Formas normales 2

Dinámica no lineal

Cátedra G. Mindlin

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Repaso

$$\frac{d\boldsymbol{x}}{dt} = DF\boldsymbol{x} + F_r(\boldsymbol{x}) + O(|x|^{r+1})$$

$$\boldsymbol{x} = \boldsymbol{y} + h_r(\boldsymbol{y})$$

$$\frac{d\boldsymbol{y}}{dt} = DF\boldsymbol{y} + F_r(\boldsymbol{y}) - L_{DF}h_r + O(|y|^{r+1})$$

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$$\boldsymbol{x} = \boldsymbol{y} + h_r(\boldsymbol{x})$$

$$\frac{d\boldsymbol{y}}{dt} = DF\boldsymbol{y} + F_r(\boldsymbol{y}) - L_{DF}h_r + O(|y|^{r+1})$$

$Si\ \exists\ h_r\ tal\ que\ L_{DF}h_r = F_r \rightarrow podemos\ eliminar\ F_r\ haciendo$

$Si\ F_r \in Imagen(L_{DF}) \rightarrow podemos\ eliminar\ F_r$

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$$L_{DF} h_r = -(-D h_r DF \boldsymbol{y} + DF h_r) \qquad h_r \in H_r = todo\ los\ términos\ posibles\ de\ orden\ r$$

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$$h_2 \in H_2 = \left\{ \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1 x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\}$$

$\bar{x}^{\bar{m}}\ \bar{e}_j$

$$L_{DF} = op.lineal \rightarrow Imagen\ generada\ por\ la\ transf.\ de\ una\ base$$

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$\bar{x}^{\bar{m}}\ \bar{e}_j$

	$\boldsymbol{v_1}$	$\boldsymbol{v_2}$	$\boldsymbol{v_3}$	$\boldsymbol{v_4}$	$\boldsymbol{v_5}$	$\boldsymbol{v_6}$
	$\begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}$	$\begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}$	$\begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}$	$\begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} x_1 x_2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} x_2^2 \\ 0 \end{pmatrix}$
m_1	2	1	0	2	1	0
m_2	0	1	2	0	1	2
j	2	2	2	1	1	1

$L_{DF} = op.\,lineal \rightarrow Imagen\ generada\ por\ la\ transf.\ de\ una\ base$

$Término\ resonante\ si\ no\ está\ en\ la\ imagen\ de\ L_{DF}$

3. Dada $A = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$, calcular $L_A \begin{pmatrix} x^m \\ 0 \end{pmatrix}$ y $L_A \begin{pmatrix} 0 \\ x^m \end{pmatrix}$, con $x^m = x_1^{m_1} x_2^{m_2}$. Obtenga la representación matricial de L_A respecto de la base

$$\left[\begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1 x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right]. \tag{3}$$

$$L_{DF} h_r = -(-Dh_r DF\boldsymbol{y} + DF h_r)$$

$$L_{DF}(\bar{x}^{\bar{m}}\, \bar{e}_j) = -\left(-D(\bar{x}^{\bar{m}}\, \bar{e}_j) \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} (\bar{x}^{\bar{m}}\, \bar{e}_j) \right)$$

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$$L_{DF} \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix} = -\left(-D \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix} \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix} \right)$$

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$$L_{DF}(\bar{x}^{\bar{m}}\, \bar{e}_j) = -\left(-D(\bar{x}^{\bar{m}}\, \bar{e}_j)\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}(\bar{x}^{\bar{m}}\, \bar{e}_j)\right)$$

$$L_{DF}\begin{pmatrix} 0 \\ x_1^2 \end{pmatrix} = -\left(-D\begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}\right)$$

$$L_{DF}\begin{pmatrix} 0 \\ x_1^2 \end{pmatrix} = -\left(-\begin{pmatrix} 0 & 0 \\ 2x_1 & 0 \end{pmatrix}\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}\right)$$

$$L_{DF}\begin{pmatrix}0\\x_1^2\end{pmatrix} = -\left(-\begin{pmatrix}0 & 0\\2x_1 & 0\end{pmatrix}\begin{pmatrix}\lambda & 1\\0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\0 & \lambda\end{pmatrix}\begin{pmatrix}0\\x_1^2\end{pmatrix}\right)$$

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$$L_{DF}\begin{pmatrix}0\\x_1^2\end{pmatrix} = \begin{pmatrix}0 & 0\\2x_1\lambda & 2x_1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} - \begin{pmatrix}x_1^2\\\lambda x_1^2\end{pmatrix}$$

$$L_{DF}\begin{pmatrix}0\\x_1^2\end{pmatrix}=-\left(-\begin{pmatrix}0&0\\2x_1&0\end{pmatrix}\begin{pmatrix}\lambda&1\\0&\lambda\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}+\begin{pmatrix}\lambda&1\\0&\lambda\end{pmatrix}\begin{pmatrix}0\\x_1^2\end{pmatrix}\right)$$

$$L_{DF}\begin{pmatrix}0\\x_1^2\end{pmatrix}=\begin{pmatrix}0&0\\2x_1\lambda&2x_1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}-\begin{pmatrix}x_1^2\\\lambda x_1^2\end{pmatrix}=\begin{pmatrix}0\\2\lambda x_1^2+2x_1x_2\end{pmatrix}-\begin{pmatrix}x_1^2\\\lambda x_1^2\end{pmatrix}=\begin{pmatrix}-x_1^2\\\lambda x_1^2+2x_1x_2\end{pmatrix}$$

$$L_{DF}\begin{pmatrix}0\\x_1^2\end{pmatrix} = -\left(-\begin{pmatrix}0 & 0\\2x_1 & 0\end{pmatrix}\begin{pmatrix}\lambda & 1\\0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\0 & \lambda\end{pmatrix}\begin{pmatrix}0\\x_1^2\end{pmatrix}\right)$$

$$L_{DF}\begin{pmatrix}0\\x_1^2\end{pmatrix} = \begin{pmatrix}0 & 0\\2x_1\lambda & 2x_1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} - \begin{pmatrix}x_1^2\\\lambda x_1^2\end{pmatrix} = \begin{pmatrix}0\\2\lambda x_1^2 + 2x_1x_2\end{pmatrix} - \begin{pmatrix}x_1^2\\\lambda x_1^2\end{pmatrix} = \begin{pmatrix}-x_1^2\\\lambda x_1^2 + 2x_1x_2\end{pmatrix}$$

$$L_{DF}\begin{pmatrix}0\\x_1^2\end{pmatrix} = -\begin{pmatrix}x_1^2\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\x_1^2\end{pmatrix} + 2\begin{pmatrix}0\\x_1x_2\end{pmatrix}$$

$$L_{DF}\begin{pmatrix}0\\x_1^2\end{pmatrix}=-\left(-\begin{pmatrix}0 & 0\\2x_1 & 0\end{pmatrix}\begin{pmatrix}\lambda & 1\\0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}+\begin{pmatrix}\lambda & 1\\0 & \lambda\end{pmatrix}\begin{pmatrix}0\\x_1^2\end{pmatrix}\right)$$

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Los monomios NO son autoestados si DF no es diagonal

$$L_{DF}h_r\neq(\bar{m}\,\bar{\lambda}-\lambda_i)$$

$$L_{DF}\begin{pmatrix}0\\x_1^2\end{pmatrix} = -\left(-\begin{pmatrix}0 & 0\\2x_1 & 0\end{pmatrix}\begin{pmatrix}\lambda & 1\\0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\0 & \lambda\end{pmatrix}\begin{pmatrix}0\\x_1^2\end{pmatrix}\right)$$

$$L_{DF}\begin{pmatrix}0\\x_1^2\end{pmatrix} = \begin{pmatrix}0 & 0\\2x_1\lambda & 2x_1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} - \begin{pmatrix}x_1^2\\\lambda x_1^2\end{pmatrix} = \begin{pmatrix}0\\2\lambda x_1^2 + 2x_1x_2\end{pmatrix} - \begin{pmatrix}x_1^2\\\lambda x_1^2\end{pmatrix} = \begin{pmatrix}-x_1^2\\\lambda x_1^2 + 2x_1x_2\end{pmatrix}$$

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$$H_2 = \left\{\begin{pmatrix}0\\x_1^2\end{pmatrix}, \begin{pmatrix}0\\x_1x_2\end{pmatrix}, \begin{pmatrix}0\\x_2^2\end{pmatrix}, \begin{pmatrix}x_1^2\\0\end{pmatrix}, \begin{pmatrix}x_1x_2\\0\end{pmatrix}, \begin{pmatrix}x_2^2\\0\end{pmatrix}\right\} \longrightarrow L_{DF}v_1 = -v_4 + \lambda v_1 + 2v_2$$

$$L_{DF}\left(\bar{x}^{\bar{m}}\ \bar{e}_j\right) = -\left(-D\left(\bar{x}^{\bar{m}}\ \bar{e}_j\right)\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\left(\bar{x}^{\bar{m}}\ \bar{e}_j\right)\right)$$

$$L_{DF}\begin{pmatrix}0\\ x_1x_2\end{pmatrix} = -\left(-D\begin{pmatrix}0\\ x_1x_2\end{pmatrix}\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}0\\ x_1x_2\end{pmatrix}\right)$$

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$$L_{DF}\begin{pmatrix}0\\ x_1x_2\end{pmatrix} = -\left(-\begin{pmatrix}0 & 0\\ x_2 & x_1\end{pmatrix}\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}0\\ x_1x_2\end{pmatrix}\right)$$

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$$L_{DF}\begin{pmatrix}0\\ x_1x_2\end{pmatrix} = -\left(-D\begin{pmatrix}0\\ x_1x_2\end{pmatrix}\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}0\\ x_1x_2\end{pmatrix}\right)$$

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$$L_{DF}\begin{pmatrix}0\\ x_1x_2\end{pmatrix} = \begin{pmatrix}0 & 0\\ x_2\lambda & x_2 + \lambda x_1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} - \begin{pmatrix}x_1x_2\\ \lambda x_1x_2\end{pmatrix}$$

$$L_{DF}(\bar{x}^{\overline{m}}\ \bar{e}_j) = -\left(-D(\bar{x}^{\overline{m}}\ \bar{e}_j)\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}(\bar{x}^{\overline{m}}\ \bar{e}_j)\right)$$

$$L_{DF}\begin{pmatrix}0\\ x_1x_2\end{pmatrix} = -\left(-D\begin{pmatrix}0\\ x_1x_2\end{pmatrix}\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}0\\ x_1x_2\end{pmatrix}\right)$$

$$L_{DF}\begin{pmatrix}0\\ x_1x_2\end{pmatrix} = -\left(-\begin{pmatrix}0 & 0\\ x_2 & x_1\end{pmatrix}\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}0\\ x_1x_2\end{pmatrix}\right)$$

$$L_{DF}\begin{pmatrix}0\\ x_1x_2\end{pmatrix} = \begin{pmatrix}0 & 0\\ x_2\lambda & x_2 + \lambda x_1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} - \begin{pmatrix}x_1x_2\\ \lambda x_1x_2\end{pmatrix} = \begin{pmatrix}0\\ 2\lambda x_1x_2 + x_2^2\end{pmatrix} - \begin{pmatrix}x_1x_2\\ \lambda x_1x_2\end{pmatrix} = \begin{pmatrix}-x_1x_2\\ \lambda x_1x_2 + x_2^2\end{pmatrix}$$

$$L_{DF}\left(\bar{x}^{\bar{m}}\ \bar{e}_j\right) = -\left(-D\left(\bar{x}^{\bar{m}}\ \bar{e}_j\right)\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\left(\bar{x}^{\bar{m}}\ \bar{e}_j\right)\right)$$

$$L_{DF}\begin{pmatrix}0\\ x_1x_2\end{pmatrix} = -\left(-D\begin{pmatrix}0\\ x_1x_2\end{pmatrix}\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}0\\ x_1x_2\end{pmatrix}\right)$$

$$L_{DF}\begin{pmatrix}0\\ x_1x_2\end{pmatrix} = -\left(-\begin{pmatrix}0 & 0\\ x_2 & x_1\end{pmatrix}\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}0\\ x_1x_2\end{pmatrix}\right)$$

$$L_{DF}\begin{pmatrix}0\\ x_1x_2\end{pmatrix} = \begin{pmatrix}0 & 0\\ x_2\lambda & x_2+\lambda x_1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} - \begin{pmatrix}x_1x_2\\ \lambda x_1x_2\end{pmatrix} = \begin{pmatrix}0\\ 2\lambda x_1x_2 + x_2^2\end{pmatrix} - \begin{pmatrix}x_1x_2\\ \lambda x_1x_2\end{pmatrix} = \begin{pmatrix}-x_1x_2\\ \lambda x_1x_2 + x_2^2\end{pmatrix}$$

$$H_2 = \left\{\begin{pmatrix}0\\ x_1^2\end{pmatrix}, \begin{pmatrix}0\\ x_1x_2\end{pmatrix}, \begin{pmatrix}0\\ x_2^2\end{pmatrix}, \begin{pmatrix}x_1^2\\ 0\end{pmatrix}, \begin{pmatrix}x_1x_2\\ 0\end{pmatrix}, \begin{pmatrix}x_2^2\\ 0\end{pmatrix}\right\} \longrightarrow L_{DF}v_2 = -v_5 + \lambda v_2 + v_3$$

$$L_{DF}\begin{pmatrix}0\\x_2^2\end{pmatrix} = -\left(-\begin{pmatrix}0 & 0\\0 & 2x_2\end{pmatrix}\begin{pmatrix}\lambda & 1\\0 & \lambda\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} + \begin{pmatrix}\lambda & 1\\0 & \lambda\end{pmatrix}\begin{pmatrix}0\\x_2^2\end{pmatrix}\right)$$

$$L_{DF}\begin{pmatrix}0\\x_2^2\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 2\lambda x_2\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} - \begin{pmatrix}x_2^2\\\lambda x_2^2\end{pmatrix} = \begin{pmatrix}0\\2\lambda x_2^2\end{pmatrix} - \begin{pmatrix}x_2^2\\\lambda x_2^2\end{pmatrix} = \begin{pmatrix}-x_2^2\\\lambda x_2^2\end{pmatrix}$$

$$H_2 = \left\{\begin{pmatrix}0\\x_1^2\end{pmatrix}, \begin{pmatrix}0\\x_1 x_2\end{pmatrix}, \begin{pmatrix}0\\x_2^2\end{pmatrix}, \begin{pmatrix}x_1^2\\0\end{pmatrix}, \begin{pmatrix}x_1 x_2\\0\end{pmatrix}, \begin{pmatrix}x_2^2\\0\end{pmatrix}\right\} \longrightarrow L_{DF} v_3 = -v_6 + \lambda v_3$$

$$L_{DF}\begin{pmatrix} x_1^2 \\ 0 \end{pmatrix} = -\left(-\begin{pmatrix} 2x_1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x_1^2 \\ 0 \end{pmatrix} \right)$$

$$L_{DF}\begin{pmatrix} x_1^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\lambda x_1 & 2x_1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} \lambda x_1^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\lambda x_1^2 + 2x_1x_2 \\ 0 \end{pmatrix} - \begin{pmatrix} \lambda x_1^2 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda x_1^2 + 2x_1x_2 \\ 0 \end{pmatrix}$$

$$H_2 = \left\{ \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\} \longrightarrow L_{DF}v_4 = \lambda v_4 + 2v_5$$

$$L_{DF}\begin{pmatrix} x_1x_2 \\ 0 \end{pmatrix} = -\left(-\begin{pmatrix} x_2 & x_1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x_1x_2 \\ 0 \end{pmatrix} \right)$$

$$L_{DF}\begin{pmatrix} x_1x_2 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda x_2 & x_2 + \lambda x_1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} \lambda x_1 x_2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\lambda x_1 x_2 + x_2^2 \\ 0 \end{pmatrix} - \begin{pmatrix} \lambda x_1 x_2 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda x_1 x_2 + x_2^2 \\ 0 \end{pmatrix}$$

$$H_2 = \left\{ \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\} \longrightarrow L_{DF}v_5 = \lambda v_5 + v_6$$

$$L_{DF}\begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} = -\left(-\begin{pmatrix} 0 & 2x_2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right)$$

$$L_{DF}\begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 2\lambda x_2 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} \lambda x_2^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2\lambda x_2^2 \\ 0 \end{pmatrix} - \begin{pmatrix} \lambda x_2^2 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda x_2^2 \\ 0 \end{pmatrix}$$

$$H_2 = \left\{ \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1 x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\} \longrightarrow L_{DF} v_6 = \lambda v_6$$

$$L_{DF} v_1 = \lambda v_1 + 2v_2 - v_4$$
$$L_{DF} v_2 = \lambda v_2 + v_3 - v_5$$
$$L_{DF} v_3 = \lambda v_3 - v_6$$
$$L_{DF} v_4 = \lambda v_4 + 2v_5$$
$$L_{DF} v_5 = \lambda v_5 + v_6$$
$$L_{DF} v_6 = \lambda v_6$$

$$L_{DF}v_1 = \lambda v_1 + 2v_2 - v_4$$

$$L_{DF}v_2 = \lambda v_2 + v_3 - v_5$$

$$L_{DF}v_3 = \lambda v_3 - v_6$$

$$L_{DF}v_4 = \lambda v_4 + 2v_5$$

$$L_{DF}v_5 = \lambda v_5 + v_6$$

$$L_{DF}v_6 = \lambda v_6$$

$$L_{DF} = \begin{pmatrix} \lambda & & & & & \\ 2 & & & & & \\ 0 & & & & & \\ -1 & & & & & \\ 0 & & & & & \\ 0 & & & & & \end{pmatrix}$$

$$L_{DF}v_1 = \lambda v_1 + 2v_2 - v_4$$

$$L_{DF}v_2 = \lambda v_2 + v_3 - v_5$$

$$L_{DF}v_3 = \lambda v_3 - v_6$$

$$L_{DF}v_4 = \lambda v_4 + 2v_5$$

$$L_{DF}v_5 = \lambda v_5 + v_6$$

$$L_{DF}v_6 = \lambda v_6$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & & & & \\ 2 & \lambda & & & & \\ 0 & 1 & & & & \\ -1 & 0 & & & & \\ 0 & -1 & & & & \\ 0 & 0 & & & & \end{pmatrix}$$

$$L_{DF}v_1 = \lambda v_1 + 2v_2 - v_4$$

$$L_{DF}v_2 = \lambda v_2 + v_3 - v_5$$

$$L_{DF}v_3 = \lambda v_3 - v_6$$

$$L_{DF}v_4 = \lambda v_4 + 2v_5$$

$$L_{DF}v_5 = \lambda v_5 + v_6$$

$$L_{DF}v_6 = \lambda v_6$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & 0 & & & \\ 2 & \lambda & 0 & & & \\ 0 & 1 & \lambda & & & \\ -1 & 0 & 0 & & & \\ 0 & -1 & 0 & & & \\ 0 & 0 & -1 & & & \end{pmatrix}$$

$$L_{DF}v_1 = \lambda v_1 + 2v_2 - v_4$$

$$L_{DF}v_2 = \lambda v_2 + v_3 - v_5$$

$$L_{DF}v_3 = \lambda v_3 - v_6$$

$$L_{DF}v_4 = \lambda v_4 + 2v_5$$

$$L_{DF}v_5 = \lambda v_5 + v_6$$

$$L_{DF}v_6 = \lambda v_6$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & 0 & 0 & & \\ 2 & \lambda & 0 & 0 & & \\ 0 & 1 & \lambda & 0 & & \\ -1 & 0 & 0 & \lambda & & \\ 0 & -1 & 0 & 2 & & \\ 0 & 0 & -1 & 0 & & \end{pmatrix}$$

$$L_{DF}v_1 = \lambda v_1 + 2v_2 - v_4$$

$$L_{DF}v_2 = \lambda v_2 + v_3 - v_5$$

$$L_{DF}v_3 = \lambda v_3 - v_6$$

$$L_{DF}v_4 = \lambda v_4 + 2v_5$$

$$L_{DF}v_5 = \lambda v_5 + v_6$$

$$L_{DF}v_6 = \lambda v_6$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & \\ 2 & \lambda & 0 & 0 & 0 & \\ 0 & 1 & \lambda & 0 & 0 & \\ -1 & 0 & 0 & \lambda & 0 & \\ 0 & -1 & 0 & 2 & \lambda & \\ 0 & 0 & -1 & 0 & 1 & \end{pmatrix}$$

$$L_{DF}v_1 = \lambda v_1 + 2v_2 - v_4$$

$$L_{DF}v_2 = \lambda v_2 + v_3 - v_5$$

$$L_{DF}v_3 = \lambda v_3 - v_6$$

$$L_{DF}v_4 = \lambda v_4 + 2v_5$$

$$L_{DF}v_5 = \lambda v_5 + v_6$$

$$L_{DF}v_6 = \lambda v_6$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 2 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 & 0 \\ -1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & -1 & 0 & 2 & \lambda & 0 \\ 0 & 0 & -1 & 0 & 1 & \lambda \end{pmatrix}$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 2 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 & 0 \\ -1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & -1 & 0 & 2 & \lambda & 0 \\ 0 & 0 & -1 & 0 & 1 & \lambda \end{pmatrix}$$

$$H_2 = \left\{ \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1 x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\}$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 2 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 & 0 \\ -1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & -1 & 0 & 2 & \lambda & 0 \\ 0 & 0 & -1 & 0 & 1 & \lambda \end{pmatrix}$$

$$H_2 = \left\{ \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1 x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\}$$

$$h_r = \begin{pmatrix} 2x_1^2 - 3x_1 x_2 \\ x_2^2 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \\ -3 \\ 0 \end{pmatrix}$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 2 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 & 0 \\ -1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & -1 & 0 & 2 & \lambda & 0 \\ 0 & 0 & -1 & 0 & 1 & \lambda \end{pmatrix} \qquad H_2 = \left\{ \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1 x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\}$$

$$h_r = \begin{pmatrix} 2x_1^2 - 3x_1 x_2 \\ x_2^2 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \\ -3 \\ 0 \end{pmatrix} \rightarrow L_{DF} h_r = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 2 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 & 0 \\ -1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & -1 & 0 & 2 & \lambda & 0 \\ 0 & 0 & -1 & 0 & 1 & \lambda \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \\ -3 \\ 0 \end{pmatrix}$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 2 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 & 0 \\ -1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & -1 & 0 & 2 & \lambda & 0 \\ 0 & 0 & -1 & 0 & 1 & \lambda \end{pmatrix} \qquad H_2 = \left\{ \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1 x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\}$$

$$h_r = \begin{pmatrix} 2x_1^2 - 3x_1 x_2 \\ x_2^2 \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \\ -3 \\ 0 \end{pmatrix} \to L_{DF} h_r = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 2 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 & 0 \\ -1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & -1 & 0 & 2 & \lambda & 0 \\ 0 & 0 & -1 & 0 & 1 & \lambda \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \\ -3 \\ 0 \end{pmatrix}$$

$$L_{DF} h_r = \begin{pmatrix} 0 \\ 0 \\ \lambda \\ 2\lambda \\ 4 - 3\lambda \\ -4 \end{pmatrix}$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 2 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 & 0 \\ -1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & -1 & 0 & 2 & \lambda & 0 \\ 0 & 0 & -1 & 0 & 1 & \lambda \end{pmatrix} \qquad H_2 = \left\{ \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1 x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\}$$

$$h_r = \begin{pmatrix} 2x_1^2 - 3x_1 x_2 \\ x_2^2 \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \\ -3 \\ 0 \end{pmatrix} \to L_{DF} h_r = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 2 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 & 0 \\ -1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & -1 & 0 & 2 & \lambda & 0 \\ 0 & 0 & -1 & 0 & 1 & \lambda \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \\ -3 \\ 0 \end{pmatrix}$$

$$L_{DF} h_r = \begin{pmatrix} 0 \\ 0 \\ \lambda \\ 2\lambda \\ 4 - 3\lambda \\ -4 \end{pmatrix} \to L_{DF} \begin{pmatrix} 2x_1^2 - 3x_1 x_2 \\ x_2^2 \end{pmatrix} = \begin{pmatrix} 2\lambda x_1^2 + (4 - 3\lambda) x_1 x_2 - 4x_2^2 \\ \lambda x_2^2 \end{pmatrix}$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 2 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 & 0 \\ -1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & -1 & 0 & 2 & \lambda & 0 \\ 0 & 0 & -1 & 0 & 1 & \lambda \end{pmatrix}$$

$$Im(L_{DF}) = ?$$

$$L_{DF} = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 & 0 \\ 2 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 0 & 0 & 0 \\ -1 & 0 & 0 & \lambda & 0 & 0 \\ 0 & -1 & 0 & 2 & \lambda & 0 \\ 0 & 0 & -1 & 0 & 1 & \lambda \end{pmatrix}$$

$$\begin{pmatrix} \lambda & 2 & 0 & -1 & 0 & 0 \\ 0 & \lambda & 1 & 0 & -1 & 0 \\ 0 & 0 & \lambda & 0 & 0 & -1 \\ 0 & 0 & 0 & \lambda & 2 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda \end{pmatrix}$$

$$Im(L_{DF}) = \langle columnas\ l.i. \rangle$$

Ejemplo: $\lambda = 0$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \cdots$$

$$Im(L_{DF}) = \langle columnas\ l.i. \rangle$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Ejemplo: $\lambda = 0$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \cdots$$

$$Im(L_{DF}) = \langle columnas\ l.i. \rangle$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Ejemplo: $\lambda = 0$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \cdots$$

$$Im(L_{DF}) = \langle columnas\ l.i. \rangle \qquad \begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad F3 \to F3 + F5$$

Ejemplo: $\lambda = 0$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \cdots$$

$$Im(L_{DF}) = \langle columnas\ l.i. \rangle$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

$$F3 \to F3 + F5$$

$$Im(L_{DF}) = \langle (0,2,0,-1,0,0), (0,0,1,0,-1,0), (0,0,0,0,2,0), (0,0,0,0,0,1) \rangle$$

$$H_2 = \left\{ \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1 x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\}$$

Ejemplo: $\lambda = 0$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \cdots$$

$$Im(L_{DF}) = \langle columnas\ l.i. \rangle$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

$$F3 \to F3 + F5$$

$$Im(L_{DF}) = \langle (0,2,0,-1,0,0), (0,0,1,0,-1,0), (0,0,0,0,2,0), (0,0,0,0,0,1) \rangle$$

$$H_2 = \left\{ \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_1 x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\}$$

Ejemplo: $\lambda = 0$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \cdots$$

$$Im(L_{DF}) = \langle columnas\ l.i. \rangle \qquad \begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix} \qquad F3 \to F3 + F5$$

$$Im(L_{DF}) = \langle (0, 2, 0, -1, 0, 0), (0, 0, 1, 0, -1, 0), (0, 0, 0, 0, 2, 0), (0, 0, 0, 0, 0, 1) \rangle$$

$$Im(L_{DF}) = \left\langle \begin{pmatrix} -x_1^2 \\ 2x_1 x_2 \end{pmatrix}, \begin{pmatrix} -x_1 x_2 \\ x_2^2 \end{pmatrix}, \begin{pmatrix} 2x_1 x_2 \\ 0 \end{pmatrix}, \begin{pmatrix} x_2^2 \\ 0 \end{pmatrix} \right\rangle$$

Ejemplo: $\lambda = 0$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \cdots$$

$$Im(L_{DF}) = \langle (0,2,0,-1,0,0),(0,0,1,0,-1,0),(0,0,0,0,2,0),(0,0,0,0,0,1) \rangle$$

Términos resonantes = complemento de la imagen

Ejemplo: $\lambda = 0$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \cdots$$

$$Im(L_{DF}) = \langle (0,2,0,-1,0,0), (0,0,1,0,-1,0), (0,0,0,0,2,0), (0,0,0,0,0,1) \rangle$$

Términos resonantes = complemento de la imagen → Agregamos l.i. hasta completar base

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} \color{red}{1} & \color{red}{0} & \color{red}{0} & \color{red}{0} & \color{red}{0} & \color{red}{0} \\ 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ \color{blue}{0} & \color{blue}{0} & \color{blue}{0} & \color{blue}{1} & \color{blue}{0} & \color{blue}{0} \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + a \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix} + b \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + a \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix} + b \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + a \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix} + b \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + a \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix} + b \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix}, \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + c \begin{pmatrix} 0 \\ x_1^2 \end{pmatrix} + d \begin{pmatrix} 0 \\ x_1 x_2 \end{pmatrix}$$

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + a\begin{pmatrix}0\\ x_1^2\end{pmatrix} + b\begin{pmatrix}x_1^2\\ 0\end{pmatrix}$$

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix} = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + c\begin{pmatrix}0\\ x_1^2\end{pmatrix} + d\begin{pmatrix}0\\ x_1 x_2\end{pmatrix}$$