of the universe is a decreasing function of Ω_0, for fixed H_0. The reason is that larger Ω_0 means larger deceleration in MAT, RAD cases, and for a fixed expansion rate today ($= H_0$) it means larger expansion rate in the past, therefore gets from $a=0$ to $a=a_0$ faster, hence the lower age.

Let's consider now the case where there is a cosmological constant on top of matter making a flat universe $\Omega_M^0 + \Omega_\Lambda^0 = 1$ $(k=0)$

$$\Rightarrow \quad \dot{a}^2 = H_0^2 a_0^3 \frac{\Omega_M^0}{a} + H_0^2 \Omega_\Lambda^0 a^2 = H_0^2 a_0^2 \left[\frac{\Omega_M^0}{x} + \Omega_\Lambda^0 x^2 \right]$$

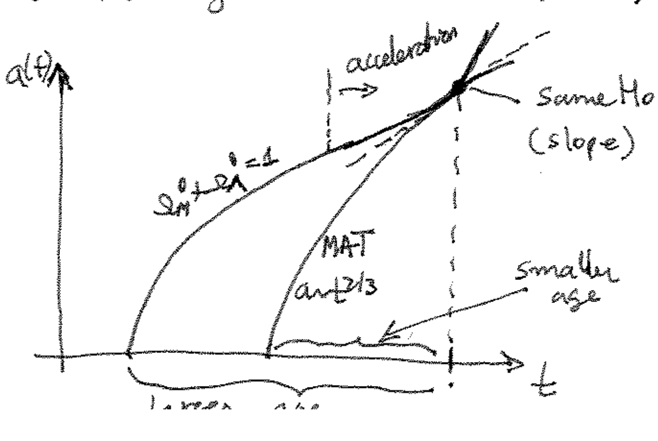
$$\Rightarrow \quad H_0 t_0 = \int_0^1 \frac{dx}{\sqrt{\frac{\Omega_M^0}{x} + \Omega_\Lambda^0 x^2}} = \frac{2}{3} \frac{1}{\sqrt{\Omega_\Lambda^0}} \ln \left[\frac{1 + \sqrt{\Omega_\Lambda^0}}{\sqrt{\Omega_M^0}} \right]$$

It's interesting to compare the age of a flat universe with only matter ($\Omega_M^0 = 1$, $\Omega_\Lambda^0 = 0$)

where $\quad H_0 t_0 = \frac{2}{3} = 0.66$

with a flat case with $\Omega_M^0 = 0.3$ $\Omega_\Lambda^0 = 0.7$ $\Rightarrow H_0 t_0 = 0.964$

$\Rightarrow$ Λ increases age for same H_0 (due to recent acceleration)



Horizon

The horizon @ ~~t=to~~ is a surface in 3D space that separates particles that have been already seen by an observer @ t_0, from those that have not yet been seen.

Since we are talking about light, consider null geodesics $ds^2=0$, for radial geodesics:

$$\text{~~coordinate~~}\left(\begin{array}{c}\text{comoving}\\ \text{distance}\end{array}\right) = \int_0^r \frac{dr}{\sqrt{1-kr^2}} = c\int_0^{t_0} \frac{dt}{a}$$

to convert from comoving distance we multiply by $a(t_0)$ to get a physical distance

$$\underline{\text{horizon}}: \quad d_H(t_0) = a(t_0) \underbrace{\int_0^{t_0} c\,\frac{dt}{a(t)}}$$

notice this is $c\,\tau_0$ ← conformal time

if $a \sim t^n$ with $n<1$ $\Rightarrow$ $d_H(t) = \dfrac{ct}{1-n}$, in particular:

$\underline{\text{MAT}}$: $a(t) \sim t^{2/3}$ $\Rightarrow$ $d_H(t) = 3ct$

$\underline{\text{RAD}}$: $a(t) \sim t^{1/2}$ $\rightarrow$ $d_H(t) = 2ct$

The fact that @ time t we can "see" up to $2ct$, $3ct$ may seem surprising, but notice that light <u>has not</u> travel that distance. The reason we get a number larger than ct is because we multiply by $a(t_0)$, the scale factor today, the comoving distance traveled by light. But $d_H(t_0)$ is the distance (physical) that an object is @ t_0, that emitted its light towards us @ the big bang (when $t=0$).

Whereas the horizon is the physical distance now (when we receive the light) for the farthest object in the universe, the past light cone $l(t)$ is the physical distance @ time t of light that reaches us @ $t = t_0$:

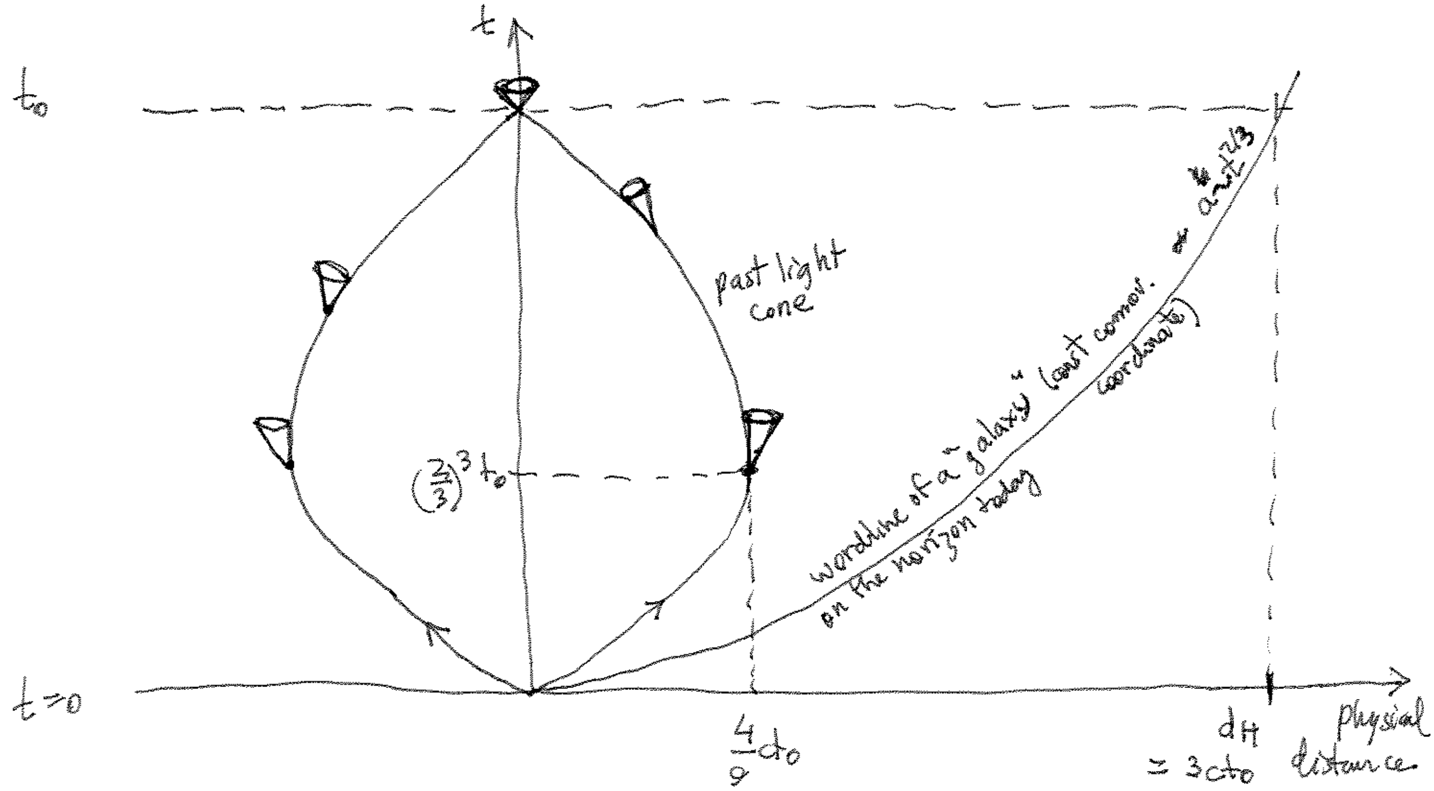
$$\text{past light cone } \; l(t) = \underbrace{a(t)}_{\substack{\text{convert to a} \\ \text{physical} \\ \text{distance @ } t}} \underbrace{\int_t^{t_0} \frac{c\,dt'}{a(t')}}_{\substack{\text{comoving distance traveled by light "emitted" @ } t \\ \text{that gets to us @ } t = t_0}}$$

For matter dominated universe: $\boxed{l(t) = 3c\left(t^{2/3}\, t_0^{1/3} - t\right)}$ (MAT)

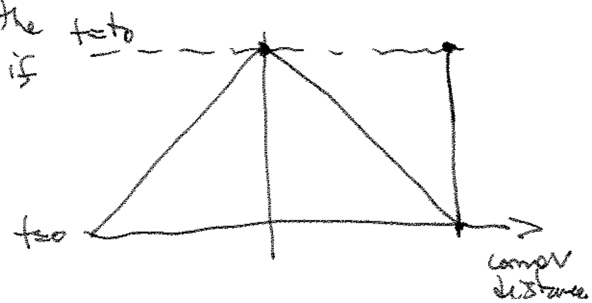
Note that this reaches a maximum $\frac{\partial l}{\partial t} = 0 \Rightarrow \frac{t}{t_0} = \left(\frac{2}{3}\right)^3 \Rightarrow l_{max} = \frac{4}{9} c t_0$

The picture of this is something like:

MAT



This looks this way due to the expansion of the universe, if we look at it in comoving coordinates it is simple:



when we multiply by $a(t)$ to convert to physical we squash the $t \to 0$ sections since $a \to 0$.

Another useful quantity is the Hubble radius, which unlike the horizon or the past light cone, is a local quantity (it does not involve integrating over the past).

Hubble radius : cH^{-1} since $H \sim \frac{1}{t}$ $H^{-1} \sim t$ is the local (at time t) time scale

(e.g. as we saw, H_0^{-1} gives the age at t_0 up to numerical coefficients)

The Hubble radius is the surface at which the recession velocity is equal to the speed of light (MAT):

$$c = v = H\, d \quad \Rightarrow \quad d = cH^{-1}$$

Note that the recession speed of the horizon is not c, but rather

$$v = H\, d_H \underset{\substack{\uparrow \\ \text{MAT}}}{=} \frac{2}{3t}\, 3ct = 2c$$

(so $d_H = 2cH^{-1}$ in MAT, in RAD $d_H = cH^{-1}$)

The importance of the Hubble radius rests on the property that it is a local quantity and therefore it is the quantity that shows up in equations of motion ~~as~~ characterizing time scales.

Another important concept is the physical distance of a wordline of an object that is at comoving coordinate = const. away from us. This is also refer to as "wavelength" because we can think of a given comoving scale λ_{comov} = const. being stretched by the expansion of the universe $\boxed{\lambda_{phys} = a(t)\, \lambda_{comov} \sim a(t)}$

like any physical distance.

Now note:

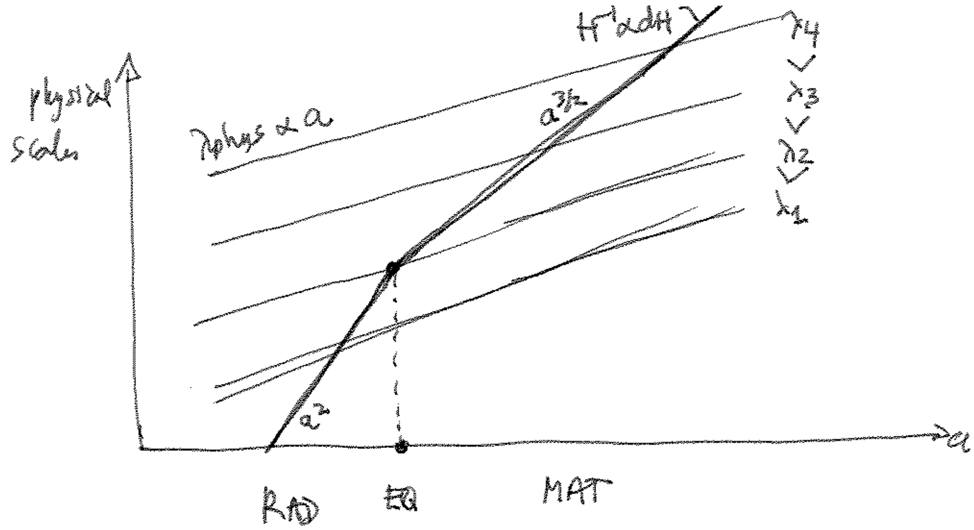
(dropping c's)

$$d_H(t) = \begin{cases} 3t & \text{MAT} \\ 2t & \text{RAD} \end{cases} \sim t \sim \begin{cases} a^{3/2} & \text{MAT} \\ a^2 & \text{RAD} \end{cases}$$

$$H^{-1} = \begin{cases} \left(\frac{2}{3t}\right)^{-1} & \text{MAT} \\ \left(\frac{1}{2t}\right)^{-1} & \text{RAD} \end{cases} \sim t \sim \begin{cases} a^{3/2} & \text{MAT} \\ a^2 & \text{RAD} \end{cases}$$

whereas $\lambda_{phys} \sim a$ always

let's plot:
(log-log)



As you wait longer and longer, more and more length scales (λ_i) become (larger and larger) visible, or in causal contact, i.e. $\lambda_{phys} < d_H \sim H^{-1}$

It is customary to say that wavelengths "cross the Hubble radius" when $\lambda_i = H^{-1}$.

Note that if $a \sim t^n$ with $n < 1$ $\Rightarrow$ $\lambda \propto a \propto t^n$

whereas $H^{-1}, d_H \sim t$ $\Rightarrow$ λ grows slower than H^{-1}, d_H

$\Rightarrow$ more and more scales become in causal contact. This is a familiar behavior, however, it introduces another puzzle in the standard big bang model.

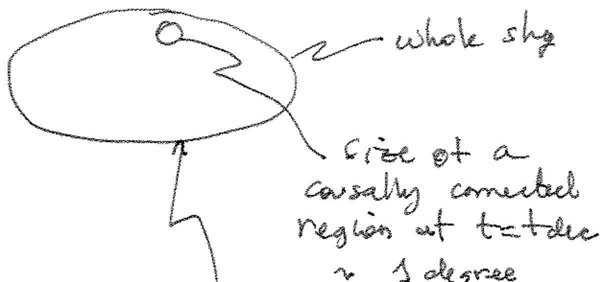
This is known as the horizon problem: Why is the temperature of the CMB the same (to within one part in 10^5) in all parts of the sky?

Lets make this more precise. Since $d_H \propto H^{-1}$ in RAD, MAT, from now on let me use just H^{-1}.

The sky we see today is roughly speaking of size H^{-1}, and T_{CMB} is pretty uniform ~~the~~ inside this volume. However the CMB was "created" or released from interaction with free electrons at the time of decoupling, which happened at about $z \sim 1100$

At that time, the size of a causally connected region was H_{dec}^{-1}, over this distance one can expect physical processes to act and make T_{CMB} the same inside this volume. However, this volume today corresponds to a very small region of our current observable sky: How did all these causally disconnected regions agree on the same temperature? This is another fine-tuning problem, known as the HORIZON PROBLEM.

In pictures:



there are 40,000 sq degrees, why all these 40,000 disconnected volumes have the same T_{CMB}?

Coming back to the plot of physical scales: