Coordenadas cartesianas (x, y, z)

Gradiente, divergencia, rotor y laplaciano

$$\nabla f = \frac{\partial f}{\partial x}\,\hat{x} + \frac{\partial f}{\partial y}\,\hat{y} + \frac{\partial f}{\partial z}\,\hat{z} \tag{1}$$

$$\nabla \cdot \mathbf{f} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} \tag{2}$$

$$\nabla \times \mathbf{f} = \left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z}\right)\hat{x} + \left(\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x}\right)\hat{y} + \left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}\right)\hat{z} \tag{3}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \tag{4}$$

$$\nabla^2 \mathbf{f} = \nabla^2 f_x\;\hat{x} + \nabla^2 f_y\;\hat{y} + \nabla^2 f_z\;\hat{z} \tag{5}$$

Velocidad de deformación, rotación y vorticidad

$$\mathbf{S} = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{xy} & S_{yy} & S_{yz} \\ S_{xz} & S_{yz} & S_{zz} \end{pmatrix} \qquad \mathbf{\Omega} = \begin{pmatrix} \Omega_{xx} & \Omega_{xy} & \Omega_{xz} \\ -\Omega_{xy} & \Omega_{yy} & \Omega_{yz} \\ -\Omega_{xz} & -\Omega_{yz} & \Omega_{zz} \end{pmatrix} \qquad \omega = 2\begin{pmatrix} -\Omega_{yz} \\ \Omega_{xz} \\ -\Omega_{xy} \end{pmatrix} \tag{6}$$

$$\begin{array}{llll} S_{xx} = \dfrac{\partial u_x}{\partial x} & S_{xy} = \dfrac{1}{2}\left(\dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x}\right) & \Omega_{xx} = 0 & \Omega_{xy} = \dfrac{1}{2}\left(\dfrac{\partial u_x}{\partial y} - \dfrac{\partial u_y}{\partial x}\right) \\ S_{yy} = \dfrac{\partial u_y}{\partial y} & S_{xz} = \dfrac{1}{2}\left(\dfrac{\partial u_x}{\partial z} + \dfrac{\partial u_z}{\partial x}\right) & \Omega_{yy} = 0 & \Omega_{xz} = \dfrac{1}{2}\left(\dfrac{\partial u_x}{\partial z} - \dfrac{\partial u_z}{\partial x}\right) \\ S_{zz} = \dfrac{\partial u_z}{\partial z} & S_{yz} = \dfrac{1}{2}\left(\dfrac{\partial u_y}{\partial z} + \dfrac{\partial u_z}{\partial y}\right) & \Omega_{zz} = 0 & \Omega_{yz} = \dfrac{1}{2}\left(\dfrac{\partial u_y}{\partial z} - \dfrac{\partial u_z}{\partial y}\right) \end{array} \tag{7}$$

Operadores sobre la velocidad y el esfuerzo[1]

$$\mathbf{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \qquad \mathbf{T} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix} \tag{8}$$

[1] No se supone ningún tipo de simetría para **T**. Por lo tanto, prestar atención al orden de los subíndices.

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(\mathbf{u} \cdot \nabla)\, \mathbf{u} = \begin{pmatrix} u_x \dfrac{\partial u_x}{\partial x} + u_y \dfrac{\partial u_x}{\partial y} + u_z \dfrac{\partial u_x}{\partial z} \\[2ex] u_x \dfrac{\partial u_y}{\partial x} + u_y \dfrac{\partial u_y}{\partial y} + u_z \dfrac{\partial u_y}{\partial z} \\[2ex] u_x \dfrac{\partial u_z}{\partial x} + u_y \dfrac{\partial u_z}{\partial y} + u_z \dfrac{\partial u_z}{\partial z} \end{pmatrix} \qquad \nabla \cdot \mathbf{T} = \begin{pmatrix} \dfrac{\partial T_{xx}}{\partial x} + \dfrac{\partial T_{xy}}{\partial y} + \dfrac{\partial T_{xz}}{\partial z} \\[2ex] \dfrac{\partial T_{yx}}{\partial x} + \dfrac{\partial T_{yy}}{\partial y} + \dfrac{\partial T_{yz}}{\partial z} \\[2ex] \dfrac{\partial T_{zx}}{\partial x} + \dfrac{\partial T_{zy}}{\partial y} + \dfrac{\partial T_{zz}}{\partial z} \end{pmatrix} \tag{9}
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