flow, meaning that w has a unique derivative with respect to z at all points of that region. Conversely, any analytic function of z can be regarded as the complex potential of a certain flow field. Thus, simply by choosing different mathematical forms of $w(z)$, we obtain possible forms of the functions ϕ and ψ, although it may not happen that the flow fields that they represent are physically interesting. A more direct way of determining irrotational flow fields is provided by the method of conformal transformation of functions of a complex variable. We shall illustrate in this section these indirect and direct procedures and other uses of the complex potential.

It will be useful as a preliminary to notice the form taken by w in the case of the following simple irrotational flow fields whose description in terms of ϕ or ψ is already known.

Uniform flow with velocity (U, V):	$w = (U-iV)z$
Simple source of strength m at the point z_0 (§2.5):	$w = \dfrac{m}{2\pi}\log(z-z_0)$
Source doublet of strength μ with direction parallel to the x-axis at z_0:	$w = -\dfrac{\mu}{2\pi(z-z_0)}$
The same, with direction parallel to the y-axis:	$w = -\dfrac{i\mu}{2\pi(z-z_0)}$
Point vortex of strength κ at z_0 (§2.6):	$w = -\dfrac{i\kappa}{2\pi}\log(z-z_0)$
Vortex doublet of strength λ with direction parallel to the x-axis at z_0:	$w = \dfrac{i\lambda}{2\pi(z-z_0)}$
Flow due to a circular cylinder of radius a moving with velocity (U, V) and circulation κ round it, centre instantaneously at z_0 (§2.10):	$w = -\dfrac{i\kappa}{2\pi}\log(z-z_0) - \dfrac{a^2(U+iV)}{z-z_0}$
Arbitrary flow outside a circle, centre at z_0, which encloses all boundaries, in fluid at rest at infinity (Laurent series, §2.10):	$w = \dfrac{m-i\kappa}{2\pi}\log(z-z_0) + \sum_{n=0}^{\infty} A_n(z-z_0)^{-n}$
Flow near a stagnation point at the origin (§2.7):	$w = \frac{1}{2}kz^2.$

Flow fields obtained by special choice of the function $w(z)$

Perhaps the simplest mathematical form for w is

$$w(z) = Az^n, \tag{6.5.1}$$

where A and n are real constants. If r, θ are polar co-ordinates in the z-plane, we have $z = re^{i\theta}$ and

$$\phi = Ar^n\cos n\theta, \quad \psi = Ar^n\sin n\theta. \tag{6.5.2}$$

The physical interest of a mathematical solution for irrotational flow