Variedad central 2

Dinámica no lineal

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Variedad central con parámetros

2. Estudie los siguientes sistemas dinámicos parametrizados por ϵ. Para $\epsilon = 0$ el origen es un punto fijo. Calcule la familia de un parámetro de variedades centrales (Que significa esta pregunta?) y describa la dinámica en la variedad. Fíjese que en $\epsilon = 0$ los sistemas coinciden con los del ejercicio 1. Discuta el rol que juega el parámetro si multiplica el término lineal o nolineal.

(c)

$$i) \quad \begin{aligned} \dot{x} &= x/2 + y + x^2 y \\ \dot{y} &= x + 2y + \epsilon y + y^2 \end{aligned} \qquad ii) \quad \begin{aligned} \dot{x} &= x/2 + y + x^2 y \\ \dot{y} &= x + 2y + \epsilon y^2 + y^2 \end{aligned} \tag{6}$$

Variedad central con parámetros

$$\dot{x} = \frac{x}{2} + y + x^2 y$$

$$\dot{y} = x + 2y + \epsilon y + y^2$$

Variedad central con parámetros

$$\begin{cases} \dot{x} = \frac{x}{2} + y + x^2 y \\ \dot{y} = x + 2y + \epsilon y + y^2 \\ \dot{\epsilon} = 0 \end{cases}$$

Variedad central con parámetros

$$\begin{cases} \dot{x} = \frac{x}{2} + y + x^2 y \\ \dot{y} = x + 2y + \epsilon y + y^2 \\ \dot{\epsilon} = 0 \end{cases} \longrightarrow \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\epsilon} \end{pmatrix} = \begin{pmatrix} 1/2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ \epsilon \end{pmatrix} + \begin{pmatrix} x^2 y \\ \epsilon y + y^2 \\ 0 \end{pmatrix}$$

Variedad central con parámetros

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Rotamos (autovalores/autovectores) el bloque (x, y)

Variedad central con parámetros

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$$\begin{pmatrix} 1/2 & 1 \\ 1 & 2 \end{pmatrix} \qquad \lambda_c = 0 \qquad \lambda_i = \frac{5}{2}$$

$$v_c = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \qquad v_i = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 1/2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x^2 y \\ \epsilon y + y^2 \end{pmatrix}$$

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$$T^{-1} = \begin{pmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{pmatrix} \qquad \begin{pmatrix} u \\ v \end{pmatrix} = T^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

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$$\dot{\epsilon}=0$$

$$\begin{pmatrix}\dot{u}\\ \dot{v}\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & 5/2\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix} + \begin{pmatrix}\frac{2}{5}(x^2y) - \frac{1}{5}(\epsilon y + y^2)\\ \frac{1}{5}(x^2y) + \frac{2}{5}(\epsilon y + y^2)\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & 5/2\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix} + \begin{pmatrix}f\\ g\end{pmatrix}$$

$$\dot{\epsilon} = 0$$

$$v = v(u, \epsilon)$$

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$$v = v(u,\epsilon) = au^2 + b\epsilon^2 + cu\epsilon + \cdots$$

$$\dot{v} = \frac{dv}{du}\dot{u} + \frac{dv}{d\epsilon}\dot{\epsilon}$$

$$\begin{pmatrix}\dot{u}\\ \dot{v}\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & 5/2\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix} + \begin{pmatrix}\frac{2}{5}(x^2y) - \frac{1}{5}(\epsilon y + y^2)\\ \frac{1}{5}(x^2y) + \frac{2}{5}(\epsilon y + y^2)\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & 5/2\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix} + \begin{pmatrix}f\\ g\end{pmatrix}$$

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$$\frac{dv(u,\epsilon)}{du}f(u,v(u,\epsilon),\epsilon) = \frac{5}{2}v(+g(u,v(u,\epsilon),\epsilon)$$

$$\frac{dv(u,\epsilon)}{du} f(u, v(u,\epsilon), \epsilon) = \frac{5}{2} v + g(u, v(u,\epsilon), \epsilon)$$

$$v = v(u,\epsilon) = au^2 + b\epsilon^2 + cu\epsilon \to 2$$

$$\frac{dv(u,\epsilon)}{du} f(u, v(u,\epsilon), \epsilon) = \frac{5}{2} v + g(u, v(u,\epsilon), \epsilon)$$

$$v = v(u,\epsilon) = au^2 + b\epsilon^2 + cu\epsilon \rightarrow \boxed{2au + c\epsilon}$$

$$\frac{dv(u,\epsilon)}{du} f(u, v(u,\epsilon), \epsilon) = \frac{5}{2} v + g(u, v(u,\epsilon), \epsilon)$$

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$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 5/2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} \frac{2}{5}(x^2y) - \frac{1}{5}(\epsilon y + y^2) \\ \frac{1}{5}(x^2y) + \frac{2}{5}(\epsilon y + y^2) \end{pmatrix}$$

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$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2u + v \\ -u + 2v \end{pmatrix}$$

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$$f = O(2)$$

$$\frac{dv(u,\epsilon)}{du} f(u, v(u,\epsilon),\epsilon) = \frac{5}{2} v + g(u, v(u,\epsilon),\epsilon)$$

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$$\frac{5}{2} v = \frac{5}{2}(au^2 + b\epsilon^2 + cu\epsilon)$$

$$\frac{dv(u,\epsilon)}{du} f(u, v(u,\epsilon), \epsilon) = \frac{5}{2} v + g(u, v(u,\epsilon), \epsilon)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2u + v \\ -u + 2v \end{pmatrix}$$

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$$f = O(2)$$

$$\frac{5}{2} v = \frac{5}{2}(au^2 + b\epsilon^2 + cu\epsilon)$$

$$g = \frac{1}{5}(x^2 y) + \frac{2}{5}(\epsilon y + y^2)$$

$$\frac{dv(u,\epsilon)}{du} f(u,v(u,\epsilon),\epsilon) = \frac{5}{2}v + g(u,v(u,\epsilon),\epsilon)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2u+v \\ -u+2v \end{pmatrix}$$

$$v = v(u,\epsilon) = au^2 + b\epsilon^2 + cu\epsilon \rightarrow 2au + c\epsilon$$

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$$g = \frac{2}{5}(-u\epsilon + u^2) + O(3)$$

$$\frac{dv(u,\epsilon)}{du} f(u, v(u,\epsilon),\epsilon) = \frac{5}{2} v + g(u, v(u,\epsilon),\epsilon)$$

$$\to O(2)$$

$$\frac{dv(u,\epsilon)}{du} f(u, v(u,\epsilon), \epsilon) = \frac{5}{2} v + g(u, v(u,\epsilon), \epsilon)$$

$$\rightarrow O(2) \qquad 0 = \frac{5}{2}(au^2 + b\epsilon^2 + cu\epsilon) + \frac{2}{5}(-u\epsilon + u^2)$$

$$\frac{dv(u,\epsilon)}{du} f(u, v(u,\epsilon), \epsilon) = \frac{5}{2} v + g(u, v(u,\epsilon), \epsilon)$$

$\rightarrow O(2)$

$$0 = \frac{5}{2}(au^2 + b\epsilon^2 + cu\epsilon) + \frac{2}{5}(-u\epsilon + u^2)$$

$$v(u,\epsilon) = -\frac{4}{25}u^2 + \frac{4}{25}u\epsilon$$

$$\frac{dv(u,\epsilon)}{du} f(u, v(u,\epsilon), \epsilon) = \frac{5}{2} v + g(u, v(u,\epsilon), \epsilon)$$

$$\rightarrow O(2) \qquad 0 = \frac{5}{2}(au^2 + b\epsilon^2 + cu\epsilon) + \frac{2}{5}(-u\epsilon + u^2)$$

$$v(u,\epsilon) = -\frac{4}{25} u^2 + \frac{4}{25} u\epsilon \qquad \dot{u}\Big|_{v(u,\epsilon)} = f(u, v(u,\epsilon), \epsilon)$$

$$\dot{u}\Big|_{v(u,\epsilon)} = f(u, v(u,\epsilon), \epsilon) = \frac{2}{5}\left((2u+v)^2(-u+2v)\right) - \frac{1}{5}\left(\epsilon(-u+2v) + (-u+2v)^2\right)$$

$$\dot{u}\Big|_{v(u,\epsilon)} = f(u, v(u,\epsilon), \epsilon) = \frac{2}{5}\big((2u+v)^2(-u+2v)\big) - \frac{1}{5}(\epsilon(-u+2v) + (-u+2v)^2)$$

$$= -\frac{1}{5}(-u\epsilon + u^2) + O(3)$$

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$$= -\frac{1}{5}(-u\epsilon + u^2) + O(3)$$

Al orden más bajo no trivial

$$\dot{u} = \frac{1}{5}u(\epsilon - u)$$

Alcanza para definir la estabilidad del pf y la bifurcación