Coordenadas cilíndricas (ρ, φ, z)

Gradiente, divergencia, rotor y laplaciano

$$\begin{aligned}
\nabla f &= \frac{\partial f}{\partial \rho}\,\hat\rho + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\,\hat\varphi + \frac{\partial f}{\partial z}\,\hat z && (1)\\
\nabla\cdot\mathbf{f} &= \frac{1}{\rho}\frac{\partial(\rho f_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial f_\varphi}{\partial \varphi} + \frac{\partial f_z}{\partial z} && (2)\\
\nabla\times\mathbf{f} &= \left(\frac{1}{\rho}\frac{\partial f_z}{\partial \varphi} - \frac{\partial f_\varphi}{\partial z}\right)\hat\rho + \left(\frac{\partial f_\rho}{\partial z} - \frac{\partial f_z}{\partial \rho}\right)\hat\varphi + \frac{1}{\rho}\left(\frac{\partial(\rho f_\varphi)}{\partial \rho} - \frac{\partial f_\rho}{\partial \varphi}\right)\hat z && (3)\\
\nabla^2 f &= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2} && (4)\\
\nabla^2\mathbf{f} &= \left(\nabla^2 f_\rho - \frac{f_\rho}{\rho^2} - \frac{2}{\rho^2}\frac{\partial f_\varphi}{\partial \varphi}\right)\hat\rho + \left(\nabla^2 f_\varphi + \frac{2}{\rho^2}\frac{\partial f_\rho}{\partial \varphi} - \frac{f_\varphi}{\rho^2}\right)\hat\varphi + \nabla^2 f_z\,\hat z && (5)
\end{aligned}$$

Velocidad de deformación, rotación y vorticidad

$$\mathbf{S} = \begin{pmatrix} S_{\rho\rho} & S_{\rho\varphi} & S_{\rho z} \\ S_{\rho\varphi} & S_{\varphi\varphi} & S_{\varphi z} \\ S_{\rho z} & S_{\varphi z} & S_{zz} \end{pmatrix} \qquad \boldsymbol{\Omega} = \begin{pmatrix} \Omega_{\rho\rho} & \Omega_{\rho\varphi} & \Omega_{\rho z} \\ -\Omega_{\rho\varphi} & \Omega_{\varphi\varphi} & \Omega_{\varphi z} \\ -\Omega_{\rho z} & -\Omega_{\varphi z} & \Omega_{zz} \end{pmatrix} \qquad \omega = 2\begin{pmatrix} -\Omega_{\varphi z} \\ \Omega_{\rho z} \\ -\Omega_{\rho\varphi} \end{pmatrix} \qquad (6)$$

$$\begin{aligned}
S_{\rho\rho} &= \frac{\partial u_\rho}{\partial \rho} & S_{\rho\varphi} &= \frac{1}{2}\left(\frac{1}{\rho}\frac{\partial u_\rho}{\partial \varphi} + \frac{\partial u_\varphi}{\partial \rho} - \frac{u_\varphi}{\rho}\right) & \Omega_{\rho\varphi} &= \frac{1}{2\rho}\left(\frac{\partial u_\rho}{\partial \varphi} - \frac{\partial(\rho u_\varphi)}{\partial \rho}\right)\\
S_{\varphi\varphi} &= \frac{1}{\rho}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_\rho}{\rho} & S_{\rho z} &= \frac{1}{2}\left(\frac{\partial u_\rho}{\partial z} + \frac{\partial u_z}{\partial \rho}\right) & \Omega_{\rho z} &= \frac{1}{2}\left(\frac{\partial u_\rho}{\partial z} - \frac{\partial u_z}{\partial \rho}\right) \qquad (7)\\
S_{zz} &= \frac{\partial u_z}{\partial z} & S_{\varphi z} &= \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial z} + \frac{1}{\rho}\frac{\partial u_z}{\partial \varphi}\right) & \Omega_{\varphi z} &= \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial z} - \frac{1}{\rho}\frac{\partial u_z}{\partial \varphi}\right)
\end{aligned}$$

$$\Omega_{\rho\rho} = \Omega_{\varphi\varphi} = \Omega_{zz} = 0$$

Operadores sobre la velocidad y el esfuerzo[1]

$$\mathbf{u} = \begin{pmatrix} u_\rho \\ u_\varphi \\ u_z \end{pmatrix} \qquad \mathbf{T} = \begin{pmatrix} T_{\rho\rho} & T_{\rho\varphi} & T_{\rho z} \\ T_{\varphi\rho} & T_{\varphi\varphi} & T_{\varphi z} \\ T_{zr} & T_{z\varphi} & T_{zz} \end{pmatrix} \qquad (8)$$

[1]No se supone ningún tipo de simetría para **T**. Por lo tanto, prestar atención al orden de los subíndices.

$$(\mathbf{u}\cdot\nabla)\,\mathbf{u} = \begin{pmatrix} u_\rho \dfrac{\partial u_\rho}{\partial \rho} + \dfrac{u_\varphi}{\rho}\dfrac{\partial u_\rho}{\partial \varphi} + u_z \dfrac{\partial u_\rho}{\partial z} - \dfrac{u_\varphi^2}{\rho} \\[2ex] u_\rho \dfrac{\partial u_\varphi}{\partial \rho} + \dfrac{u_\varphi}{\rho}\dfrac{\partial u_\varphi}{\partial \varphi} + u_z \dfrac{\partial u_\varphi}{\partial z} + \dfrac{u_\rho u_\varphi}{\rho} \\[2ex] u_\rho \dfrac{\partial u_z}{\partial \rho} + \dfrac{u_\varphi}{\rho}\dfrac{\partial u_z}{\partial \varphi} + u_z \dfrac{\partial u_z}{\partial z} \end{pmatrix} \tag{9}$$

$$\nabla\cdot\mathbf{T} = \begin{pmatrix} \dfrac{\partial T_{\rho\rho}}{\partial \rho} + \dfrac{T_{\rho\rho}}{\rho} + \dfrac{1}{\rho}\dfrac{\partial T_{\varphi\rho}}{\partial \varphi} + \dfrac{\partial T_{z\rho}}{\partial z} - \dfrac{T_{\varphi\varphi}}{\rho} \\[2ex] \dfrac{1}{\rho}\dfrac{\partial T_{\varphi\varphi}}{\partial \varphi} + \dfrac{\partial T_{\rho\varphi}}{\partial \rho} + \dfrac{T_{\rho\varphi}}{\rho} + \dfrac{T_{\varphi\rho}}{\rho} + \dfrac{\partial T_{z\varphi}}{\partial z} \\[2ex] \dfrac{\partial T_{zz}}{\partial z} + \dfrac{\partial T_{\rho z}}{\partial \rho} + \dfrac{T_{\rho z}}{\rho} + \dfrac{1}{\rho}\dfrac{\partial T_{\varphi z}}{\partial \varphi} \end{pmatrix} \tag{10}$$