Estructura de la Materia 2

Clase 12 - Teoría

Docentes

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Web: http://materias.df.uba.ar/edlm2a2020c2

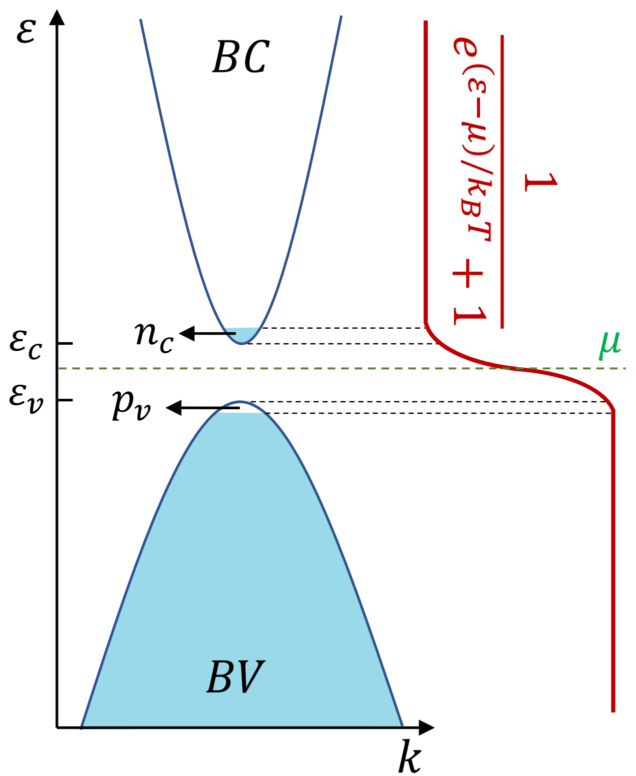
Repaso

Semiconductor en equilibrio térmico y caso intrínseco

$$\begin{cases} n_c(T) = \int_{\varepsilon_c}^{\infty} f(\varepsilon) g_c(\varepsilon) d\varepsilon = \int_{\varepsilon_c}^{\infty} \left[e^{(\varepsilon-\mu)/k_BT} + 1\right]^{-1} g_c(\varepsilon) d\varepsilon \\ p_v(T) = \int_{-\infty}^{\varepsilon_v} \left(1 - f(\varepsilon)\right) g_v(\varepsilon) d\varepsilon = \int_{-\infty}^{\varepsilon_v} \left[e^{(\mu-\varepsilon)/k_BT} + 1\right]^{-1} g_v(\varepsilon) d\varepsilon \end{cases}$$

$$\begin{cases} \varepsilon_c - \mu \gg k_BT \\ \mu - \varepsilon_v \gg k_BT \end{cases}$$

(Condición de no-degeneración)



$$\begin{cases} n_c(T) = N_c(T) e^{-\frac{\varepsilon_c-\mu}{k_BT}} \\ p_v(T) = P_v(T) e^{-\frac{\mu-\varepsilon_v}{k_BT}} \end{cases}$$

$$\begin{cases} N_c(T) = \int_{\varepsilon_c}^{\infty} e^{-\frac{\varepsilon-\varepsilon_c}{k_BT}} g_c(\varepsilon) d\varepsilon = \frac{1}{4}\left(\frac{2m_ck_BT}{\pi\hbar^2}\right)^{3/2} \\ P_v(T) = \int_{-\infty}^{\varepsilon_v} e^{-\frac{\varepsilon_v-\varepsilon}{k_BT}} g_v(\varepsilon) d\varepsilon = \frac{1}{4}\left(\frac{2m_vk_BT}{\pi\hbar^2}\right)^{3/2} \end{cases}$$

$$\boxed{n_cp_v = N_cP_v e^{-\frac{\varepsilon_c-\varepsilon_v}{k_BT}} = N_cP_v e^{-\frac{E_g}{k_BT}} = n_i^2}$$

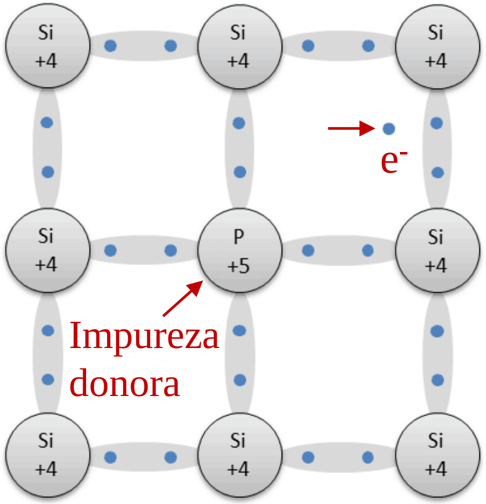
Ley de acción de masas

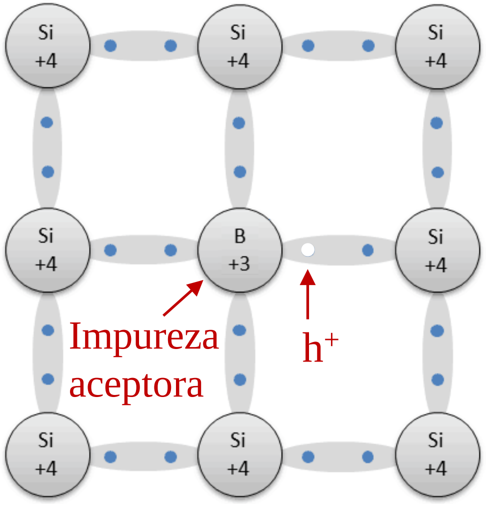
$$\begin{cases} n_i(T) = [N_cP_v]^{1/2} e^{-\frac{E_g}{2k_BT}} = \frac{1}{4}\left(\frac{2k_BT}{\pi\hbar^2}\right)^{3/2} (m_cm_v)^{3/4} e^{-\frac{E_g}{2k_BT}} = n_c^{(i)} = p_v^{(i)} \\ \mu_i = \varepsilon_v + \frac{E_g}{2} + \frac{1}{2}k_BT \ln\left(\frac{P_v}{N_c}\right) = \varepsilon_v + \frac{E_g}{2} + \frac{3}{4}k_BT \ln\left(\frac{m_v}{m_c}\right) \end{cases}$$

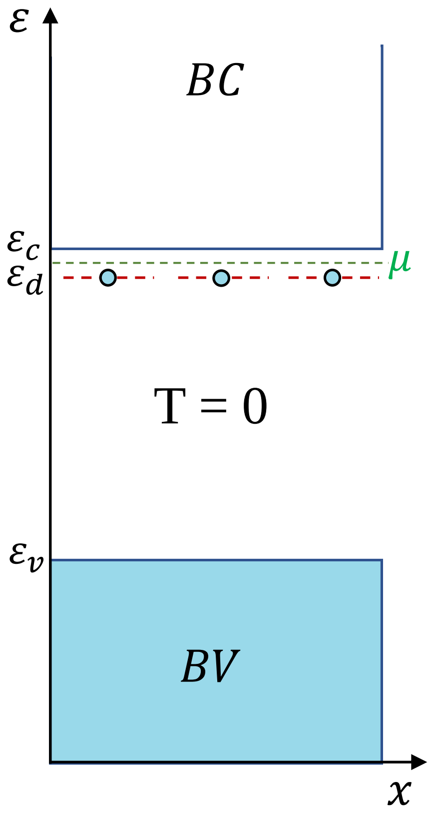
Repaso

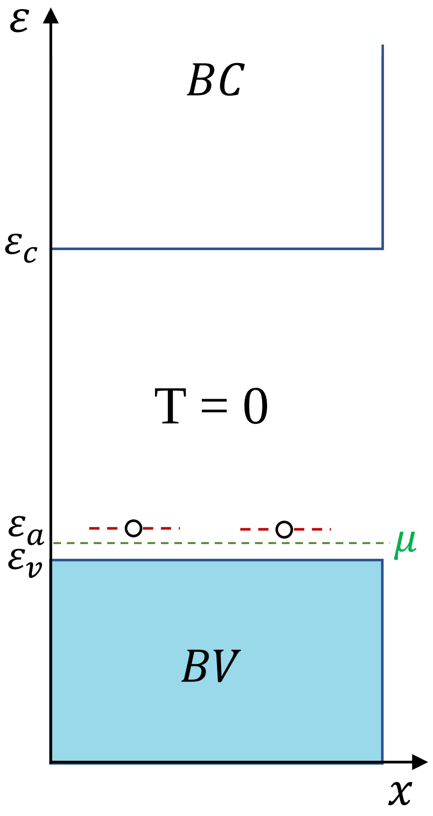
Semiconductor extrínseco

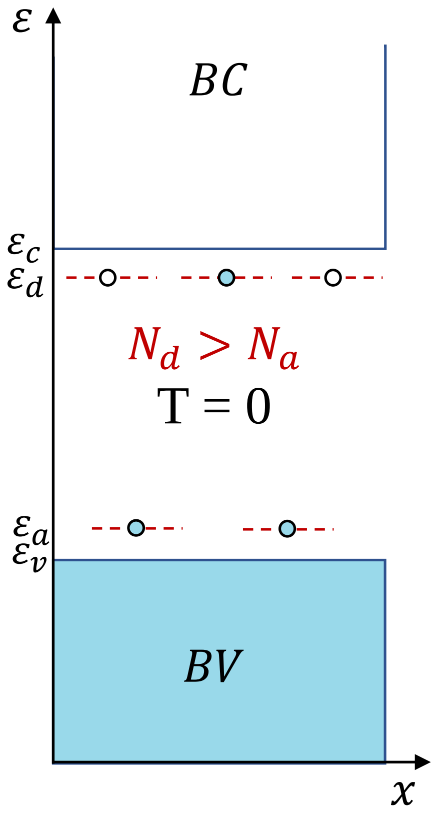
Dopamos al semiconductor con una baja concentración de impurezas donoras o aceptoras.











$$n_d = \frac{N_d}{\frac{1}{2}e^{\beta(\varepsilon_d-\mu)}+1}$$

$$p_a = \frac{N_a}{\frac{1}{2}e^{\beta(\mu-\varepsilon_a)}+1}$$

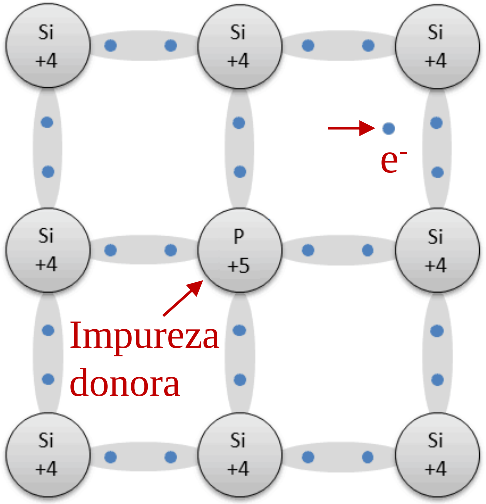
$$n_c + n_d = N_d - N_a + p_v + p_a$$

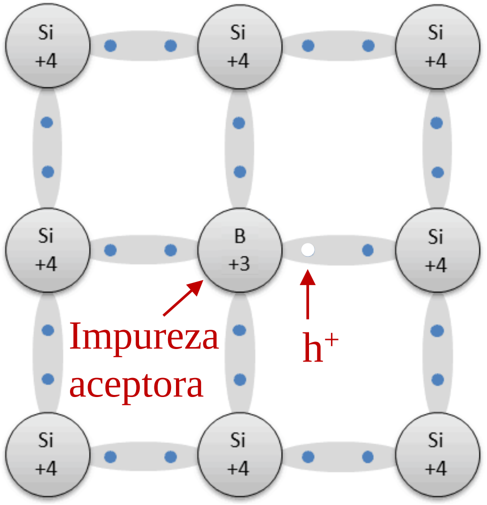
Balance de carga

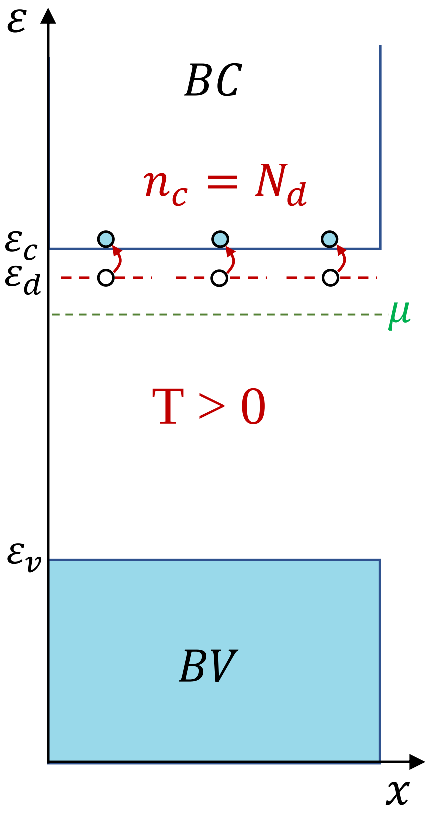
Repaso

Semiconductor extrínseco

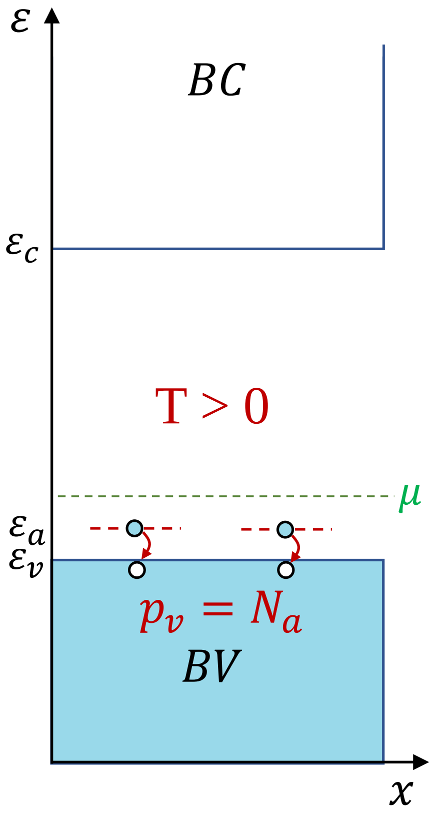
Dopamos al semiconductor con una baja concentración de impurezas donoras o aceptoras.



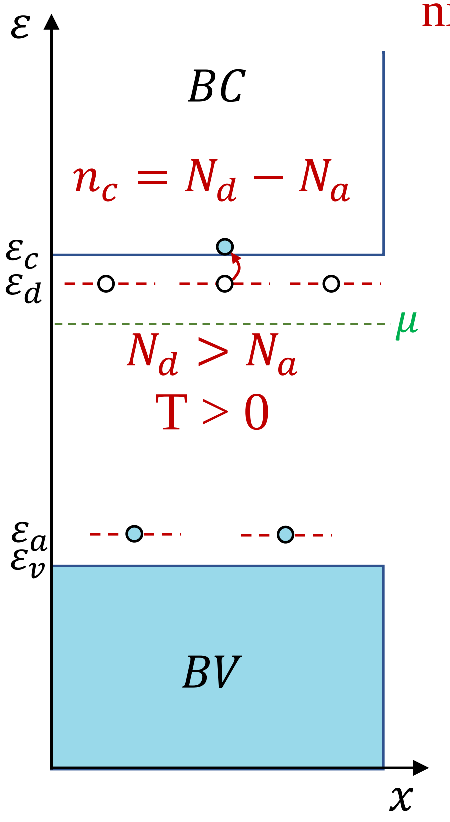




$$n_d = \frac{N_d}{\frac{1}{2} e^{\beta(\varepsilon_d - \mu)} + 1}$$



$$p_a = \frac{N_a}{\frac{1}{2} e^{\beta(\mu - \varepsilon_a)} + 1}$$



Cond. de no-deg. para niveles de impurezas

$$\begin{cases} \varepsilon_d - \mu \gg k_B T \\ \mu - \varepsilon_a \gg k_B T \end{cases}$$

$$\Delta n = n_c - p_v = N_d - N_a$$

Impurezas totalmente ionizadas

$$\boxed{n_c + n_d = N_d - N_a + p_v + p_a}$$

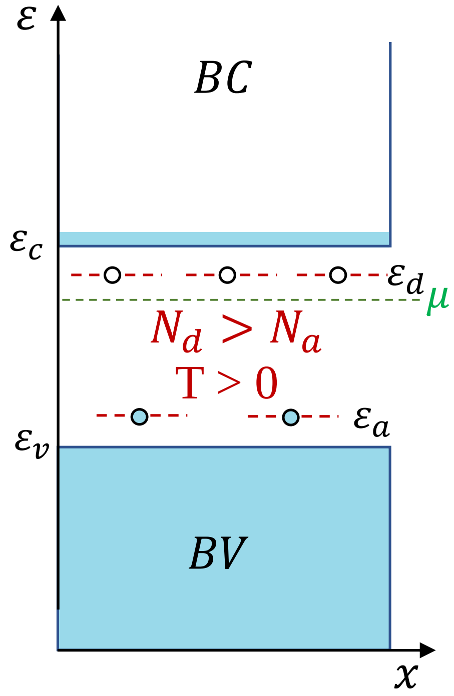
Balance de carga

Repaso

Semiconductor extrínseco

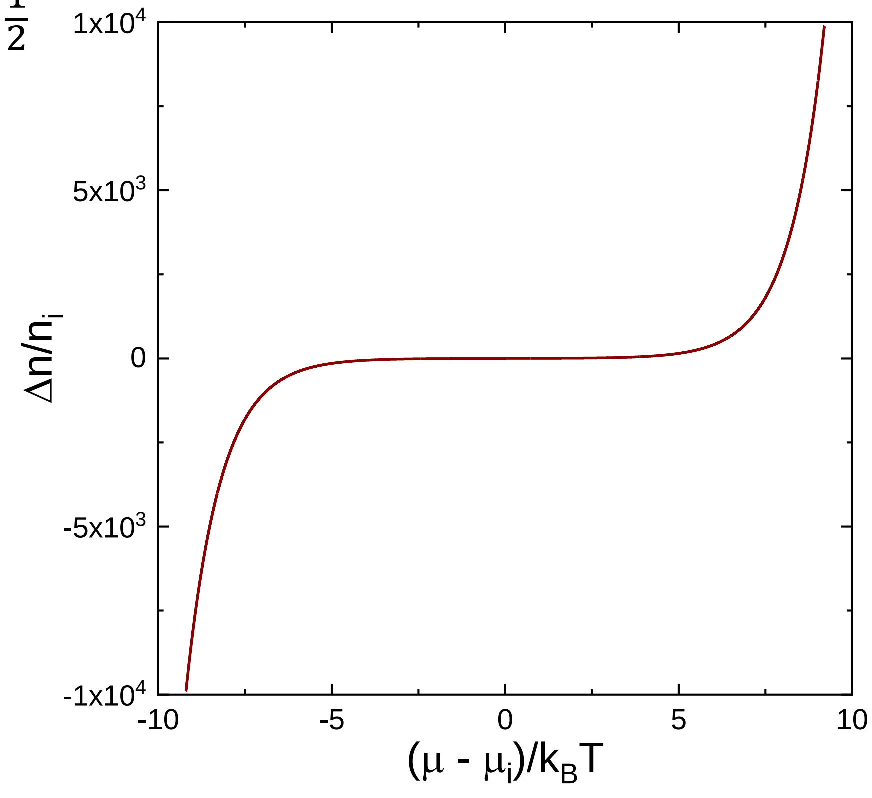
$$\begin{cases} n_c p_v = n_i^2 \\ n_c - p_v = \Delta n \end{cases} \rightarrow \begin{Bmatrix} n_c \\ p_v \end{Bmatrix} = \pm\frac{1}{2}(\Delta n) + \frac{1}{2}\left[(\Delta n)^2 + 4n_i^2\right]^{\frac{1}{2}};\quad \begin{cases} n_c = e^{\beta(\mu-\mu_i)} n_i \\ p_v = e^{-\beta(\mu-\mu_i)} n_i \end{cases} \rightarrow \frac{\Delta n}{n_i} = 2\sinh(\beta(\mu-\mu_i))$$

$$\begin{cases} \varepsilon_d - \mu \gg k_B T \\ \mu - \varepsilon_a \gg k_B T \end{cases} \rightarrow \begin{Bmatrix} n_c \\ p_v \end{Bmatrix} = \pm\frac{1}{2}(N_d - N_a) + \frac{1}{2}\left[(N_d - N_a)^2 + 4n_i^2\right]^{\frac{1}{2}}$$



$$\xrightarrow{n_i \gg |N_d - N_a|} \begin{Bmatrix} n_c \\ p_v \end{Bmatrix} \approx \pm\frac{1}{2}(N_d - N_a) + n_i$$

$$\underset{\begin{cases} \varepsilon_d - \mu \gg k_B T \\ \mu - \varepsilon_a \gg k_B T \end{cases}}{\xrightarrow{n_i \ll |N_d - N_a|}} \begin{cases} N_d > N_a & \begin{cases} n_c \approx N_d - N_a \\ p_v \approx \dfrac{n_i^2}{N_d - N_a} \end{cases} \\ N_a > N_d & \begin{cases} n_c \approx \dfrac{n_i^2}{N_a - N_d} \\ p_v \approx N_a - N_d \end{cases} \end{cases}$$

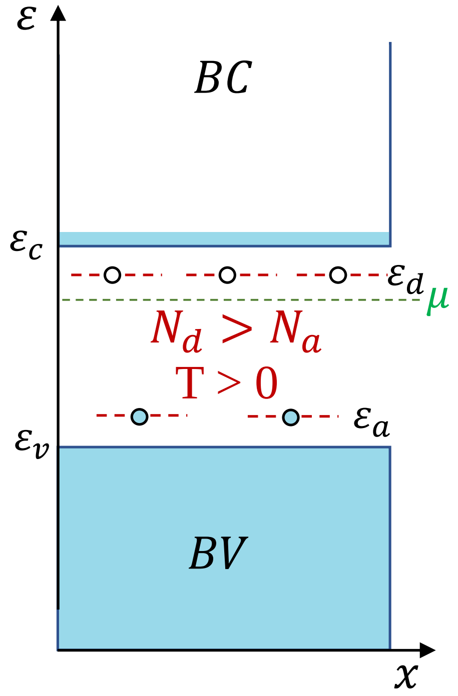


Repaso

Semiconductor extrínseco

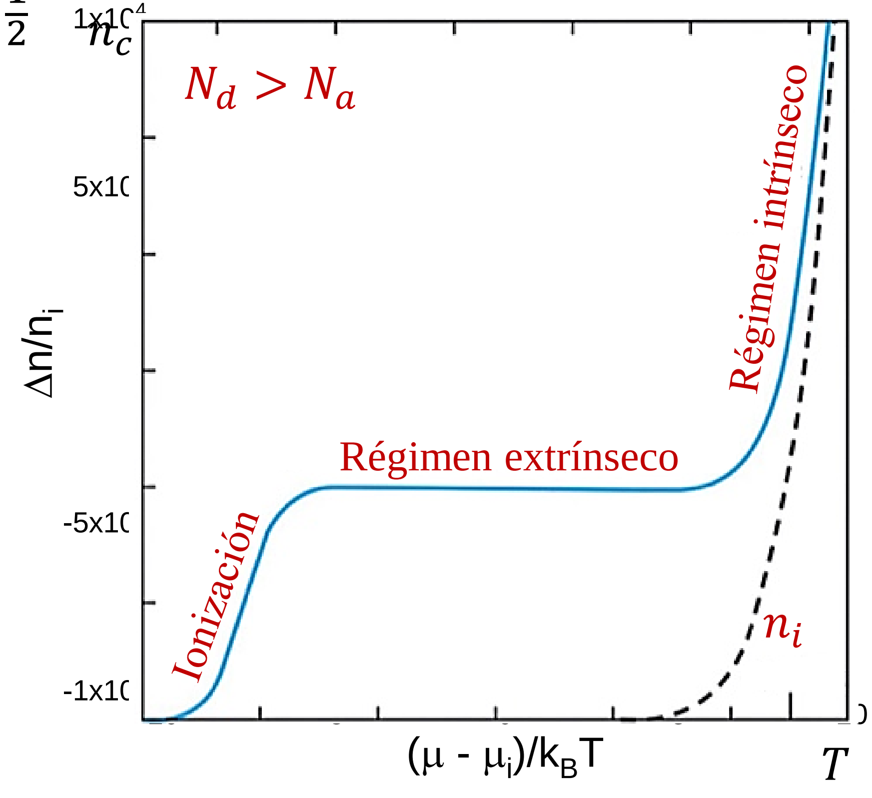
$$\begin{cases} n_c p_v = n_i^2 \\ n_c - p_v = \Delta n \end{cases} \rightarrow \begin{Bmatrix} n_c \\ p_v \end{Bmatrix} = \pm\frac{1}{2}(\Delta n) + \frac{1}{2}\left[(\Delta n)^2 + 4n_i^2\right]^{\frac{1}{2}}\,; \quad \begin{cases} n_c = e^{\beta(\mu-\mu_i)} n_i \\ p_v = e^{-\beta(\mu-\mu_i)} n_i \end{cases} \rightarrow \frac{\Delta n}{n_i} = 2\sinh(\beta(\mu-\mu_i))$$

$$\begin{cases} \varepsilon_d - \mu \gg k_B T \\ \mu - \varepsilon_a \gg k_B T \end{cases} \rightarrow \begin{Bmatrix} n_c \\ p_v \end{Bmatrix} = \pm\frac{1}{2}(N_d - N_a) + \frac{1}{2}\left[(N_d - N_a)^2 + 4n_i^2\right]^{\frac{1}{2}}$$



$$\xrightarrow{n_i \gg |N_d - N_a|} \begin{Bmatrix} n_c \\ p_v \end{Bmatrix} \approx \pm\frac{1}{2}(N_d - N_a) + n_i$$

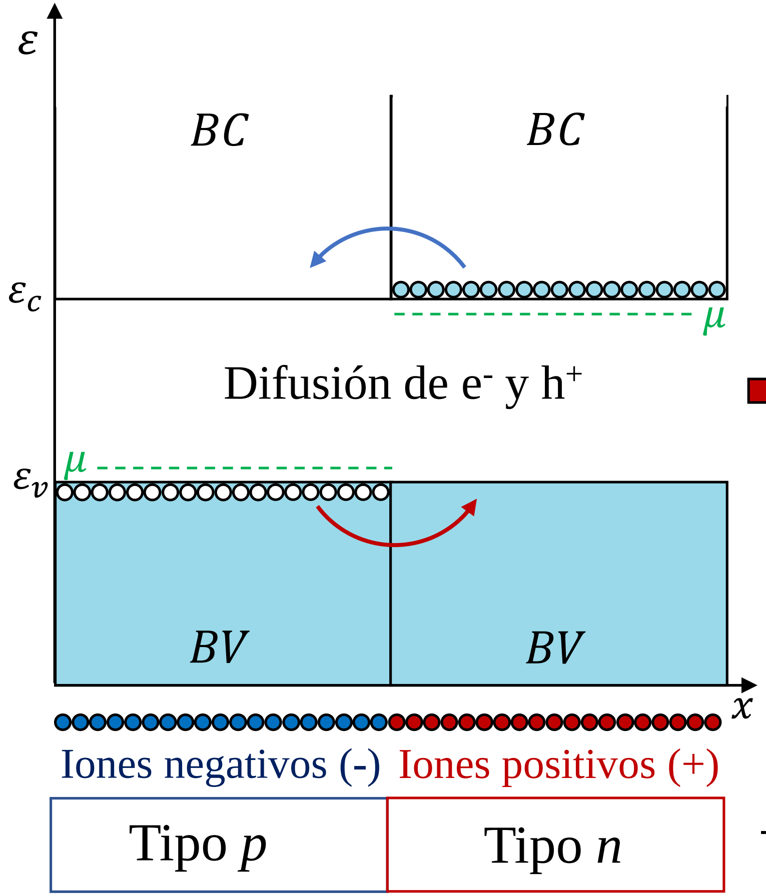
$$\xrightarrow[\begin{cases} \varepsilon_d - \mu \gg k_B T \\ \mu - \varepsilon_a \gg k_B T \end{cases}]{n_i \ll |N_d - N_a|} \begin{cases} N_d > N_a & \begin{cases} n_c \approx N_d - N_a \\ p_v \approx \dfrac{n_i^2}{N_d - N_a} \end{cases} \\ N_a > N_d & \begin{cases} n_c \approx \dfrac{n_i^2}{N_a - N_d} \\ p_v \approx N_a - N_d \end{cases} \end{cases}$$

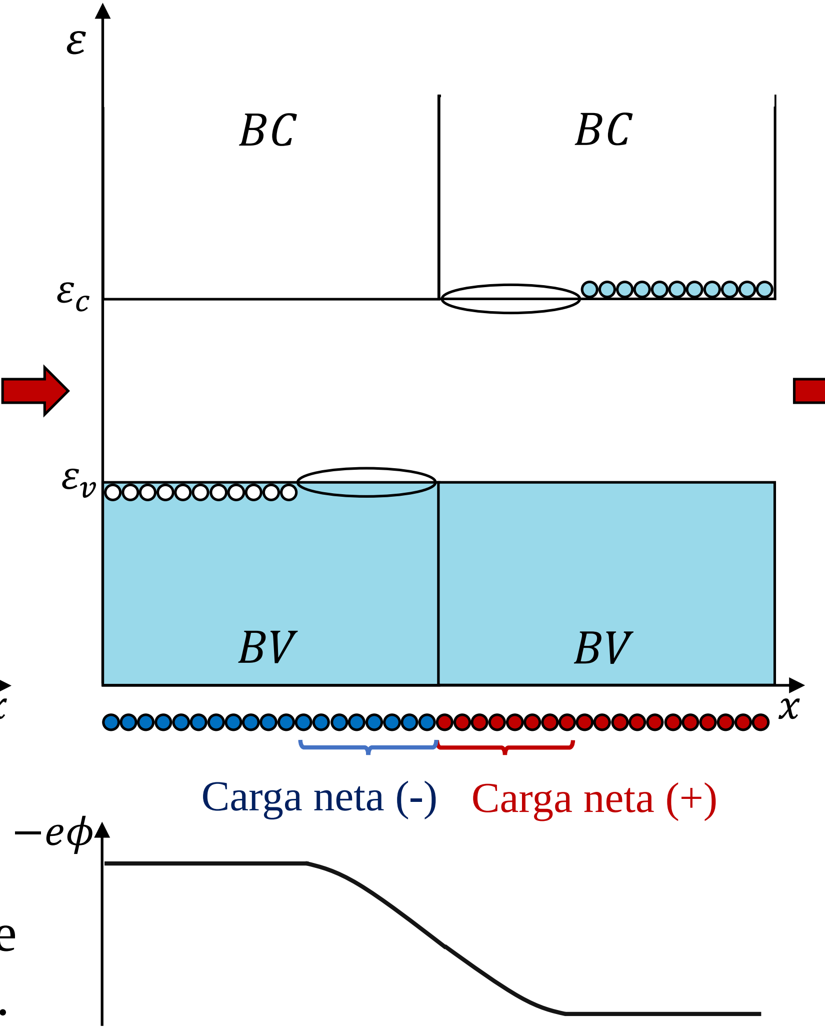


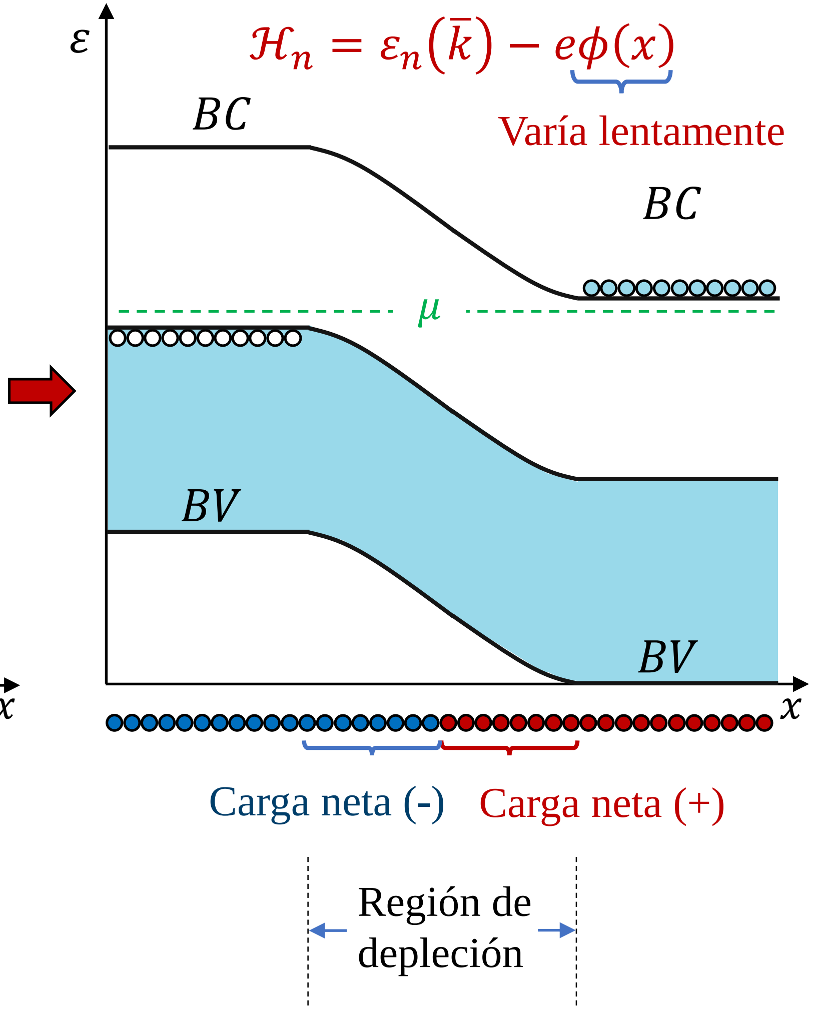
Juntura semiconductora *p-n*

Juntura semiconductora *p-n*

Construimos una juntura *p-n* poniendo semiconductores tipo *p* y *n* en contacto directo.



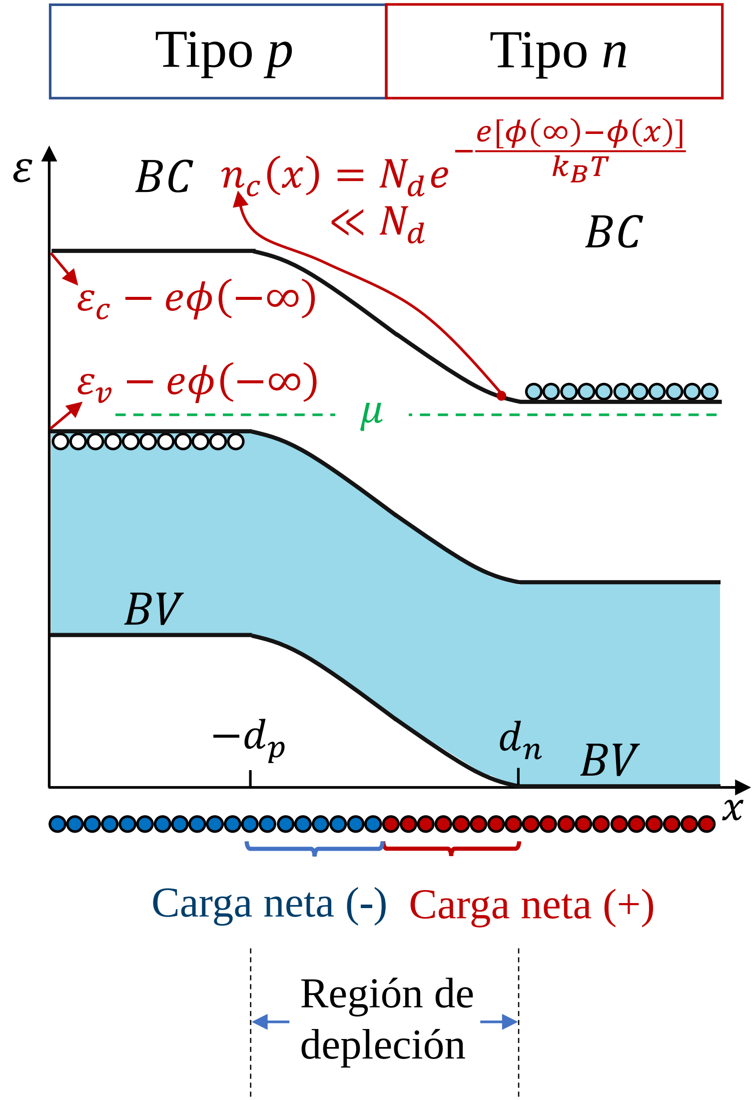




Suponemos impurezas totalmente ionizadas en cada semiconductor.

Juntura semiconductora *p-n*

<u>Juntura semiconductora *p-n*</u>: Región de depleción



$$\mathcal{H}_n = \varepsilon_n(\bar{k}) - e\phi(x)$$

$$\begin{cases} n_c(x) = N_c(T)e^{-\frac{[\varepsilon_c - e\phi(x) - \mu]}{k_B T}} \\ p_v(x) = P_v(T)e^{-\frac{[\mu - \varepsilon_v + e\phi(x)]}{k_B T}} \end{cases} \qquad \begin{cases} n_c(\infty) = N_c(T)e^{-\frac{[\varepsilon_c - e\phi(\infty) - \mu]}{k_B T}} = N_d \\ p_v(-\infty) = P_v(T)e^{-\frac{[\mu - \varepsilon_v + e\phi(-\infty)]}{k_B T}} = N_a \end{cases}$$

$$\rightarrow \mu = k_B T \ln \frac{N_d}{N_c} + \varepsilon_c - e\phi(\infty) = k_B T \ln \frac{P_v}{N_a} + \varepsilon_v - e\phi(-\infty)$$

$$\rightarrow e[\phi(\infty) - \phi(-\infty)] = \boxed{e\Delta\phi = E_g + k_B T \ln \left[\frac{N_d N_a}{N_c P_v}\right]}$$

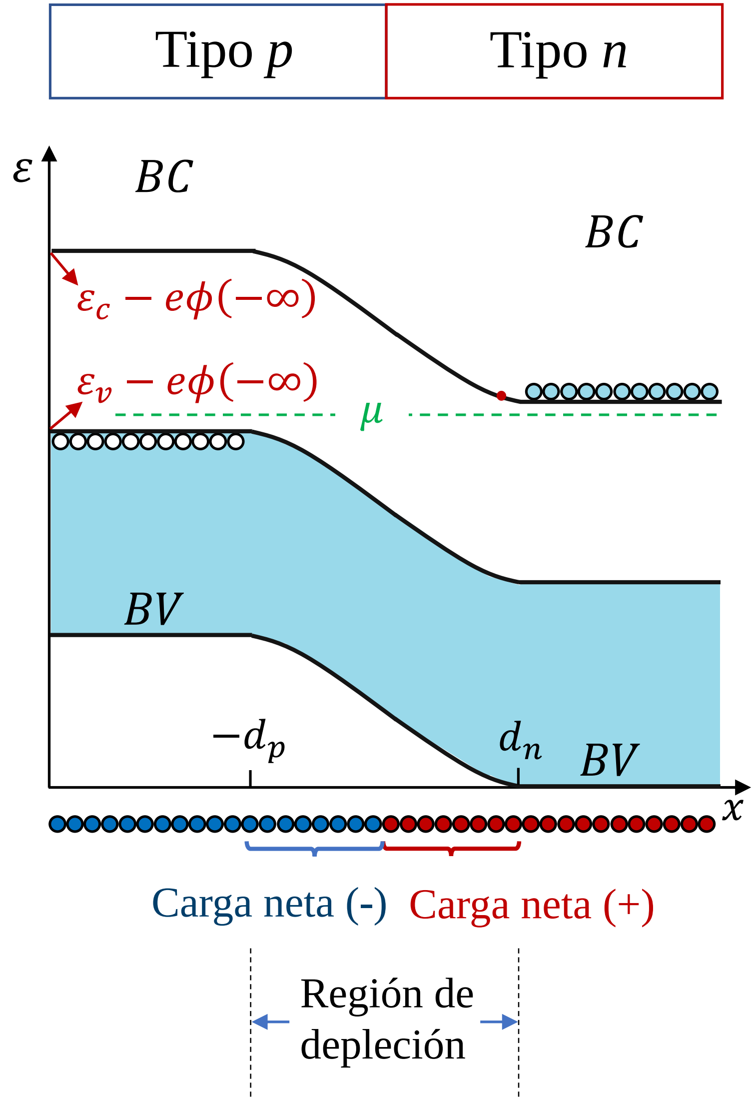
Nos sirve como condición de contorno para determinar $\phi(x)$ de la ecuación de Poisson: $\nabla^2\phi = \frac{d^2\phi}{dx^2} = -\frac{4\pi\rho(x)}{\epsilon}$

Fuera de la región de depleción: $\rho(x) = 0$

Dentro de la región de depleción: $\rho(x) = \begin{cases} eN_d, & x > 0 \\ -eN_a, & x < 0 \end{cases}$

Juntura semiconductora *p-n*

Juntura semiconductora *p-n*: Región de depleción



$$\phi''(x) = \begin{cases} 0, & x > d_n \\ -\dfrac{4\pi e N_d}{\epsilon}, & d_n > x > 0 \\ \dfrac{4\pi e N_a}{\epsilon}, & 0 > x > -d_p \\ 0, & -d_p > x \end{cases} \qquad \phi(x) = \begin{cases} \phi(\infty) \\ \phi(\infty) - \dfrac{2\pi e N_d}{\epsilon}(x - d_n)^2 \\ \phi(-\infty) + \dfrac{2\pi e N_a}{\epsilon}(x + d_p)^2 \\ \phi(-\infty) \end{cases}$$

Continuidad de ϕ en $x = 0$ → $\Delta\phi = \dfrac{2\pi e}{\epsilon}\left(N_d {d_n}^2 + N_a {d_p}^2\right)$

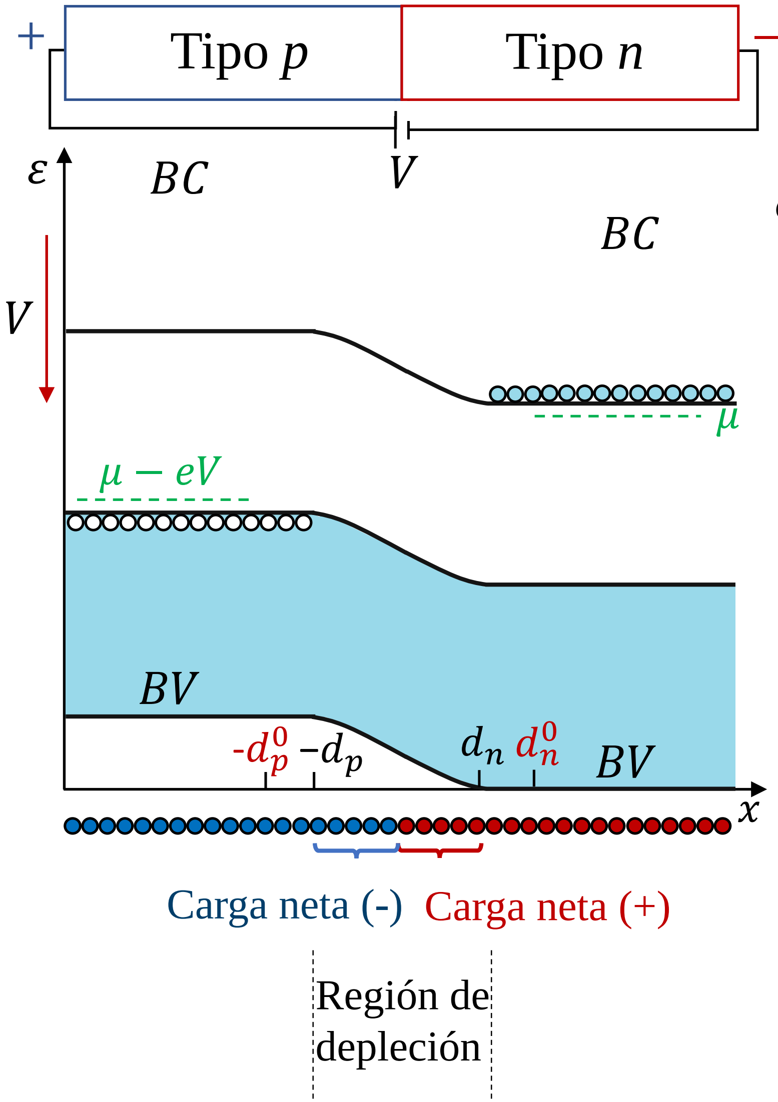
Continuidad de ϕ' en $x = 0$ → $N_d d_n = N_a d_p$

→ $d_{n,p} = \left[\dfrac{(N_a/N_d)^{\pm 1}}{N_d + N_a}\dfrac{\epsilon\Delta\phi}{2\pi e}\right]^{1/2}$ ($e\epsilon\Delta\phi \sim 1eV$, $N_a, N_d \sim 10^{16}$-10^{18} cm^{-3} → $d_{n,p} \sim 10^4$-10^2 Å)

$V \neq 0$ → $\Delta\phi = (\Delta\phi)_0 - V$ → $d_{n,p}(V) = \underbrace{d_{n,p}(0)}_{d^0_{n,p}}\left[1 - \dfrac{V}{(\Delta\phi)_0}\right]^{1/2}$

Juntura semiconductora p-n

Juntura semiconductora p-n: Región de depleción



$$\phi''(x) = \begin{cases} 0, & x > d_n \\ -\dfrac{4\pi e N_d}{\epsilon}, & d_n > x > 0 \\ \dfrac{4\pi e N_a}{\epsilon}, & 0 > x > -d_p \\ 0, & -d_p > x \end{cases} \qquad \phi(x) = \begin{cases} \phi(\infty) \\ \phi(\infty) - \dfrac{2\pi e N_d}{\epsilon}(x - d_n)^2 \\ \phi(-\infty) + \dfrac{2\pi e N_a}{\epsilon}(x + d_p)^2 \\ \phi(-\infty) \end{cases}$$

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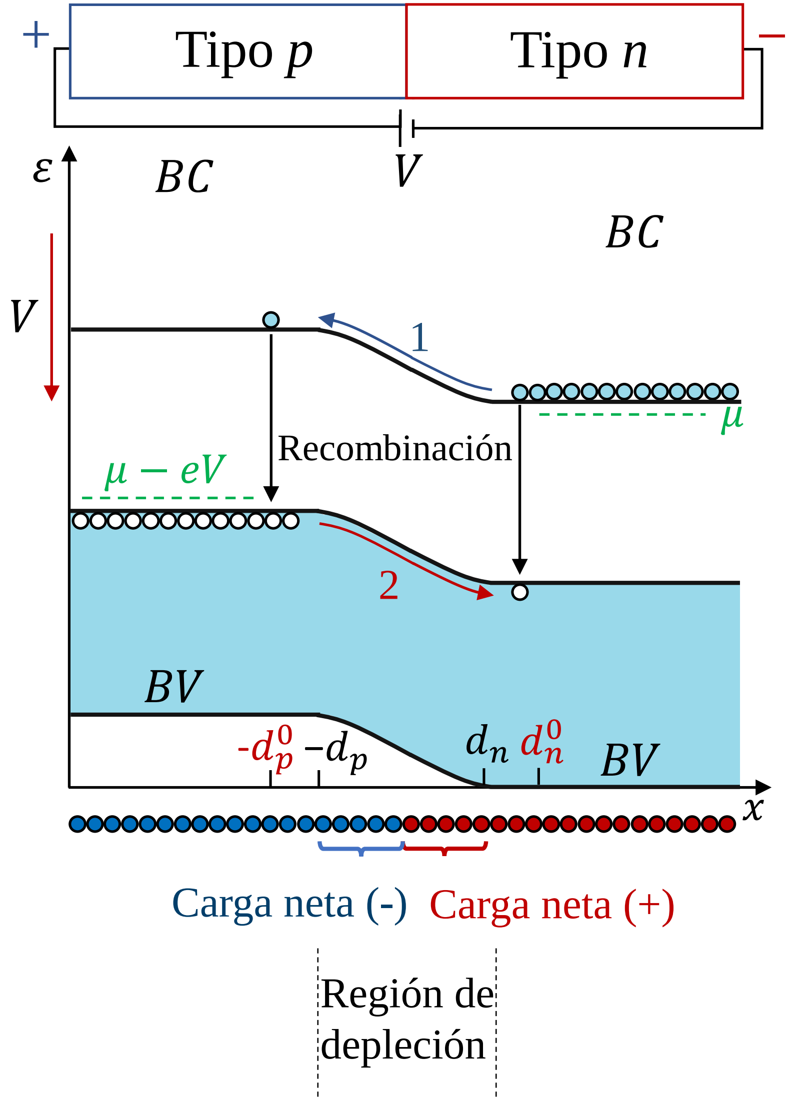
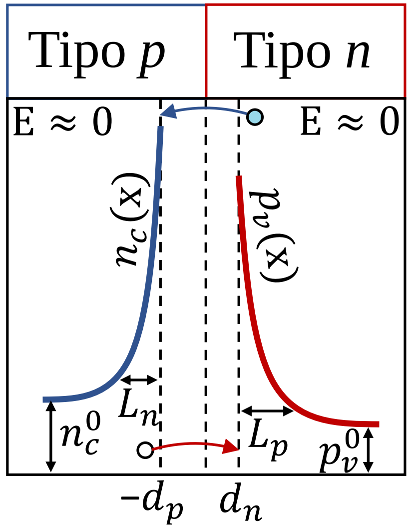
Juntura semiconductora *p-n*

<u>Juntura semiconductora *p-n*</u>: Corriente a través de la juntura



Definimos: $j_e = -eJ_e$; $j_h = eJ_h$ ($J_{e,h}$: Densidades de corriente numéricas)



Escribimos $J_{e,h}$ como la suma de la corriente de deriva ($\propto$ E $= -d\phi/dx$) y la corriente de difusión ($\propto$ gradiente de la densidad de portadores):

$$J_e = -\mu_n n_c E - D_n \frac{dn_c}{dx}; \quad J_h = \mu_p p_v E - D_p \frac{dp_v}{dx}$$

($\mu_{n,p}$: Movilidad de e⁻, h⁺; $D_{n,p}$: Cte. de difusión de e⁻, h⁺)

Escribimos también las ecuaciones de continuidad:

Diferencia entre el flujo de salida y entrada de portadores

$$\frac{\partial n_c}{\partial t} = \underbrace{G_e - R_e} - \overbrace{\frac{\partial J_e}{\partial x}}; \quad \frac{\partial p_v}{\partial t} = \underbrace{G_h - R_h} - \overbrace{\frac{\partial J_h}{\partial x}}$$

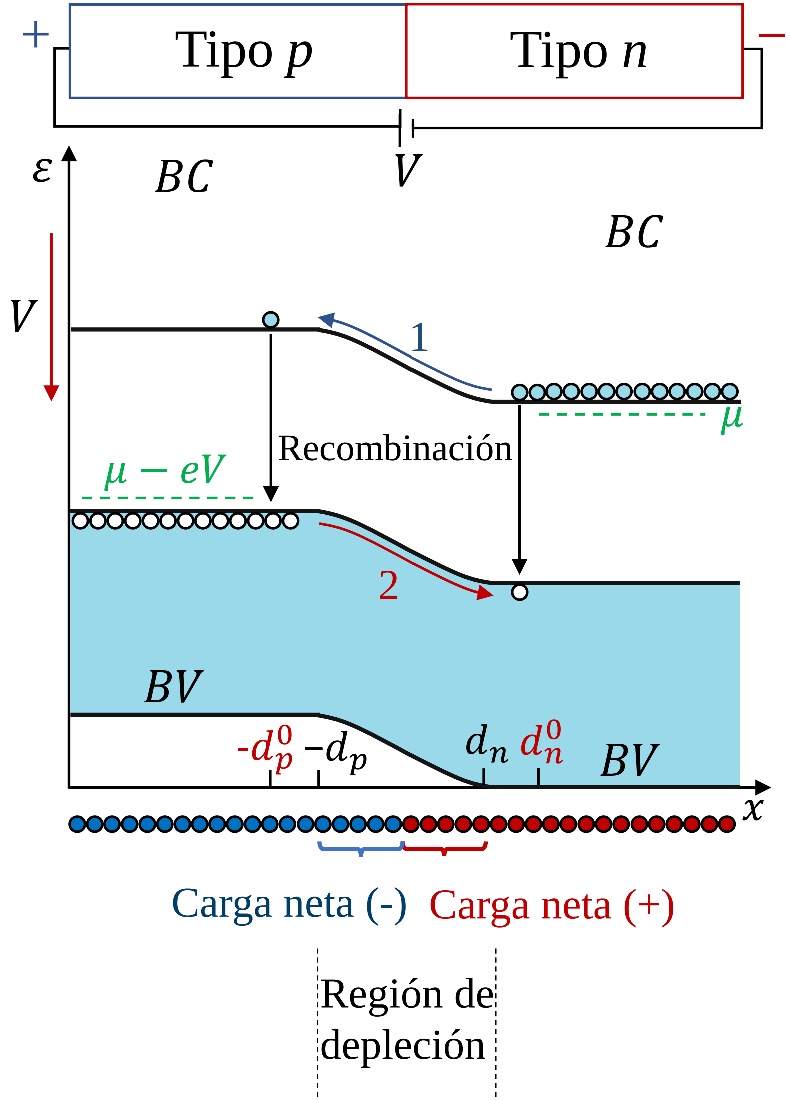
Diferencia entre tasa de generación y recombinación de portadores

Suponemos que en la región de depleción $G_{e,h}, R_{e,h} \approx 0$, con lo cual en el estacionario J_e y J_h resultan constantes a lo largo de la misma.

→ $j = -eJ_e(-d_p) + eJ_h(d_n)$

Juntura semiconductora *p-n*

<u>Juntura semiconductora *p-n*</u>: Corriente a través de la juntura



<u>Lado *p*</u>

Escribimos: $R_e = \alpha n_c p_v;\ G_e = \alpha n_c^0 p_v^0$

(En eq. térmico → $G_e = R_e = \alpha n_c^0 p_v^0$)

→ $R_e - G_e = \alpha n_c p_v - \alpha n_c^0 p_v^0 \approx \alpha p_v^0 (n_c - n_c^0)$

$$= \frac{n_c - n_c^0}{\tau_n}$$

$n_c^0 = n_i^2/N_a$

$\tau_n = 1/\alpha p_v^0$ (Tiempo de vida e$^-$)

En el estacionario: $\dfrac{\partial n_c}{\partial t} = 0 = -\dfrac{n_c - n_c^0}{\tau_n} - \dfrac{\partial J_e}{\partial x}$

$J_e = -\mu_n n_c E - D_n dn_c/dx$

→ $-\dfrac{\partial J_e}{\partial x} = \dfrac{n_c - n_c^0}{\tau_n} = D_n \dfrac{d^2 n_c}{d^2 x}$ → $n_c = n_c^0 + \left(n_c(-d_p) - n_c^0\right) e^{\frac{x+d_p}{\sqrt{D_n \tau_n}}}$

E ≈ 0

$L_n = \sqrt{D_n \tau_n}$

Longitudes de difusión

→ $J_e(-d_p) = -\dfrac{D_n}{L_n}\left(n_c(-d_p) - n_c^0\right)$

<u>Lado *n*</u>

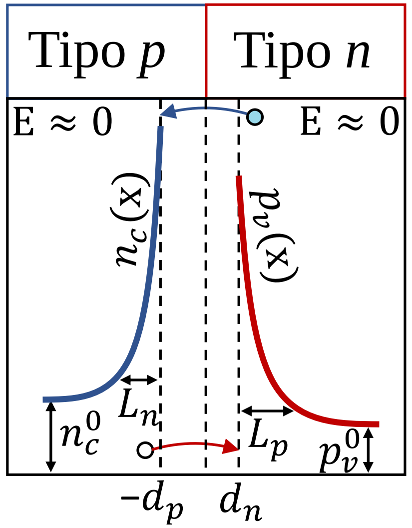
Análogamente:

$p_v^0 = \dfrac{n_i^2}{N_d}$

Tiempo de vida h$^+$

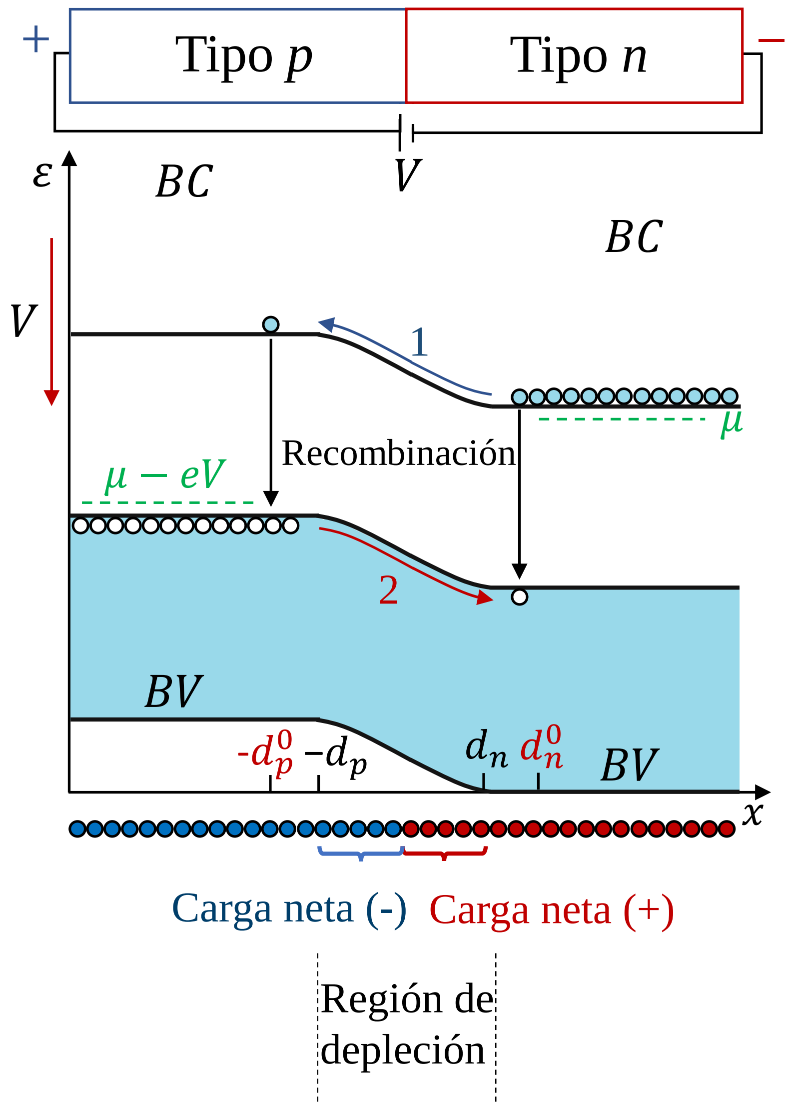
$L_p = \sqrt{D_p \tau_p}$

$p_v = p_v^0 + \left(p_v(d_n) - p_v^0\right) e^{-\frac{x-d_n}{\sqrt{D_p \tau_p}}}$ → $J_h(d_n) = \dfrac{D_p}{L_p}\left(p_v(d_n) - p_v^0\right)$



Juntura semiconductora *p-n*

Juntura semiconductora *p-n*: Corriente a través de la juntura



Lado p

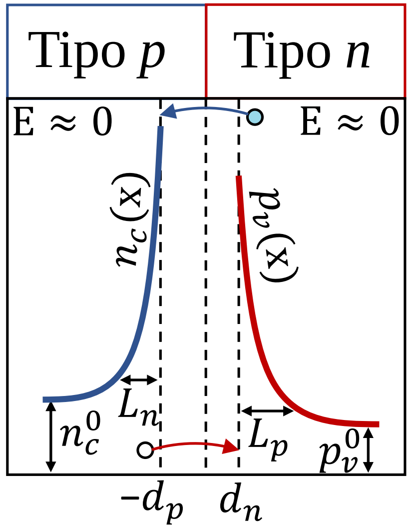
$$V = 0:\ n_c(x) = N_c(T)e^{-\frac{[\varepsilon_c - e\phi(x) - \mu]}{k_B T}}$$

$$\begin{cases} n_c(d_n^0) = N_c(T)e^{-\frac{[\varepsilon_c - e\phi(\infty) - \mu]}{k_B T}} = N_d \\ n_c(-d_p^0) = N_c(T)e^{-\frac{[\varepsilon_c - e\phi(-\infty) - \mu]}{k_B T}} = N_d e^{-\frac{e(\Delta\phi)_0}{k_B T}} \end{cases}$$

$$V \neq 0:\ n_c(-d_p) = N_d e^{-\frac{e(\Delta\phi)_0}{k_B T}} e^{\frac{V}{k_B T}} = \frac{n_i^2}{N_a} e^{\frac{V}{k_B T}}$$

n_c^0

$$\rightarrow J_e(-d_p) = -\frac{D_n}{L_n}\left(n_c(-d_p) - n_c^0\right) = -\frac{D_n}{L_n N_a} n_i^2 \left(e^{\frac{V}{k_B T}} - 1\right)$$



Lado n

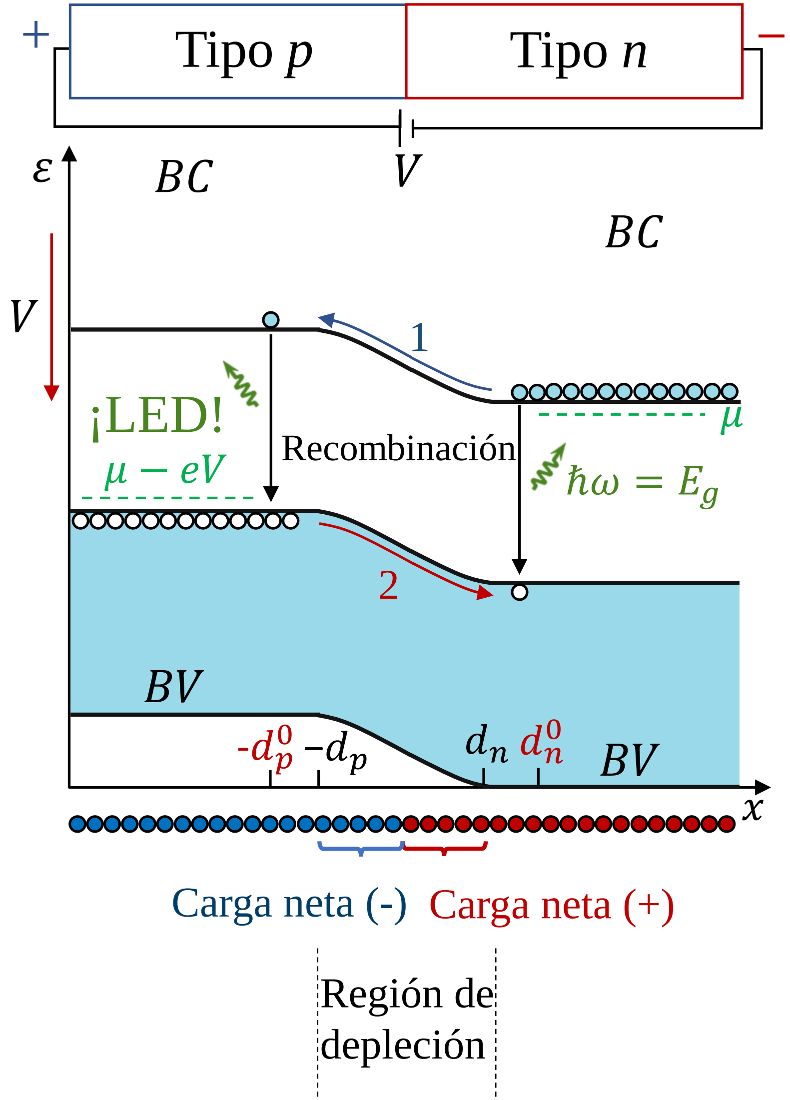
$$\text{Análogamente: } p_v(d_n) = \frac{n_i^2}{N_d} e^{\frac{V}{k_B T}} \rightarrow J_h(d_n) = \frac{D_p}{L_p N_d} n_i^2 \left(e^{\frac{V}{k_B T}} - 1\right)$$

Corriente total

$$j = -eJ_e(-d_p) + eJ_h(d_n) = e n_i^2 \left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d}\right)\left(e^{\frac{V}{k_B T}} - 1\right)$$

Juntura semiconductora *p-n*

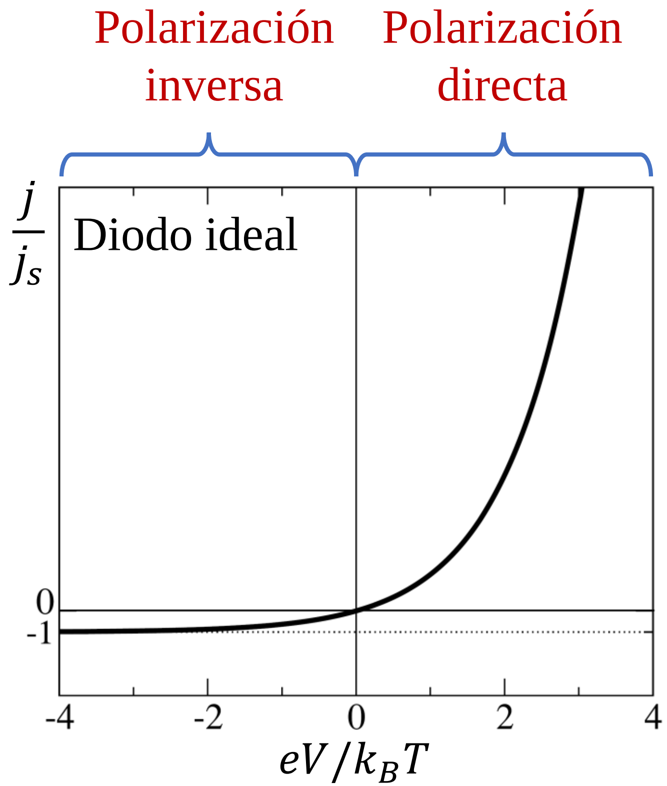
Juntura semiconductora *p-n*: Corriente a través de la juntura

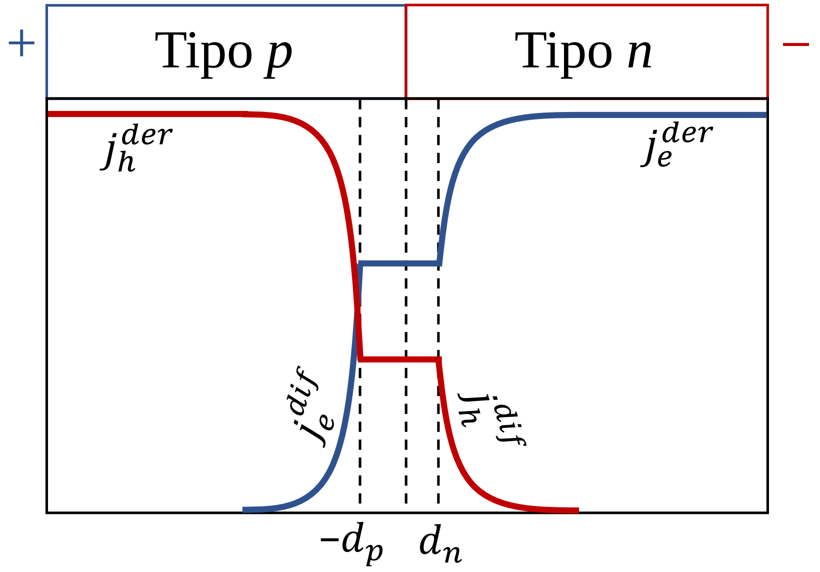


$$j = e n_i^2 \underbrace{\left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d}\right)}_{\text{Corriente de saturación: } j_s \,\propto\, e^{-E_g/k_BT}} \left(e^{\frac{V}{k_BT}} - 1\right) = e n_i^2 \left(\frac{L_n}{N_a \tau_n} + \frac{L_p}{N_d \tau_p}\right)\left(e^{\frac{V}{k_BT}} - 1\right)$$

$L_{n,p} = \sqrt{D_{n,p}\tau_{n,p}}$

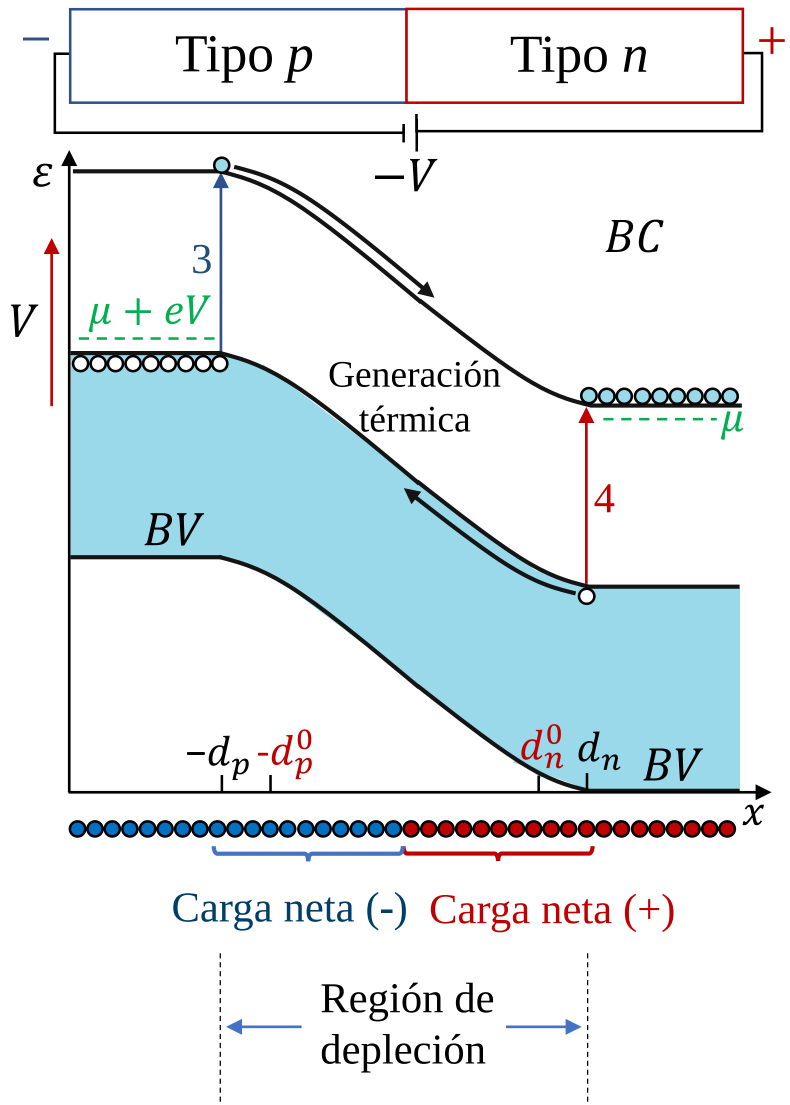
$$= e\left(J_e^{gen} + J_h^{gen}\right)\left(e^{\frac{V}{k_BT}} - 1\right)$$





Juntura semiconductora *p-n*

Juntura semiconductora *p-n*: Corriente a través de la juntura



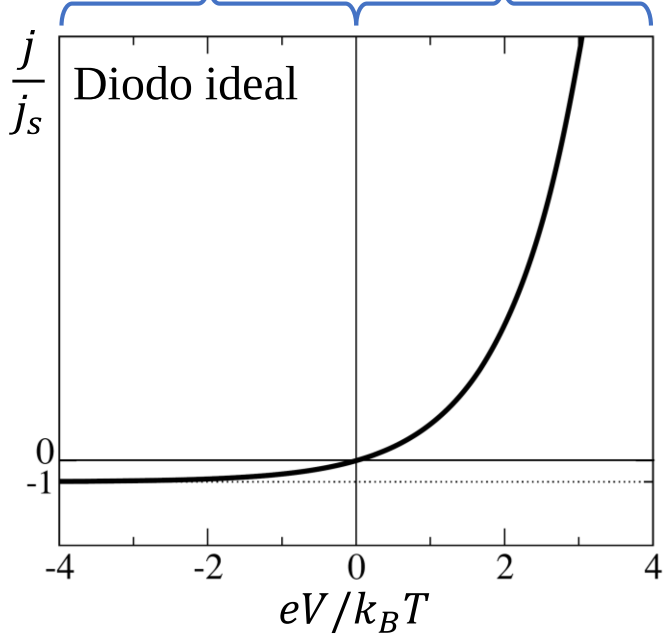
$$j = en_i^2\underbrace{\left(\frac{D_n}{L_nN_a}+\frac{D_p}{L_pN_d}\right)}_{\text{Corriente de saturación: } j_s \propto e^{-E_g/k_BT}}\left(e^{\frac{V}{k_BT}}-1\right) = en_i^2\left(\frac{L_n}{N_a\tau_n}+\frac{L_p}{N_d\tau_p}\right)\left(e^{\frac{V}{k_BT}}-1\right)$$

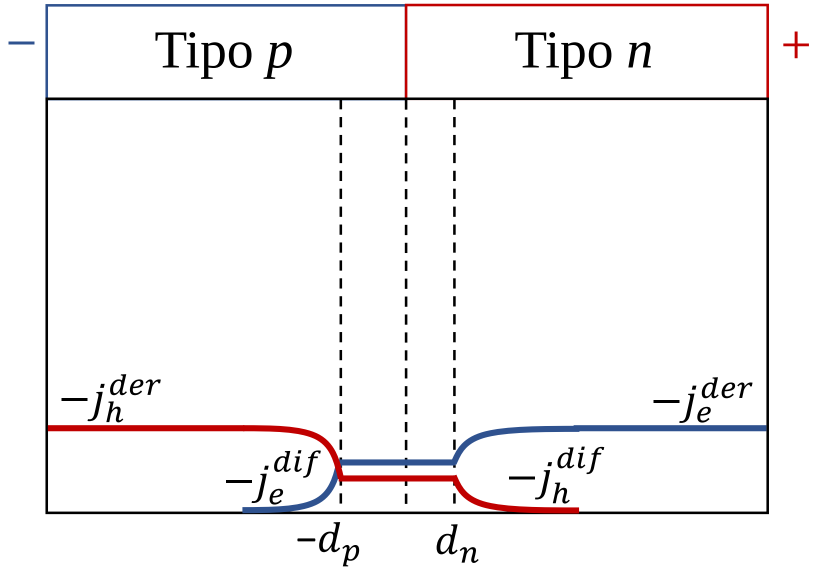
$L_{n,p} = \sqrt{D_{n,p}\tau_{n,p}}$

$$= e\left(J_e^{gen}+J_h^{gen}\right)\left(e^{\frac{V}{k_BT}}-1\right)$$

Polarización inversa

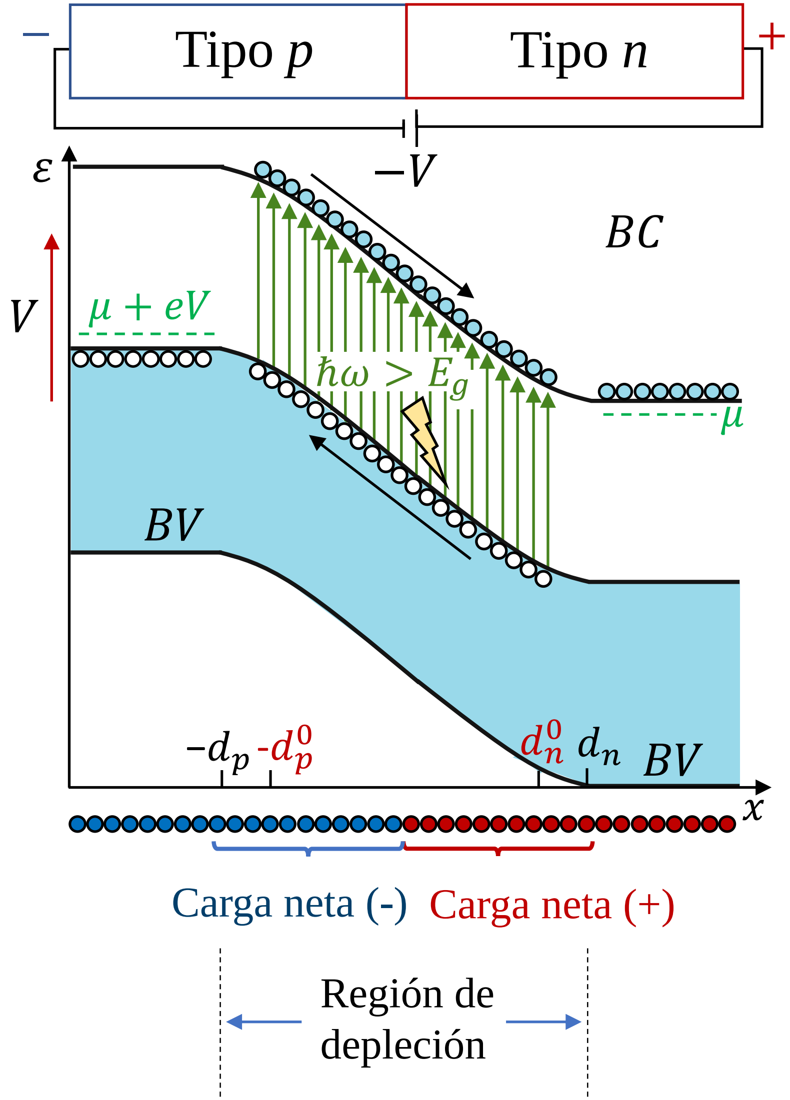
Polarización directa





Juntura semiconductora *p-n*

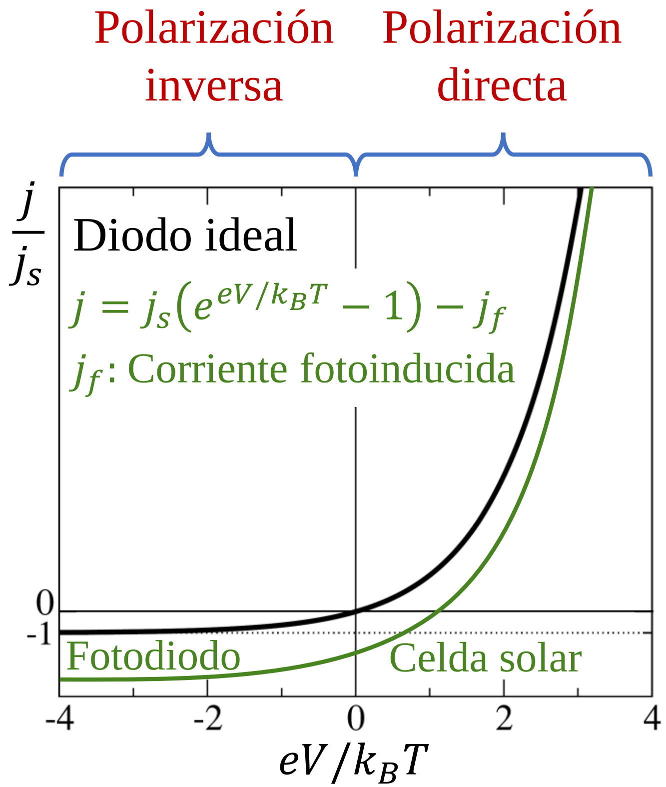
<u>Juntura semiconductora *p-n*</u>: Corriente a través de la juntura

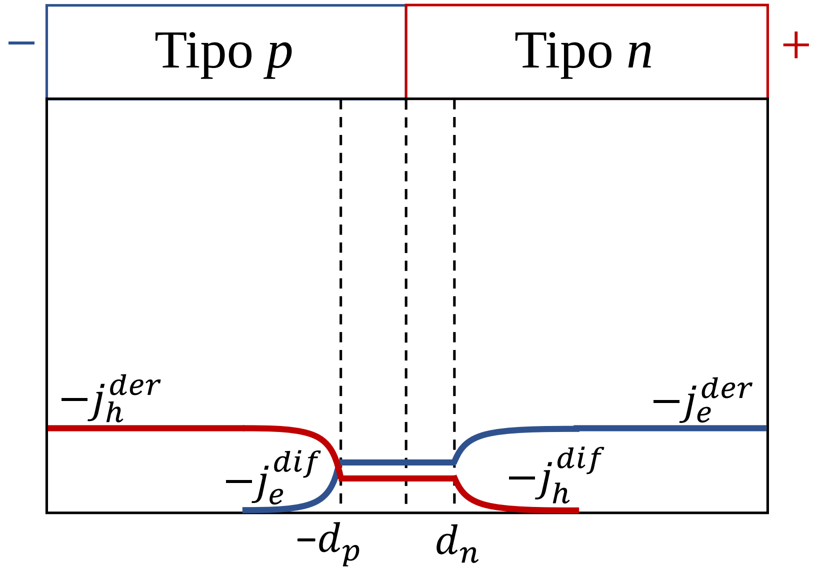


$$j = \underbrace{e n_i^2 \left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d} \right)}_{\text{Corriente de saturación: } j_s \,\propto\, e^{-E_g/k_BT}} \left(e^{\frac{V}{k_BT}} - 1 \right) = e n_i^2 \left(\frac{L_n}{N_a \tau_n} + \frac{L_p}{N_d \tau_p} \right) \left(e^{\frac{V}{k_BT}} - 1 \right)$$

$$L_{n,p} = \sqrt{D_{n,p}\tau_{n,p}}$$

$$= e\left(J_e^{gen} + J_h^{gen} \right) \left(e^{\frac{V}{k_BT}} - 1 \right)$$

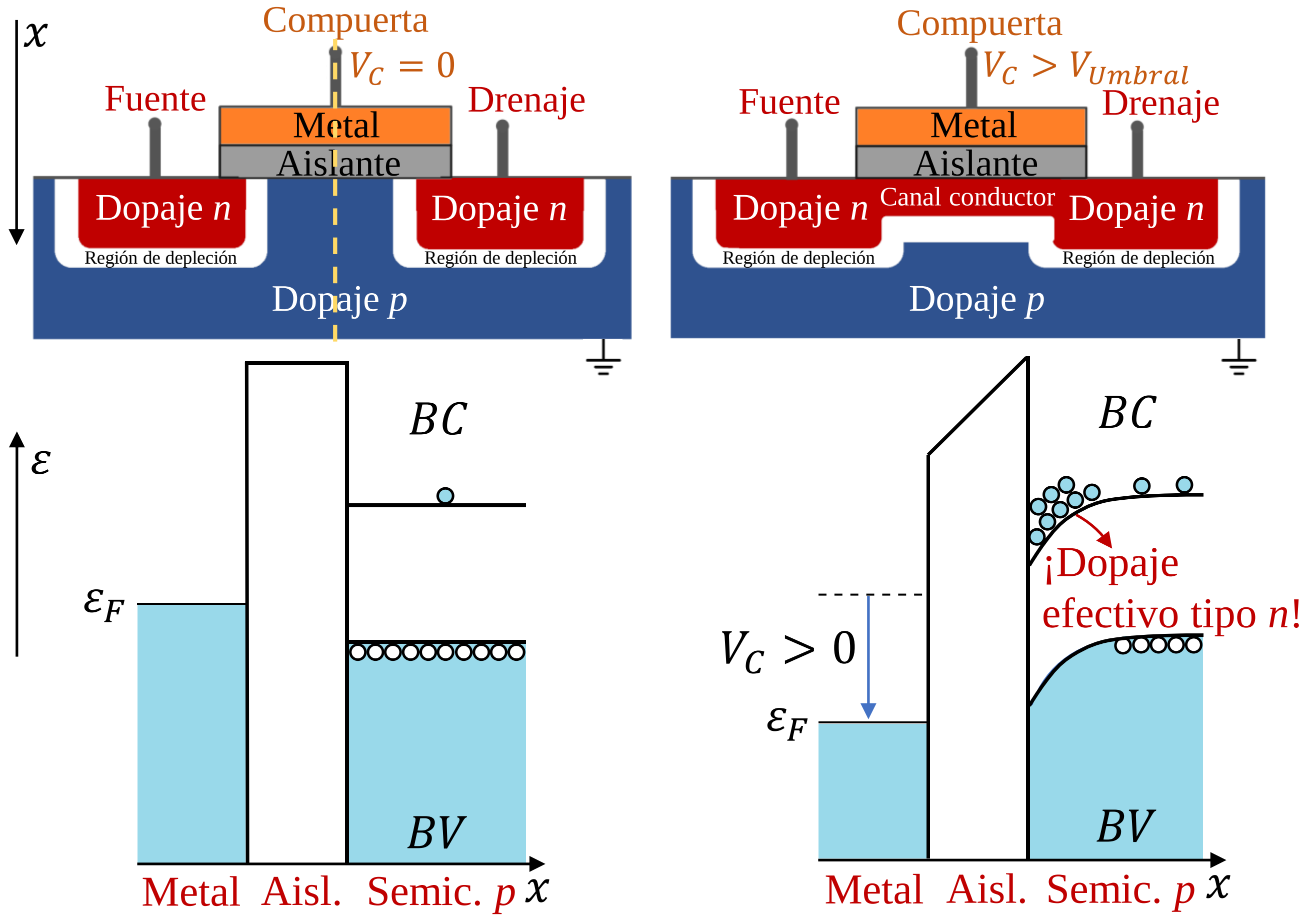




Transistor y pozos cuánticos

Transistor de efecto de campo (MOSFET)

Se utiliza principalmente como interruptor o amplificador.

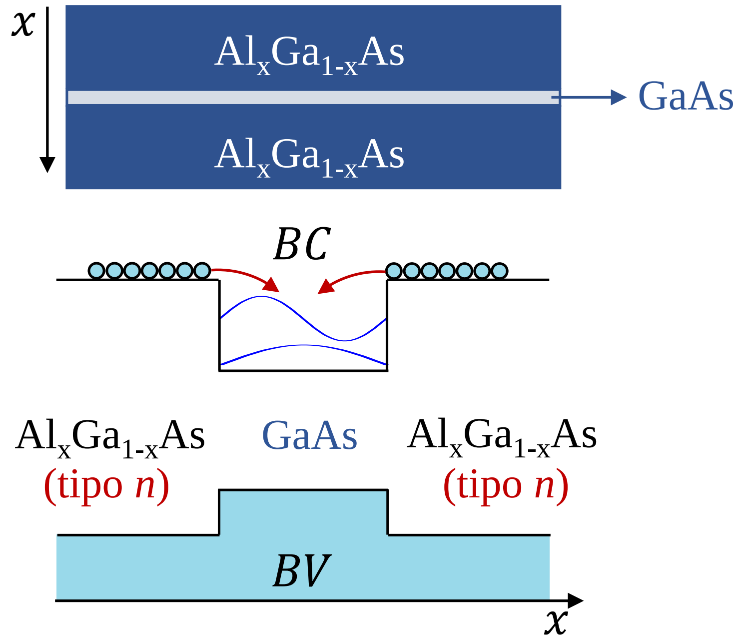


Al aumentar V_C por encima de V_{umbral}, sea crea un canal de conducción, cuyo tamaño (y resistencia) depende de V_C.

Pozos cuánticos

GaAs y AlAs son semiconductores de *gap* directo con $E_g^{GaAs} = 1.4$ eV y $E_g^{AlAs} = 2.7$ eV. Aleaciones $Al_xGa_{1-x}As$ tienen E_g intermedios.

→ Heteroestructuras de $Al_xGa_{1-x}As/GaAs/Al_xGa_{1-x}As$ pueden formar pozos cuánticos.



Utilizando $Al_xGa_{1-x}As$ tipo $n(p)$, $e^-(h^+)$ “caen” al pozo creando un gas bidimensional de $e^-(h^+)$.

Resumen

- Región de depleción en la juntura *p-n*
- Respuesta ante un voltaje aplicado
- Corriente a través de la juntura
- Polarización directa e inversa
- Aplicaciones de junturas *p-n*