

$$[\hat{\Theta}_1, \hat{A}] = 0 \quad \text{Problem 5} \quad \text{remember } |D_S\rangle = \hat{A} |HP\rangle$$

↓

$$\text{key: } [\hat{\Theta}_1, \hat{P}_x] = 0$$

$$\hat{\Theta}_1 \hat{A} |HP\rangle \stackrel{?}{=} \hat{A} \hat{\Theta}_1 |HP\rangle$$

↓

$$= \hat{A} (\sum \epsilon_i) |HP\rangle$$

$$= (\sum \epsilon_i) \hat{A} |HP\rangle \quad \textcircled{I}$$

$$\hat{\Theta}_1 \hat{A} |HP\rangle = \hat{\Theta}_1 |D_S\rangle \quad \text{verlo p/ 2 part.}$$

$$\hat{\Theta}_1 |D_S\rangle = (\hat{\Theta}_1 |1\rangle \otimes |12\rangle \oplus |11\rangle \otimes \hat{\Theta}_1 |2\rangle) \frac{1}{\sqrt{2}} (\phi_2^{(1)} \phi_6^{(2)} - \phi_6^{(1)} \phi_2^{(2)})$$

$$= \frac{1}{\sqrt{2}} (\epsilon_2 \phi_2 \phi_6 - \epsilon_6 \phi_6^{(1)} \phi_2 + \epsilon_6 \phi_2 \phi_6 - \epsilon_2 \phi_6 \phi_2)$$

$$= \frac{1}{\sqrt{2}} [\epsilon_2 (\phi_2 \phi_6 - \phi_6 \phi_2) + \epsilon_6 (\phi_2 \phi_6 - \phi_6 \phi_2)]$$

$$= \frac{1}{\sqrt{2}} (\epsilon_2 + \epsilon_6) (\phi_2 \phi_6 - \phi_6 \phi_2)$$

$$\equiv (\epsilon_2 + \epsilon_6) |D_S\rangle \quad \textcircled{II} \quad (= (\epsilon_2 + \epsilon_6) \hat{A} |HP\rangle)$$

$$\downarrow \textcircled{I} \rightarrow \textcircled{II} \quad \hat{\Theta}_1 \hat{A} |HP\rangle = (\sum \epsilon_i) \hat{A} |HP\rangle = \hat{A} \hat{\Theta}_1 |HP\rangle$$