Redshift-space distortions

In galaxy surveys we don't observe true distances, but rather angles on the sky and redshifts.

Redshifts are "contaminated" by peculiar velocities, so while in ~~a~~ universe that is completely smooth recession velocities are solely due to the expansion of the universe,

$$\vec{V} = H\vec{r} \qquad \text{(Hubble)} \qquad \Rightarrow$$

redshifts are perfect indicators or radial distances

$$\vec{x} = \frac{\vec{r}}{a} = \frac{\vec{V}}{Ha} = \frac{\vec{V}}{\mathcal{H}}$$

along the line of sight

in the presence of inhomogeneities we have $\vec{V} \to \vec{V} + \vec{v}_{pec}$

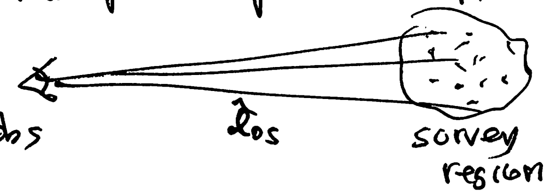
$$\Rightarrow \underset{\substack{\text{redshift}\\ \text{comov position}}}{\vec{s}} = \underset{\substack{\text{true}\\ \text{comoving}\\ \text{position}}}{\vec{x}} + \underbrace{\left(\frac{\vec{v}_{pec}\cdot\hat{l}_{os}}{\mathcal{H}}\right)\hat{l}_{os}}_{\substack{\text{contamination}\\ \text{along the}\\ \text{line of sight}}}$$

We derived that $\vec{v}_{pec} = -\mathcal{H} f\, \vec{u}$

where $\vec{u}$ can be calculated in perturbation theory (PT). In linear PT $\Theta \equiv \nabla\cdot\vec{u} = \delta_0$, in general

$$\underset{(k,z)}{\Theta} = \sum_n D_+^n(z) \int G_n(\vec{q}_1, \dots, \vec{q}_n)\, \delta_D(\vec{k} - \Sigma\vec{q}_i)\, \delta_0(\vec{q}_1)\cdots\delta_0(\vec{q}_n)\, d^3\vec{q}_1\cdots d^3\vec{q}_n$$

if we take the observer to be very far away from the region observed, we can work in the plane parallel approx.



where $\hat{l}_{os} \approx$ fixed.

say, $\hat{l}_{os} \simeq \hat{z}$

$$\Rightarrow \boxed{\vec{s} = \vec{x} - f u_z \hat{z}}$$ redshift-space mapping

lets work out what density fluctuations we see in redshift-space.

$$[1+\delta_s(\vec{s})]\, d^3s = [1+\delta(\vec{x})]\, d^3x$$ (map conserves number of objects)

$$\Downarrow \int \frac{e^{-i\vec{k}\cdot\vec{s}}}{(2\pi)^3}$$

$$\int e^{-i\vec{k}\cdot\vec{s}} \frac{d^3s}{(2\pi)^3} (1+\delta_s) = \delta_D(\vec{k}) + \delta_s(\vec{k}) = \int \frac{d^3x}{(2\pi)^3} e^{-i\vec{k}\cdot(\vec{x}-fu_z\hat{z})} (1+\delta)$$

$$\Rightarrow \boxed{\delta_D(\vec{k}) + \delta_s(\vec{k}) = \int e^{-i\vec{k}\cdot\vec{x}} \frac{d^3x}{(2\pi)^3} [1+\delta(\vec{x})]\, e^{i f k_z u_z(\vec{x})}}$$

therefore δ_s is given by a nonlinear map from δ and u_z.

Let's be naive and expand in PT the exponential factor,

$$\delta_D + \delta_s(\vec{k}) = \int e^{-i\vec{k}\cdot\vec{x}} \frac{d^3x}{(2\pi)^3} [1+\delta] (1 + i f k_z u_z(\vec{x}) + \cdots)$$

$$\simeq \delta_D + \delta(\vec{k}) + i f k_z u_z(\vec{k})$$

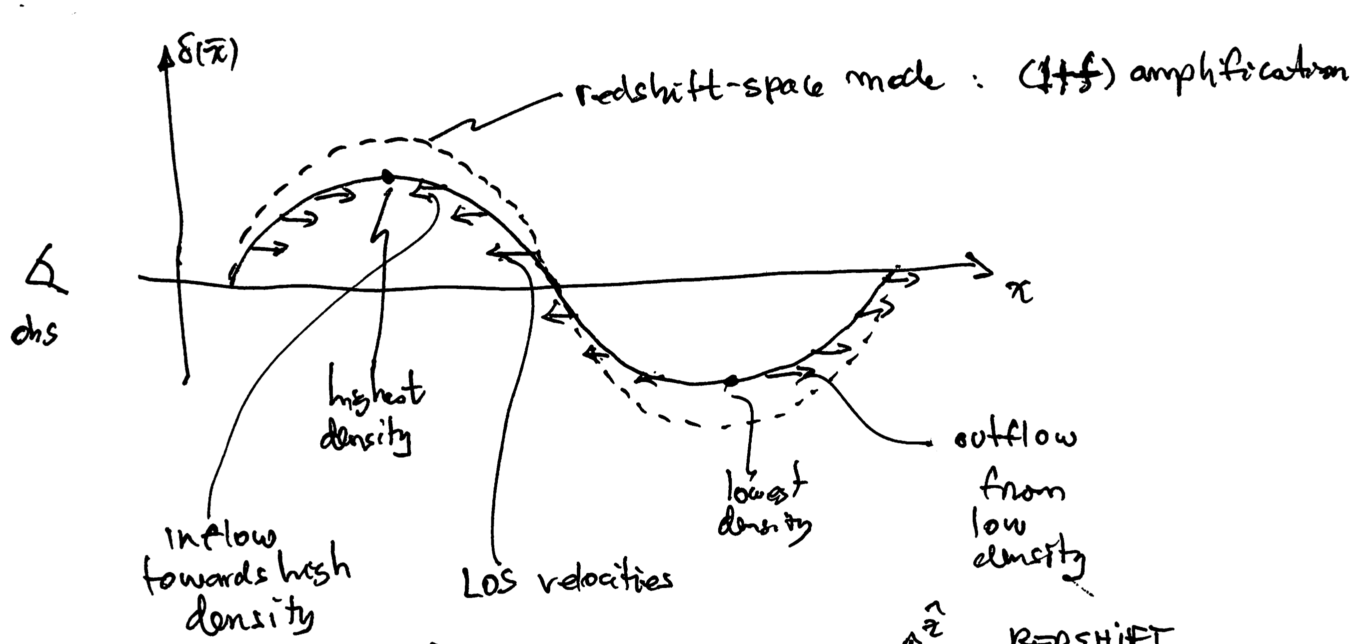
or $$\delta_s(\vec{k}) = \delta(\vec{k}) + i f k_z u_z(\vec{k})$$

in linear PT $\nabla\cdot\vec{u} = \delta \Rightarrow$ $$\vec{u}(\vec{k}) = \frac{-i\vec{k}}{k^2}\Theta(\vec{k}) = \frac{-i\vec{k}}{k^2}\delta(\vec{k})$$

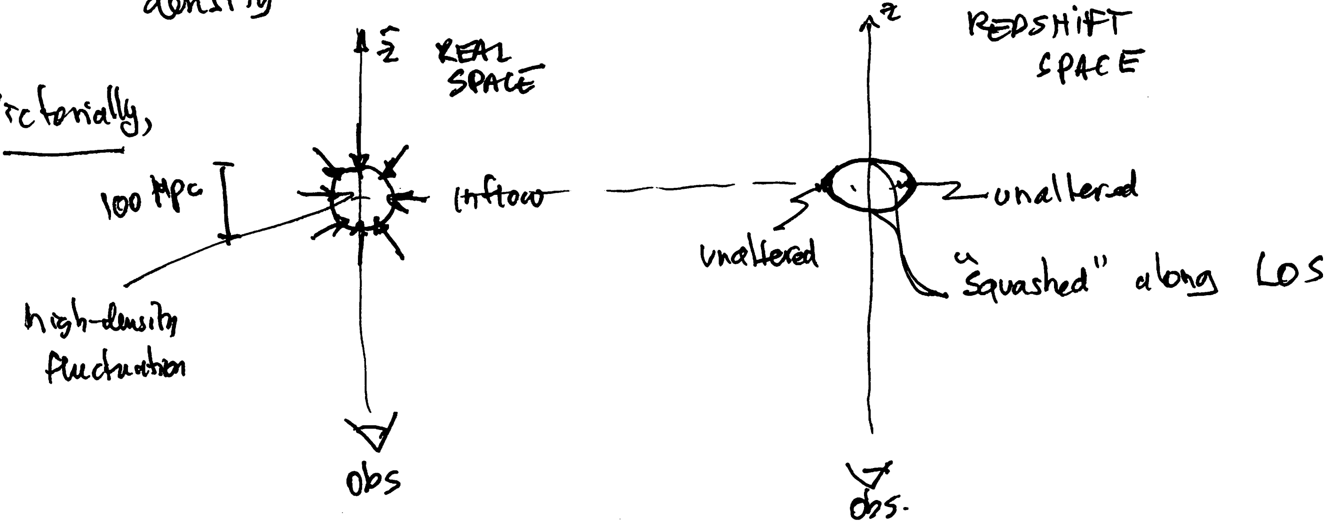
$$\Rightarrow \boxed{\delta_s(\vec{k}) = \delta(\vec{k}) + f\, i k_z \frac{-i k_z}{k^2} \delta(\vec{k}) \equiv (1+f\mu^2)\,\delta(\vec{k})}$$ (Kaiser 87)

where $\mu \equiv \frac{k_z}{k}$ is the cosine of wave dir wrt LOS ($\hat{z}$).

This says that $\perp$ to the LOS $\delta_s = \delta$ ($\mu=0$) but $\parallel$ to LOS ($\mu=1$) the density fluctuations are enhanced by $1+f$. To understand this, let us look at a single k mode parallel to LOS:



Pictorially,



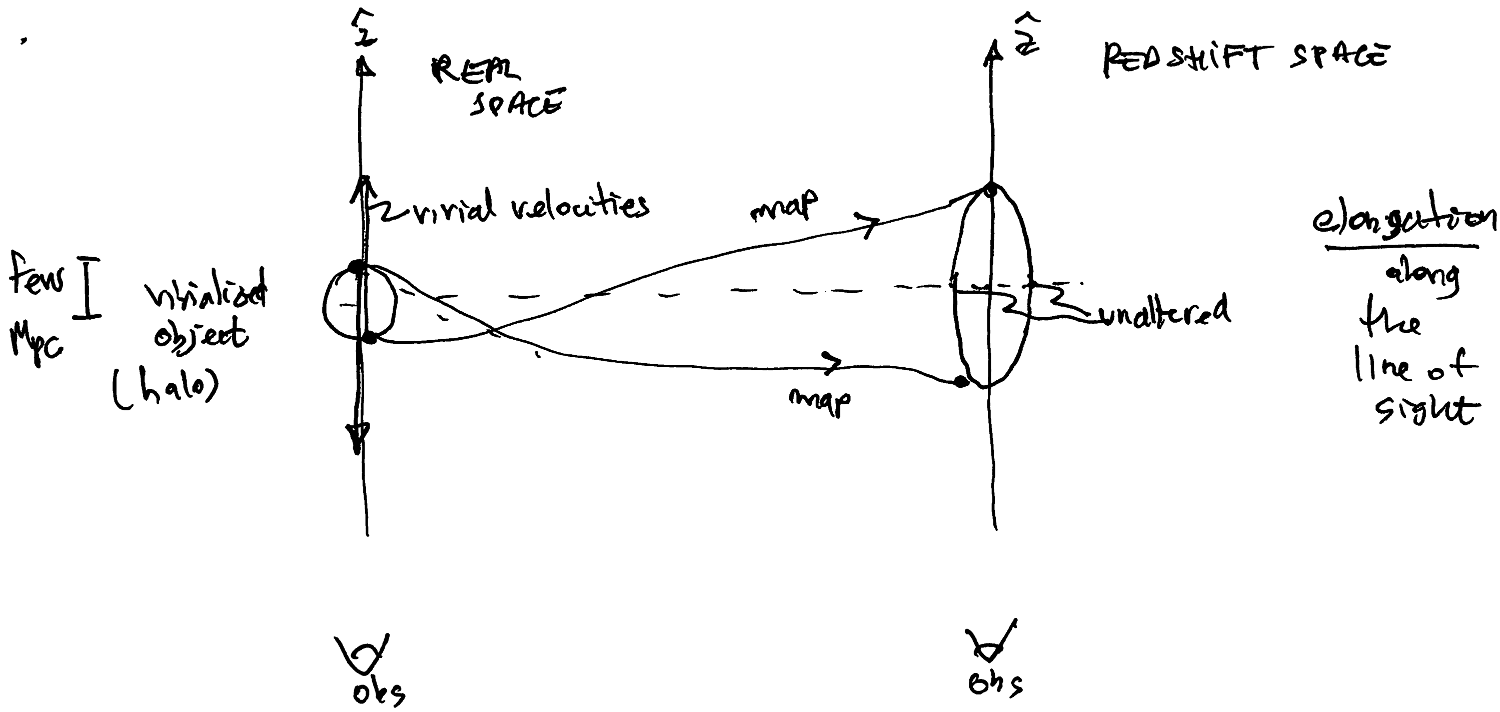
that means that a spherical overdensity contour gets squashed along the line of sight $\Rightarrow$ quadrupole gets generated in redshift space.

So far we assume velocities are small (by expanding the exponential). Eventually, at small scales corresponding to virialized structures (halos of size few Mpc, velocity dispersions of 10^3 km/s) the velocities are larger than the typical size of an object, i.e.

$$\underbrace{\frac{10^3\,\mathrm{km/s}}{100\,\frac{\mathrm{km}}{\mathrm{s}}\frac{h}{\mathrm{Mpc}}} \sim 10\,\mathrm{Mpc}/h}_{\text{z-space displacements inside } 10^3 \frac{\mathrm{km}}{\mathrm{s}} \text{ halo}} > \underbrace{3\,\mathrm{Mpc}}_{\text{size of halo}}$$

[This is a property of virialized systems as $v^2 \sim \frac{GM}{R} \sim R^2$ $\Rightarrow \frac{v}{H} \sim R$ and prefactor is >1 $\simeq$ indep. of halo size]

So we have instead the following picture



So now we get elongation along the line of sight, i.e. a quadrupole of opposite sign than at large scales. ("Finger of God" effect)

Let us work out the power spectrum in redshift space. First in linear theory is easy,

$$\underbrace{\langle \delta_s(\vec{k})\, \delta_s(\vec{k}') \rangle}_{P_s(\vec{k})\, \delta_D(\vec{k}+\vec{k}')} = (1+f\mu^2)(1+f\mu'^2) \underbrace{\langle \delta(\vec{k})\, \delta(\vec{k}') \rangle}_{P(k)\, \delta_D(\vec{k}+\vec{k}')} \Rightarrow \mu^2 = \mu'^2$$

$$\Rightarrow \boxed{P_s(\vec{k}) = P(k)\,(1+f\mu^2)^2}$$

or we can write it in terms of multipoles

$$P_s(\vec{k}) = \sum_{\ell=0,2,4} \mathcal{L}_\ell(\mu)\, P^{(\ell)}_{(k)} \qquad P^{(\ell)}_{(k)} = (2\ell+1)\int_{-1}^{1} P_s(\vec{k})\, \mathcal{L}_\ell(\mu)\, \frac{d\mu}{2}$$

$$P^{(0)}_{(k)} = \left(1 + \frac{2}{3} f + \frac{f^2}{5}\right) P(k) \qquad P^{(4)}_{(k)} = \frac{8}{35} f^2\, P(k)$$

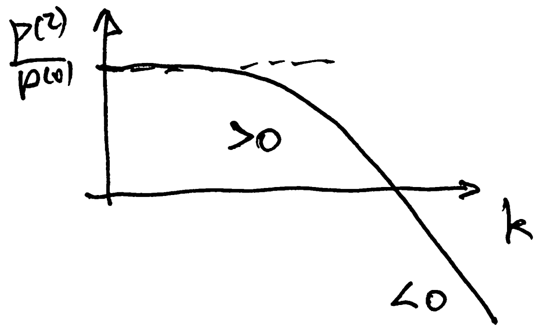
$$P^{(2)}_{(k)} = \left(\frac{4}{3} f + \frac{4}{7} f^2\right) P(k)$$

Thus, the quadrupole to monopole ratio,

$$R \equiv \frac{P^{(2)}_{(k)}}{P^{(0)}_{(k)}} = \frac{\frac{4}{3} f + \frac{4}{7} f^2}{1+\frac{2}{3} f + \frac{f^2}{5}}$$

is independent of k (at large scales) and measures f. In practice, things are not so easy, in particular due to

i) deviations from Kaiser (eventually quadrupole must change sign)



ii) galaxies are biased – let's work things out for linear bias, assuming galaxies velocities agree with matter velocities. Then:

$$(b_1 + f\mu^2)\,\delta^{mass}_{(\vec k)} \sim (b_1\sigma_8 + f\sigma_8\mu^2)$$

$$\delta_S^{gal}(\vec k) = \delta^{gal}_{(\vec k)} + f\,\frac{k_z^2}{k^2}\,\delta^{mass}_{(\vec k)} = \left(1 + \frac{f}{b_1}\mu^2\right)\underbrace{\delta^{gal}_{(\vec k)}}_{}{}^{\propto b_1\sigma_8}$$

where $\beta \equiv f/b_1$ is the new measure of squashing of galaxy fluctuations. $\Rightarrow R = R(\beta)$ [We could measure $b_1\sigma_8$ & β; or $b_1\sigma_8$ and $f\sigma_8$]

A simple phenomenological model often used to incorporate both these effects is the so-called "dispersion model"

$$P_S^{gal}(\vec k) = \underbrace{P_g(k)}_{\substack{\text{real-space}\\\text{galaxy}\\\text{power}}}\;\underbrace{(1+\beta\mu^2)^2}_{\substack{\text{squashing}\\\text{(enhances}\\\mu=1)}}\;\underbrace{\frac{1}{1+k^2\mu^2\sigma_p^2/2}}_{\substack{\text{suppresses}\\\mu=1\text{ at}\\\text{high-}k}}$$

σ_p : pairwise velocity dispersion (from halos)

To understand how this result might arise, we go back to page 2 and write a slightly different expression for $\delta_s(\vec{k})$:

$$[1+\delta_s(\vec{s})]\, d^3s = [1+\delta(\vec{x})]\, d^3x$$

for the mapping $\vec{s} = \vec{x} - f u_z \hat{z}$, the Jacobian is $\left|\frac{d^3s}{d^3x}\right| = J$

with $J = |1 - f\nabla_z u_z|$ so we write ~~[illegible]~~ $J \simeq 1 - f\nabla_z u_z$

$$1+\delta_s(\vec{s}) = \frac{1+\delta(\vec{x})}{J} \Rightarrow \delta_s(\vec{s}) = \frac{1+\delta(\vec{x})-J}{J} = \frac{\delta(\vec{x})+f\nabla_z u_z}{J}$$

$$\Rightarrow \delta_s(\vec{k}) = \int \delta_s(\vec{s}) \frac{d^3s}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{s}} = \int (\delta + f\nabla_z u_z) \frac{d^3s}{(2\pi)^3 J} e^{-i\vec{k}\cdot(\vec{x}-fu_z\hat{z})}$$

d^3x

$$\Rightarrow \delta_s(\vec{k}) = \int \frac{d^3x}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} [\delta(\vec{x}) + f\nabla_z u_z(\vec{x})]\, e^{if k_z u_z}$$

We want the power spectrum, so putting two such fields together and using translation invariance we have

$$P_s(\vec{k}) = \int \frac{d^3r}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{r}} \left\langle e^{ifk_z \Delta u_z} [\delta(\vec{x}) + f\nabla_z u_z(\vec{x})][\delta(\vec{x}') + f\nabla_z' u_z(\vec{x}')] \right\rangle$$

Where $\Delta u_z \equiv u_z(\vec{x}) - u_z(\vec{x}')$ and $r \equiv \vec{x} - \vec{x}'$.

Let's make some drastic approximations: i) assume that the exponential factor can be factored out, treated as independent from [ ]'s

then

$$P_s(\vec{k}) = \int \frac{d^3r}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{r}} \langle e^{ifk_z \Delta u_z}\rangle \langle [\delta + f\nabla_z u_z][\delta' + f\nabla_z u_z'] \rangle$$

ii) for a Gaussian field we know that

$$\langle e^{\lambda \Delta u_z}\rangle = e^{\frac{\lambda^2}{2}\langle \Delta u_z^2\rangle} = e^{-f^2 k^2 \mu^2 \langle \Delta u_z^2\rangle/2}$$

which gives qualitatively the expected suppression at high-k by velocities - Of course assuming a Gaussian field for small-scale velocities its dangerous, but motivated by simulations iii) a Lorentzian suppression with scale-independent dispersion $f^2 \langle \Delta v_z^2 \rangle \rightarrow \sigma_p^2$ works well, leading to

$$P_s(\vec{k}) = \int \frac{d^3r}{(2\pi)^3} \frac{1}{1+k^2\mu^2\sigma_p^2/2} \langle [\delta + f\nabla_z v_z][\delta' + f\nabla_z v_z'] \rangle$$

$$\underset{\substack{\uparrow \\ \sigma_p^2 \text{ indep of } r}}{=} \frac{1}{1+k^2\mu^2\sigma_p^2/2} (1+f\mu^2)^2 P(k)$$

Adding linear bias $f \rightarrow \frac{f}{b_1} = \beta$ $P \rightarrow b_1^2 P = P_g$ gives:

$$P_s(\vec{k}) = P_g(k) \frac{(1+\beta\mu^2)^2}{1+k^2\mu^2\sigma_p^2/2}$$

Given the drastic approximations this cannot be expected to work quantitatively to better than 10-20%, but it gives a qualitative understanding of what's going on -