DNL/MCA 2020
Práctica 12/06/2020
Cátedra Mindlin

Orador hoy: Santiago Boari

Formas Normales: The End

Forma normal para sistemas linealmente oscilatorios (repaso)

$$DF = \begin{pmatrix} 0 & -\beta \\ \beta & 0 \end{pmatrix}, \qquad \text{con autovalores} \quad \lambda = \pm i\beta.$$

$$v_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 1 \\ i \end{pmatrix}$$

de modo que la transformación T puede escribirse como:

$$T = \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}, \text{por lo que } T^{-1} = \begin{pmatrix} 1/2 & i/2 \\ 1/2 & -i/2 \end{pmatrix}.$$

Forma normal para sistemas linealmente oscilatorios (repaso)

$$X = TZ,$$

o sea que

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix},$$

o

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 1/2 & i/2 \\ 1/2 & -i/2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Rightarrow$$

$$z_1 = {z_2}^*$$

Forma normal para sistemas linealmente oscilatorios (repaso)

$$X = TZ,$$
$$T^{-1}X = Z$$

Quiero entonces ecuaciones en mis nuevas variables Z.

$$\dot{X} = \begin{pmatrix} 0 & -\beta \\ \beta & 0 \end{pmatrix} X + NL$$

$$\dot{Z} = T^{-1}\dot{X} = T^{-1}\begin{pmatrix} 0 & -\beta \\ \beta & 0 \end{pmatrix} TZ + T^{-1}NL(Z)$$

$$\dot{Z} = T^{-1}\dot{X} = \begin{pmatrix} i\beta & 0 \\ 0 & -i\beta \end{pmatrix} Z + T^{-1}NL(Z)$$

$$(\overline{m}.\overline{\lambda} - \lambda_{\mathrm{i}}) = (m_1\lambda_1 - m_2\lambda_1 - \lambda_{\mathrm{i}}) = 0,$$

Forma normal para sistemas linealmente oscilatorios (repaso)

$$\frac{d}{dt}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} i\beta & 0 \\ 0 & -i\beta \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \sum_{1}^{N} a_k \begin{pmatrix} {z_1}^{k+1}{z_2}^{k} \\ 0 \end{pmatrix} + b_k \begin{pmatrix} 0 \\ {z_1}^{k}{z_2}^{k+1} \end{pmatrix}$$

A orden 3:

$$\boxed{\frac{dz}{dt} = i\beta z + a_k z^2 z^*.}$$

Formas normales: el ejercicio que quedó

Ejercicio: Sea el Sistema de ecuaciones:

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -k\gamma^2(x + \epsilon x^3) + \beta\gamma y - \gamma x^2 y$$

Mostrar que cuando nacen las oscilaciones,

$$\omega = \gamma\sqrt{k}(1 + 3\epsilon\beta)$$

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -k\gamma^2(x + \epsilon x^3) + \beta\gamma y - \gamma x^2 y$$

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esta perfecto, pero el determinanten queda con raiz :)

From Lorenzo Gutierrez to Everyone:
Raiz va me parece

From Sol Luque to Everyone:
si

From Lorenzo Gutierrez to Everyone:
sisi

From Valentín Agulló to Everyone:
yo compro

From Valentín Agulló to Everyone:
genial

From Eric Lescano to Everyone:
joya

To: Facundo Fainstein (Privately)

Type message here...

$\beta = 0$

Hopf

$$\begin{cases} x' = \bar{y} \\ \bar{y}' = -(x+\epsilon x^3) + \frac{\beta}{\sqrt{k}}\bar{y} - \frac{1}{\sqrt{k}}x^2\bar{y} \end{cases}$$

$$\begin{pmatrix} x' \\ \bar{y}' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & \beta/\sqrt{k} \end{pmatrix}\begin{pmatrix} x \\ \bar{y} \end{pmatrix} + \begin{pmatrix} \\ -\epsilon x^3 - \frac{1}{\sqrt{k}}x^2\bar{y} \end{pmatrix}$$

$DF(0,0)$

$\tau = \beta/\sqrt{k}$

$\Delta = 1$

$$\boxed{\lambda_\pm = \frac{\beta}{2\sqrt{k}} \pm i \quad \beta \sim 0}$$

$$\frac{\Gamma\, dy}{d\tau} = -k\gamma^2\Gamma(x+\epsilon x^3) + \beta\gamma\Gamma y - \gamma\Gamma x^2 y$$

$$\begin{pmatrix} 0 & -\beta \\ \beta & 0 \end{pmatrix} \quad \pm i\beta$$

$$\lambda_\pm = \frac{\tau}{2} \pm \frac{\sqrt{\tau^2 - 4\Delta}}{2}$$

$\tau \sim 0$

$\tau^2 < 4\Delta$

$$\lambda_\pm = \frac{\tau}{2} \pm \frac{i\sqrt{4\Delta - \tau^2}}{2}$$

$$\lambda_\pm = \frac{\tau}{2} \pm i\sqrt{\Delta}\sqrt{1 - \frac{\tau^2}{4\Delta}}$$

$\tau \sim 0$

$$\boxed{\lambda_\pm \simeq \frac{\tau}{2} \pm i\sqrt{\Delta}} \quad \sim (1 - (\tau)^2)$$

$$\sqrt{1-x^2} \simeq 1 - \frac{x^2}{2} + \dots$$

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$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -k\gamma^2(x + \epsilon x^3) + \beta\gamma y - \gamma x^2 y$$

$\beta = 0$ Hopf

$$\begin{cases} x' = \bar{y} \\ \bar{y}' = -(x + \epsilon x^3) + \frac{\beta}{\sqrt{k}}\bar{y} - \frac{1}{\sqrt{k}}x^2\bar{y} \end{cases}$$

$$\begin{pmatrix} x' \\ \bar{y}' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ & 0 \end{pmatrix}\begin{pmatrix} x \\ \bar{y} \end{pmatrix} + \begin{pmatrix} 0 \\ -\epsilon x^3 - \frac{1}{\sqrt{k}}x^2\bar{y} \end{pmatrix}$$

$= \pm i$

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \qquad T^{-1} = \frac{1}{\det(T)}\begin{pmatrix} -i & -1 \\ -i & 1 \end{pmatrix}$$

$$T^{-1} = \begin{pmatrix} 1/2 & -i/2 \\ 1/2 & i/2 \end{pmatrix}$$

→ cambio de variables $X = TZ$

$$\begin{pmatrix} x \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\begin{pmatrix} z \\ z^* \end{pmatrix} \rightarrow \begin{pmatrix} x \\ \bar{y} \end{pmatrix} = \begin{pmatrix} z + z^* \\ i(z - z^*) \end{pmatrix}$$

$$Z' = T^{-1}X' = T^{-1}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}TZ + T^{-1}\begin{pmatrix} 0 \\ -\epsilon(z+z^*)^3 - \frac{i}{\sqrt{k}}(z+z^*)^2(z-z^*) \end{pmatrix}$$

Zoom Group Chat

From Lorenzo Gutierrez to Everyone:
viene re bien! gracias

From Debora Copacopa to Everyone:
ahora Tz y evaluar la parte no lineal
si

From Rosario Zimmermann to Everyone:
si

From Lorenzo Gutierrez to Everyone:
sip

To: Facundo Fainstein (Privately)

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Talking:

$$\frac{dx}{dt} = y$$

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$$\frac{dy}{dt} = -k\gamma^2(x + \epsilon x^3) + \beta\gamma y - \gamma x^2 y$$

$\beta = 0$ Hopf

$$\begin{cases} x' = \bar{y} \\ \bar{y}' = -(x + \epsilon x^3) + \frac{\beta}{\sqrt{k}}\bar{y} - \frac{1}{\sqrt{k}} x^2 \bar{y} \end{cases}$$

$$\begin{pmatrix} x' \\ \bar{y}' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ \bar{y} \end{pmatrix} + \begin{pmatrix} 0 \\ -\epsilon x^3 - \frac{1}{\sqrt{k}} x^2 \bar{y} \end{pmatrix}$$

$\rightarrow$ cambio de variables $X = TZ$

$$T = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \qquad T^{-1} = \frac{1}{\det(T)}$$

$$\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\boxed{\pm i}$$

$$Z' = T^{-1}X' = \boxed{T^{-1}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} T} Z + T^{-1}\begin{pmatrix} 0 \\ -\epsilon(z+z^*)^3 - \frac{i}{\sqrt{k}}(z+z^*)^2(z-z^*) \end{pmatrix}$$

$$T^{-1} = \begin{pmatrix} 1/2 & -i/2 \\ 1/2 & i/2 \end{pmatrix}$$

$$\begin{pmatrix} z' \\ z^{*\prime} \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} z \\ z^* \end{pmatrix} + \begin{pmatrix} 1/2 & -i/2 \\ 1/2 & i/2 \end{pmatrix}\begin{pmatrix} 0 \\ -\epsilon(z+z^*)^3 - \frac{i}{\sqrt{k}}(z+z^*)^2(z-z^*) \end{pmatrix}$$

Zoom Group Chat

ni se hace :P

From Valentín Agulló to Everyone:
totalmente

From Sol Luque to Everyone:
si

From Alejandro Martinez to Everyone:
sis

From Rosario Zimmermann to Everyone:
si

To: Facundo Fainstein (Privately)

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$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -k\gamma^2(x+\epsilon x^3) + \beta\gamma y - \gamma x^2 y$$

$\beta = 0$ Hopf

$$\begin{cases} x' = \bar{y} \\ \bar{y}' = -(x+\epsilon x^3) + \frac{\beta}{\sqrt{k}}\bar{y} - \frac{1}{\sqrt{k}} x^2\bar{y} \end{cases}$$

$$\begin{pmatrix} x' \\ \bar{y}' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ \bar{y} \end{pmatrix} + \begin{pmatrix} 0 \\ -\epsilon x^3 - \frac{1}{\sqrt{k}} x^2 \bar{y} \end{pmatrix}$$

$\rightarrow$ cambio de variables $X = TZ$

$$Z' = T^{-1}X' = T^{-1}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} T Z + T^{-1}\begin{pmatrix} 0 \\ -\epsilon(z+z^*)^3 - \frac{i}{\sqrt{k}}(z+z^*)^2(z-z^*) \end{pmatrix}$$

$$\begin{pmatrix} z' \\ z^{*\prime} \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} z \\ z^* \end{pmatrix} + \begin{pmatrix} +\frac{i}{2}\left(+\epsilon(z+z^*)^3 + \frac{i}{\sqrt{k}}(z+z^*)^2(z-z^*)\right) \\ \frac{i}{2}\left(-\epsilon(z+z^*)^3 - \frac{i}{\sqrt{k}}(z+z^*)^2(z-z^*)\right) \end{pmatrix}$$

$$z' = iz + \frac{i}{2}\left[\epsilon\left(z^3 + 3z^2z^* + 3zz^{*2} + z^{*3}\right) + \frac{i}{\sqrt{k}}\left(z^3 + z^2z^* - zz^{*2} - z^{*3}\right)\right]$$

$$z' = iz + \frac{i}{2}\left[3\epsilon + \frac{i}{\sqrt{k}}\right] z|z|^2 + O(5)$$

$\lambda = \pm i$

$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \qquad T^{-1} = \frac{1}{\det(T)}\begin{pmatrix} -i & -1 \\ -i & 1 \end{pmatrix}$$

$$T^{-1} = \begin{pmatrix} 1/2 & -i/2 \\ 1/2 & i/2 \end{pmatrix}$$

$$\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ -\epsilon(z+z^*)^3 - \frac{i}{\sqrt{k}}(z+z^*)^2(z-z^*) \end{pmatrix}$$

$z^2 z^* \quad z|z|^2$

$zz^* = |z|^2$

$z^{k+1} z^{*k}$

Zoom Group Chat

From Eric Lescano to Everyone:
jajajaj

From Valentín Agulló to Everyone:
eso lleva una cte no?

From Sol Luque to Everyone:
si

From Valentín Agulló to Everyone:
del cambio de variables ?
okas

To: Facundo Fainstein (Privately)

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Fri 12:35 PM Santiago Boari

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perfecto

From Eric Lescano to Everyone:
bien

From Lorenzo Gutierrez to Everyone:
si joya

From Felipe Cignoli to Everyone:
si

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To: Facundo Fainstein (Privately)

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$t = \Gamma \tau$

$dt = \Gamma d\tau$

$z' = iz + \frac{i}{2}\left[3\epsilon + \frac{i}{\sqrt{k}}\right] |z|^2$

$z = \rho e^{i\phi} \qquad |z| = \rho$

$z' = \rho' e^{i\phi} + i\rho\phi' e^{i\phi}$

$\rho' \cancel{e^{i\phi}} + i\rho\phi' \cancel{e^{i\phi}} = \left(\frac{\beta}{2\sqrt{k}} + i\right)\rho \cancel{e^{i\phi}} + \frac{1}{2}\left(3\epsilon i - \frac{1}{\sqrt{k}}\right)\rho^3 \cancel{e^{i\phi}}$

Re $\left\{ \rho' = \frac{\beta}{2\sqrt{k}}\rho - \frac{1}{2\sqrt{k}}\rho^3 \right.$

Im $\left\{ \rho\phi' = \chi + \frac{3}{2}\epsilon\rho^{2} \right.$

$\longrightarrow \quad \frac{1}{\Gamma\sqrt{k}}$

$' \to \frac{d}{d\tau} \quad dt \quad \Gamma$

$\dot{\rho} = \Gamma\sqrt{k}\left(\frac{\beta\rho}{2\sqrt{k}} - \frac{1}{2\sqrt{k}}\rho^3\right)$

$\dot{\phi} = \Gamma\sqrt{k}\left(1 + \frac{3}{2}\epsilon\rho^2\right)$

$\lambda_\pm = \pm i$

$A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\beta = 0$

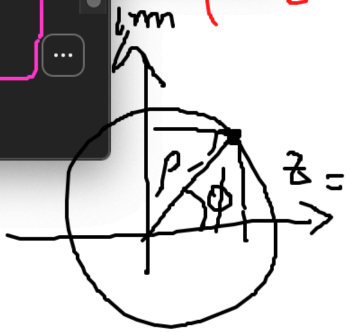
$\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} z \\ z^* \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\beta \sim 0$

$\begin{pmatrix} \frac{\beta}{2\sqrt{k}} + i & 0 \\ 0 & \frac{\beta}{2\sqrt{k}} - i \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ -1 & \beta/\sqrt{k} \end{pmatrix}$



$z = \rho e^{i\phi}$

$\cos\phi + i\sin\phi$

$\lambda_\pm = \frac{\beta}{2\sqrt{k}} \pm i$

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Fri 12:37 PM Santiago Boari

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From Eric Lescano to Everyone:
sisi
vamos!

From Alejandro Pérez Velilla to Everyone:
medio quemado jaja

From Eric Lescano to Everyone:
se pudo

From Facundo Fainstein to Me: (Privately)
impecable

To: Facundo Fainstein (Privately)

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$$dt = \Gamma d\tau$$

$$z' = iz + \frac{i}{2}\left[3\epsilon + \frac{i}{\sqrt{k}}\right] z|z|^2$$

$$\boxed{z' = \left(\frac{\beta}{2\sqrt{k}} + i\right) z + \frac{i}{2}\left[3\epsilon + \frac{i}{\sqrt{k}}\right] z|z|^2}$$

$$z = \rho e^{i\phi} \quad |z| = \rho$$

$$z' = \rho' e^{i\phi} + i\rho\phi' e^{i\phi}$$

$$\rho' \cancel{e^{i\phi}} + i\rho\phi' \cancel{e^{i\phi}} = \left(\frac{\beta}{2\sqrt{k}} + i\right)\rho \cancel{e^{i\phi}} + \frac{1}{2}\left(3\epsilon i - \frac{1}{\sqrt{k}}\right)\rho^3 \cancel{e^{i\phi}}$$

Re $\left\{ \rho' = \frac{\beta}{2\sqrt{k}}\rho - \frac{1}{2\sqrt{k}}\rho^3 \right.$

Im $\left\{ \rho\phi' = \chi + \frac{3}{2}\epsilon\rho^2 \right.$

$$' \to \frac{d}{d\tau} \quad dt \quad \Gamma \qquad \frac{1}{\Gamma\sqrt{k}}$$

$$\begin{cases} \dot{\rho} = \Gamma\sqrt{k}\left(\frac{\beta\rho}{2\sqrt{k}} - \frac{1}{2\sqrt{k}}\rho^3\right) \\ \dot{\phi} = \Gamma\sqrt{k}\left(1 + \frac{3}{2}\epsilon\rho^2\right) \end{cases}$$

$$\boxed{\begin{cases} \dot{\rho} = \frac{\Gamma}{2}\rho(\beta - \rho^2) \\ \dot{\phi} = \Gamma\sqrt{k}\left(1 + \frac{3}{2}\epsilon\rho^2\right) \end{cases}}$$

$$\boxed{\lambda_\pm = \pm i}$$

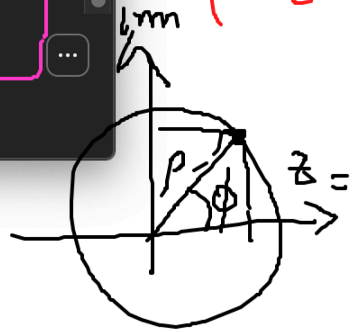
$$A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\beta = 0$$

$$\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} z \\ z^* \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\beta \sim 0$$

$$\begin{pmatrix} \frac{\beta}{2\sqrt{k}} + i & 0 \\ 0 & \frac{\beta}{2\sqrt{k}} - i \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ -1 & \beta/\sqrt{k} \end{pmatrix}$$



$$z = \rho e^{i\phi} \qquad \cos\phi + i\sin\phi$$

$$\lambda_\pm = \frac{\beta}{2\sqrt{k}} \pm i$$

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$t = \Gamma \tau$

$dt = \Gamma d\tau$

$\frac{d}{d\tau}$ $\quad \Gamma \frac{d}{dt}$ $\quad \frac{1}{\Gamma}\frac{d}{d\tau}$

$\beta = 0$

$\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} z \\ z^* \end{pmatrix}$

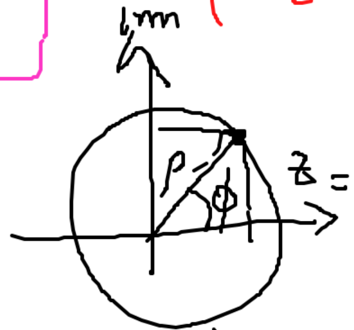
$z' = iz + \frac{i}{2}\left[3\epsilon + \frac{i}{\sqrt{k}}\right] z|z|^2$

$$z' = \left(\frac{\beta}{2\sqrt{k}} + i\right) z + \frac{i}{2}\left[3\epsilon + \frac{i}{\sqrt{k}}\right] z|z|^2$$

$\beta \sim 0$

$\begin{pmatrix} \frac{\beta}{2\sqrt{k}}+i & 0 \\ 0 & \frac{\beta}{2\sqrt{k}}-i \end{pmatrix}$

$z = \rho e^{i\phi}$ $\qquad |z| = \rho$



$z = \rho e^{i\phi}$

$\cos\phi + i\sin\phi$

¿Con qué ω nacen las oscilaciones? $\to \beta \sim 0$

(Si st c/ bif Hopf $\beta = 0$)

$\rho = cte \to$ ciclo límite

$\dot\rho = 0 \to 0 = \cancel{\frac{\gamma}{2}\rho}(\beta - \rho^2) \to \boxed{\rho^2 = \beta}$

$$\dot\phi\big|_{\rho^2=\beta} = \boxed{\gamma\sqrt{k}\left(1 + \frac{3}{2}\epsilon\beta\right) = \omega}$$

llegamos!

$\beta = 0$

$$\begin{cases} \dot\rho = \frac{\gamma}{2}\rho(\beta - \rho^2) \\ \dot\phi = \gamma\sqrt{k}\left(1 + \frac{3}{2}\epsilon\rho^2\right) \end{cases}$$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Zoom Group Chat

consulta santi, no me queda claro el tema de que nacen significa que estas parado en ropunto=0

From Enzo Gaggioli to Everyone:
digo, que phi es constante

From Eric Lescano to Everyone:
necesariamente rho tiene que tender a cero en este ciclo límite xq todo todo esto vale para z cerca de cero, no? Osea, es todo consistente

From Enzo Gaggioli to Everyone:
ok

To: Facundo Faistein (Privately)

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