where $\chi_{\text{P}} = \mu_{\text{B}}^2\, g(\varepsilon_{\text{F}})$ is the *Pauli susceptibility*. The Pauli susceptibility is positive, and hence is *paramagnetic*.

Using the formula for the density of states,

$$g(\varepsilon_{\text{F}}) = \frac{m^* k_{\text{F}}}{\pi^2 \hbar^2} \ , \tag{4.71}$$

we find

$$\chi_{\text{P}} = \frac{1}{4\pi^2}\,\frac{m^*}{m}\left(\frac{e^2}{\hbar c}\right)^2 (k_{\text{F}} a_{\text{B}}) \ . \tag{4.72}$$

Using $e^2/\hbar c \simeq 1/137.036$ and assuming $k_{\text{F}} a_{\text{B}} \approx 1$, we find $\chi_{\text{P}} \approx 10^{-6}$, which is comparable in magnitude (though opposite in sign) from the Larmor susceptibility of closed shells.

4.5.2 Landau Diamagnetism

Next, we investigate the orbital contribution. We assume a parabolic band, in which case

$$\mathcal{H} = \frac{1}{2m^*}\big(\boldsymbol{p} + \tfrac{e}{c}\boldsymbol{A}\big)^2 + \mu_{\text{B}}\boldsymbol{\sigma}\cdot\boldsymbol{H} \ . \tag{4.73}$$

Appealing to the familiar results of a quantized charged particle in a uniform magnetic field, the energy levels are given by

$$\varepsilon(n, k_z, \sigma) = (n + \tfrac{1}{2})\,\hbar\omega_{\text{c}} + \sigma\mu_{\text{B}} H + \frac{\hbar^2 k_z^2}{2m^*} \ , \tag{4.74}$$

where $\omega_{\text{c}} = eH/m^* c$ is the *cyclotron frequency*. Note that $\mu_{\text{B}} H = \big(\frac{m^*}{m}\big)\cdot\frac{1}{2}\hbar\omega_{\text{c}}$. The three-dimensional density of states is a convolution of the two-dimensional density of states,

$$g_{\text{2d}}(\varepsilon) = \frac{1}{2\pi\ell^2}\sum_{n=0}^{\infty}\delta\big(\varepsilon - (n+\tfrac{1}{2})\,\hbar\omega_{\text{c}}\big) \ , \tag{4.75}$$

where $\ell = \sqrt{\hbar c/eH}$ is the magnetic length, and the one-dimensional density of states,

$$g_{\text{1d}}(\varepsilon) = \frac{1}{\pi}\frac{dk}{d\varepsilon} = \frac{\sqrt{m^*}}{\sqrt{2}\,\pi\hbar}\,\frac{1}{\sqrt{\varepsilon}} \ . \tag{4.76}$$

Thus,

$$g(\varepsilon) = \frac{\sqrt{m^*}}{\sqrt{2}\,\pi\hbar}\,\frac{1}{2\pi\ell^2}\sum_{n=0}^{\infty}\sum_{\sigma=\pm 1}\frac{\Theta\big(\varepsilon - \varepsilon_{n\sigma}\big)}{\sqrt{\varepsilon - \varepsilon_{n\sigma}}} \ . \tag{4.77}$$

Thus, the grand potential,

$$\Omega(T, V, \mu, H) = -V k_{\text{B}} T \int\limits_{-\infty}^{\infty} d\varepsilon\, g(\varepsilon)\,\ln\Big\{1 + e^{(\mu-\varepsilon)/k_{\text{B}}T}\Big\} \tag{4.78}$$

may be written as the sum,

$$\Omega(T,V,\mu,H) = -V k_{\mathrm{B}} T \frac{\sqrt{m^*}eH}{\sqrt{8}\,\pi^2\hbar^2 c} \sum_{n=0}^{\infty} \sum_{\sigma=\pm 1} F(\mu_\sigma - n\hbar\omega_{\mathrm{c}}) \ , \tag{4.79}$$

with $\lambda = m^*/m$,

$$\mu_\sigma \equiv \mu - \tfrac{1}{2}(1+\sigma\lambda)\hbar\omega_{\mathrm{c}} \ , \tag{4.80}$$

and

$$F(\nu) = \int\limits_0^\infty \frac{d\omega}{\sqrt{\omega}} \, \ln\Big\{1 + e^{(\nu-\omega)/k_{\mathrm{B}}T}\Big\} \ . \tag{4.81}$$

We now invoke the Euler-MacLaurin formula,

$$\sum_{n=0}^{\infty} f(n) = \int\limits_0^\infty \! dx\, f(x) + \tfrac{1}{2} f(0) - \tfrac{1}{12}\, f'(0) + \tfrac{1}{720}\, f'''(0) + \ldots \ , \tag{4.82}$$

which gives

$$\Omega = -\frac{V k_{\mathrm{B}} T\, m^{*3/2}}{2\sqrt{2}\,\pi^2\hbar^3} \sum_{\sigma=\pm} \left\{ \int\limits_{-\infty}^{\mu_\sigma} \! d\varepsilon\, F(\varepsilon) + \tfrac{1}{2}\hbar\omega_{\mathrm{c}}\, F(\mu_\sigma) + \tfrac{1}{12}(\hbar\omega_{\mathrm{c}})^2 F'(\mu_\sigma) + \ldots \right\} . \tag{4.83}$$

We now sum over σ and perform a Taylor expansion in $\hbar\omega_{\mathrm{c}} \propto H$, yielding

$$\begin{aligned}\Omega(T,V,\mu,H) &= -\frac{V k_{\mathrm{B}} T\, m^{*3/2}}{2\sqrt{2}\,\pi^2\hbar^3} \sum_{\sigma=\pm} \left\{ \int\limits_{-\infty}^{\mu} \! d\varepsilon\, F(\varepsilon) + \tfrac{1}{8}\big(\lambda^2 - \tfrac{1}{3}\big)(\hbar\omega_{\mathrm{c}})^2\, F'(\mu) + \mathcal{O}(H^4) \right\} \\ &= \left\{ 1 + \tfrac{1}{2}\big(1 - \tfrac{1}{3\lambda^2}\big)(\mu_{\mathrm{B}} H)^2\, \frac{\partial^2}{\partial\mu^2} + \mathcal{O}(H^4) \right\} \Omega(T,V,\mu,0) \ . \end{aligned} \tag{4.84}$$

Thus,

$$M = -\frac{1}{V}\big(1 - \tfrac{1}{3\lambda^2}\big)\, \mu_{\mathrm{B}}^2\, H\, \frac{\partial^2\Omega}{\partial\mu^2}\bigg|_{H=0} \ , \tag{4.85}$$

and the zero field magnetic susceptibility is

$$\chi = \left(1 - \frac{1}{3\lambda^2}\right) \mu_{\mathrm{B}}^2\, \frac{\partial n}{\partial \mu} \ . \tag{4.86}$$

The quantity $\chi_{\mathrm{P}} = \mu_{\mathrm{B}}^2\,(\partial n/\partial\mu)$ is simply the finite temperature Pauli susceptibility. The orbital contribution is negative, *i.e.* diamagnetic. Thus, $\chi = \chi_{\mathrm{P}} + \chi_{\mathrm{L}}$, where

$$\chi_{\mathrm{L}} = -\tfrac{1}{3}\,(m/m^*)^2\, \chi_{\mathrm{P}} \tag{4.87}$$

is the *Landau diamagnetic susceptibility*. For free electrons, $\lambda = m/m^* = 1$ and $\chi_{\mathrm{L}} = -\frac{1}{3}\chi_{\mathrm{P}}$ resulting in a reduced – but still paramagnetic – total susceptibility. However, in