

Problema 3

Tenemos 2 grupos, cada uno o.n. $\left. \begin{array}{l} \{ \phi_i^\alpha \}_{i=1}^n, \{ \phi_i^\beta \}_{i=1}^n \\ \langle \phi_i^\alpha | \phi_j^\alpha \rangle = \delta_{ij} \\ \langle \phi_i^\beta | \phi_j^\beta \rangle = \delta_{ij} \end{array} \right\}$

Pero! $\langle \phi_i^\alpha | \phi_j^\beta \rangle = \delta_{ij}$

$$\chi_i^\alpha = \phi_i^\alpha \alpha / \omega$$

Armo $\{ \chi_i^\alpha, \chi_j^\beta \}_{i,j=1}^n \rightarrow \chi_j^\beta = \phi_j^\beta \beta / \omega$

$\nexists \nexists \nexists \{ \chi \}_{i=1}^n$ es o.n. Hay 3 casos para $\langle \chi_a | \chi_b \rangle$

(1) $a, b \in$ conjunto $\{ \phi_i^\alpha \}_{i=1}^n$

(2) $a, b \in$ conjunto $\{ \phi_j^\beta \}_{j=1}^n$

(3) $a, b \in$ cada uno a un grupo \neq .

(1) $\langle \chi_a | \chi_b \rangle = \langle \phi_a^\alpha | \phi_b^\alpha \rangle \langle \alpha | \alpha \rangle = \delta_{ab} \cdot 1 = \delta_{ab}$

(2) $\langle \chi_a | \chi_b \rangle = \langle \phi_a^\beta | \phi_b^\beta \rangle \langle \beta | \beta \rangle = \delta_{ab} \cdot 1 = \delta_{ab}$

(3) $\langle \chi_a | \chi_b \rangle = \langle \phi_a^\alpha | \phi_b^\beta \rangle \underbrace{\langle \alpha | \beta \rangle}_0 = \delta_{ab} \cdot 0 = 0$

$\Rightarrow \forall a, b \quad \langle \chi_a | \chi_b \rangle = \delta_{ab}$

\therefore el grupo $\{ \chi_i \}_{i=1}^n$ es o.n.