$$i\hbar |\dot{\Psi}(t)\rangle = \tilde{H}_{int} |\Psi(t)\rangle$$

$$i\hbar\left(\dot{c}_g(t)|g\rangle + \dot{c}_e(t)|e\rangle\right) = \frac{\hbar\Omega}{2}\left(e^{i\Delta t - i\varphi} c_g(t)|e\rangle + e^{-i\Delta t + i\varphi} c_e(t)|g\rangle\right)$$

$$\left| \begin{array}{l} i\,\dot{c}_g(t) = \frac{\Omega}{2} e^{-i\Delta t + i\varphi} c_e(t) \\ i\,\dot{c}_e(t) = \frac{\Omega}{2} e^{i\Delta t - i\varphi} c_g(t) \end{array} \right.$$

$$\boxed{\begin{array}{ll} \dot{c}_g(t) = -i\frac{\Omega}{2} e^{-i\Delta t + i\varphi} c_e(t) & (1) \\ \dot{c}_e(t) = -i\frac{\Omega}{2} e^{i\Delta t - i\varphi} c_g(t) & (2) \end{array}}$$

Derivando (2)

$$\ddot{c}_e(t) = \frac{\Omega}{2}\Delta e^{i\Delta t - i\varphi} c_g(t) - i\frac{\Omega}{2} e^{i\Delta t - i\varphi} \dot{c}_g(t)$$

$$c_g(t) = i\frac{2}{\Omega} e^{-i\Delta t + i\varphi} \dot{c}_e(t) \quad (2) \qquad \dot{c}_g(t) = i\frac{\Omega}{2} e^{-i\Delta t + i\varphi} c_e(t) \quad (2)$$

$$\ddot{c}_e(t) = i\Delta \dot{c}_e(t) - \left(i\frac{\Omega}{2}\right)^2 c_e(t)$$

$$\boxed{\ddot{c}_e(t) - i\Delta \dot{c}_e(t) + \left(i\frac{\Omega}{2}\right)^2 c_e(t) = 0 \quad (3)}$$

esta ecuación diferencial tiene coeficientes constantes!

$$\Rightarrow c_e(t) = A_1 e^{i\alpha_1 t} + A_2 e^{i\alpha_2 t}$$

con $\alpha_{1,2}$ las raíces de $\alpha^2 + \Delta\alpha - \frac{\Omega^2}{4} = 0$

$$\alpha_{1,2} = \frac{\Delta}{2} \pm \frac{1}{2}\sqrt{\Delta^2 + \Omega^2}$$

$$\boxed{c_e(t) = e^{i\frac{\Delta}{2}t}\left(A_1 e^{i\sqrt{\Delta^2+\Omega^2}\,t} + A_2 e^{-i\sqrt{\Delta^2+\Omega^2}\,t}\right)}$$

Por su parte, $c_g(t)$ puede encontrarse volviendo a (1)

$$\dot{c}_g(t) = -i\frac{\Omega}{2} e^{-i\Delta t + i\varphi} e^{i\frac{\Delta}{2}t}\left(A_1 e^{i\sqrt{\Delta^2+\Omega^2}\,t} + A_2 e^{-i\sqrt{\Delta^2+\Omega^2}\,t}\right)$$

$$= -i\frac{\Omega}{2} e^{i\varphi} e^{-i\frac{\Delta}{2}t}\left(A_1 e^{i\sqrt{\Delta^2+\Omega^2}\,t} + A_2 e^{-i\sqrt{\Delta^2+\Omega^2}\,t}\right)$$

$$\Rightarrow \boxed{c_g(t) = e^{i\varphi} e^{-i\frac{\Delta}{2}t} \frac{\Omega}{2}\left\{ \frac{A_1 e^{i\sqrt{\Delta^2+\Omega^2}\,t}}{\frac{\Delta}{2} - \sqrt{\Delta^2+\Omega^2}} + \frac{A_2 e^{-i\sqrt{\Delta^2+\Omega^2}\,t}}{\frac{\Delta}{2} + \sqrt{\Delta^2+\Omega^2}} \right\}}$$

Las constantes $A_{1,2}$ se fijan recurriendo a la condición inicial

$$c_e(0) = 1$$
$$c_g(0) = 0$$

$$c_e(0) = A_1 + A_2 = 1 \qquad (4)$$
$$c_g(0) = \frac{\Omega}{2}\left\{ \frac{A_1}{\frac{\Delta}{2} - \sqrt{\Delta^2+\Omega^2}} + \frac{A_2}{\frac{\Delta}{2} + \sqrt{\Delta^2+\Omega^2}} \right\} = 0 \qquad (5)$$

$$(5) \Rightarrow \frac{A_2}{\frac{\Delta}{2} + \sqrt{\Delta^2+\Omega^2}} = - \frac{A_1}{\frac{\Delta}{2} - \sqrt{\Delta^2+\Omega^2}}$$
$$A_2 = - \frac{\Delta + 2\sqrt{\Delta^2+\Omega^2}}{\Delta - 2\sqrt{\Delta^2+\Omega^2}} A_1$$

$$(4) \quad A_1 - \frac{\Delta + 2\sqrt{\Delta^2+\Omega^2}}{\Delta - 2\sqrt{\Delta^2+\Omega^2}} A_1 = 1$$
$$A_1 \frac{-4\sqrt{\Delta^2+\Omega^2}}{\Delta - 2\sqrt{\Delta^2+\Omega^2}} = 1 \Rightarrow \boxed{\begin{array}{l} A_1 = -\dfrac{\frac{\Delta}{2} - \sqrt{\Delta^2+\Omega^2}}{\sqrt{\Delta^2+\Omega^2}} \\ A_2 = \dfrac{\frac{\Delta}{2} + \sqrt{\Delta^2+\Omega^2}}{\sqrt{\Delta^2+\Omega^2}} \end{array}}$$

$$c_e(t) = e^{i\frac{\Delta}{2}t}\left(A_1 e^{i\sqrt{\Delta^2+\Omega^2}\,t} + A_2 e^{-i\sqrt{\Delta^2+\Omega^2}\,t}\right)$$
$$= \frac{e^{i\frac{\Delta}{2}t}}{\sqrt{\Delta^2+\Omega^2}}\left(-\left(\frac{\Delta}{2} - \sqrt{\Delta^2+\Omega^2}\right)e^{i\sqrt{\Delta^2+\Omega^2}\,t} + \left(\frac{\Delta}{2} + \sqrt{\Delta^2+\Omega^2}\right)e^{-i\sqrt{\Delta^2+\Omega^2}\,t}\right)$$

$$c_e(t) = e^{i\frac{\Delta}{2}t}\left\{\cos\left(\sqrt{\Delta^2+\Omega^2}\,t\right) - i\frac{\Delta}{\sqrt{\Delta^2+\Omega^2}}\sin\left(\sqrt{\Delta^2+\Omega^2}\,t\right)\right\}$$

$$c_g(t) = e^{i\varphi} e^{-i\frac{\Delta}{2}t}\frac{\Omega}{2}\left\{\frac{A_1 e^{i\sqrt{\Delta^2+\Omega^2}\,t}}{\frac{\Delta}{2} - \sqrt{\Delta^2+\Omega^2}} + \frac{A_2 e^{-i\sqrt{\Delta^2+\Omega^2}\,t}}{\frac{\Delta}{2} + \sqrt{\Delta^2+\Omega^2}}\right\}$$

$$c_g(t) = -e^{i\varphi} e^{-i\frac{\Delta}{2}t}\frac{\Omega}{\sqrt{\Delta^2+\Omega^2}}\sin\left(\sqrt{\Delta^2+\Omega^2}\,t\right)$$