28|10

$$H = H_0 + H_I$$

$$H_0 = \frac{\hbar\omega_0}{2}\sigma_z \qquad \hbar\omega \updownarrow \quad \begin{matrix} \underline{\quad\quad} \ |e\rangle \\ \\ \underline{\quad\quad} \ |g\rangle \end{matrix}$$

$H_I = -\bar{d}\cdot\bar{E}$ aprox. dipolar $\qquad \bar{E} = E_0\,\check{\varepsilon}\cos(\omega_L t)$

$\bar{d}$ sabemos que es antidiag. $\langle g|\bar{d}|g\rangle = \langle e|\bar{d}|e\rangle = 0$

$$\langle e|\bar{d}|g\rangle = \langle g|\bar{d}|e\rangle \Rightarrow \bar{d} = \langle g|\bar{d}|e\rangle(\sigma^+ + \sigma^-) = \langle g|\bar{d}|e\rangle\,\sigma_x$$

$$H_I = -\bar{d}\cdot\bar{E} = -\langle g|\bar{d}\cdot\check{\varepsilon}|e\rangle E_0 \cos(\omega_L t)(\sigma^+ + \sigma^-)$$

$$\Omega_0 = \frac{\langle g|\bar{d}\cdot\check{\varepsilon}|e\rangle E_0}{\hbar} \Rightarrow H_I = -\hbar\Omega_0\cos(\omega_L t)(\sigma^+ + \sigma^-)$$

$$\Rightarrow \boxed{H = \frac{\hbar\omega_0}{2}\sigma_z - \hbar\Omega_0\cos(\omega_L t)(\sigma^+ + \sigma^-)}$$

Rep. de interacción: busco sols. $|\psi(t)\rangle = c_1(t)e^{-i\omega_1 t}|g\rangle + c_2(t)e^{-i\omega_2 t}|e\rangle$

donde $\begin{cases} \hbar\omega_1 = -\dfrac{\hbar\omega_0}{2} \\ \hbar\omega_2 = \dfrac{\hbar\omega_0}{2} \end{cases}$

S: schrödinger

Más Formal:

$$|\psi^I(t)\rangle = e^{\frac{iH_0 t}{\hbar}} |\psi^S(t)\rangle$$

Ventaja: $i\hbar \frac{\partial}{\partial t} |\psi^I(t)\rangle = H_I^I |\psi^I(t)\rangle$

$$O^I(t) = e^{iH_0 t/\hbar} O^S e^{-i\frac{H_0 t}{\hbar}}$$

Busco ecuaciones para c_1 y c_2 con ec. de Shrödinger:

$$\dot{c}_1 = i\Omega_0 e^{-i\omega_0 t} \cos(\omega_L t) c_2$$

$$\dot{c}_2 = i\Omega_0 e^{i\omega_0 t} \cos(\omega_L t) c_1$$

RWA: $|\omega_0 - \omega_L| \ll |\omega_0 + \omega_L|$ $\quad \boxed{\omega_0 - \omega_L = \delta}$

$$\Rightarrow \boxed{\begin{aligned} \dot{c}_1 &= \frac{i\Omega_0}{2} e^{i\delta t} c_2 \\ \dot{c}_2 &= \frac{i\Omega_0}{2} e^{-i\delta t} c_1 \end{aligned}}$$

Caso $\delta = 0$:

$$\dot{c}_1 = \frac{i\Omega_0}{2} c_2$$

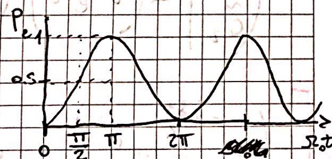
$$\dot{c}_2 = \frac{i\Omega_0}{2} c_1$$

$$\Rightarrow \quad \ddot{c}_1 = -\left(\frac{\Omega_0}{2}\right)^2 c_1 \Rightarrow \boxed{\ddot{c}_1 + \frac{\Omega_0^2}{4} c_1 = 0}$$

•) $c_1(0) = 1$, $c_2(0) = 0$

$$c_1(t) = \cos\left(\frac{\Omega_0 t}{2}\right) \qquad c_2(t) = i\,\mathrm{sen}\left(\frac{\Omega_0 t}{2}\right)$$

$$P_e(t) = |c_2(t)|^2 = \mathrm{sen}^2\left(\frac{\Omega_0 t}{2}\right)$$



↳ **osc. de Rabi**

Caso $\delta \neq 0$: defino nuevas variables:

$$\tilde{c}_1 = c_1 e^{-i\frac{\delta t}{2}} \qquad \tilde{c}_2 = c_2 e^{i\frac{\delta t}{2}}$$

$$\Rightarrow \quad \dot{\tilde{c}}_1 = -i\frac{\delta}{2}\tilde{c}_1 + \frac{i\Omega_0}{2}\tilde{c}_2$$

$$\dot{\tilde{c}}_2 = \frac{i\Omega_0}{2}\tilde{c}_1 + \frac{i\delta}{2}\tilde{c}_2$$

$$\boxed{\frac{d}{dt}\begin{pmatrix}\tilde{c}_1 \\ \tilde{c}_2\end{pmatrix} = \begin{pmatrix}-\delta & \Omega_0 \\ \Omega_0 & \delta\end{pmatrix}\begin{pmatrix}\tilde{c}_1 \\ \tilde{c}_2\end{pmatrix}}$$

Sol. con condiciones iniciales arbitrarias:

$$\tilde{C}_1(t) = \tilde{C}_1(0)\cos($$

$$\tilde{C}_2(t) =$$

$$\begin{pmatrix} \tilde{C}_1 \\ \tilde{C}_2 \end{pmatrix} = \begin{pmatrix} e^{-\frac{i\delta t}{2}} & 0 \\ 0 & e^{\frac{i\delta t}{2}} \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

$$\begin{pmatrix} e^{-\frac{i\delta t}{2}} & 0 \\ 0 & e^{\frac{i\delta t}{2}} \end{pmatrix} = \mathbb{1}\cos\left(\frac{\delta t}{2}\right) + i\,\mathrm{sen}\left(\frac{\delta t}{2}\right)\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \mathbb{1}\cos\left(\frac{\delta t}{2}\right) + i\,\mathrm{Sen}\left(\frac{\delta t}{2}\right)\vec{n}\cdot\vec{\sigma}$$

$$\vec{n} = (0,0,-1) \qquad = e^{-i\left(\frac{\delta t}{2}\right)\vec{n}\cdot\vec{\sigma}} = e^{-i(-\delta t)\frac{\vec{n}\cdot\vec{S}}{\hbar}}$$

$$= D(\vec{n}, \phi = -\delta t)$$

$$e^{i\alpha(\vec{n}\cdot\vec{\sigma})} = \mathbb{1}\cos(\alpha) + i(\vec{n}\cdot\vec{\sigma})\,\mathrm{sen}(\alpha)$$

$$D(\vec{n},\phi) = e^{-i\phi\frac{\vec{n}\cdot\vec{S}}{\hbar}}$$ 2 niveles: $\vec{J} = \vec{S} = \frac{\hbar}{2}\vec{\sigma}$

Los $\tilde{c}_1$ y $\tilde{c}_2$ dan las soluciones en

un marco rotante con el láser!

(1^ero^ pasamos a un marco rotante con ω_0, luego

a uno con δ, eso me da uno que rote

con ω_L)

Caso $\delta = 0$:

Sol general; $\tilde{c}_1(t) = c_1(t) = c_1(0)\cos\left(\frac{\Omega_0 t}{2}\right) + i c_2(0)\,\mathrm{sen}\left(\frac{\Omega_0 t}{2}\right)$

$\tilde{c}_2(t) = c_2(t) =$ $i c_1(0)\,\mathrm{sen}\left(\frac{\Omega_0 t}{2}\right) + c_2(0)\cos\left(\frac{\Omega_0 t}{2}\right)$

$$|g\rangle \xrightarrow{\pi/2} \frac{|g\rangle + i|e\rangle}{\sqrt{2}} \xrightarrow{\pi/2} i|e\rangle$$

$$i|e\rangle \xrightarrow{\pi/2} \frac{-|g\rangle + i|e\rangle}{\sqrt{2}} \xrightarrow{\pi/2} -|g\rangle$$

$$\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = \begin{pmatrix} \cos\frac{\Omega_0 t}{2} & i\,\mathrm{sen}() \\ i\,\mathrm{sen}() & \cos() \end{pmatrix}\begin{pmatrix} c_1(0) \\ c_2(0) \end{pmatrix} = \left[\mathbb{1}\cos\left(\frac{\Omega_0 t}{2}\right) + i\,\mathrm{sen}\left(\frac{\Omega_0 t}{2}\right)\sigma_x\right]\begin{pmatrix} c_1(0) \\ c_2(0) \end{pmatrix}$$

Rotación $D\left[\check{n} = \check{x},\ \varphi = \Omega_0 t\right]$

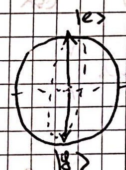
Esfera de Bloch:

Un estado arbitrario $|\psi\rangle$ de un sist. de 2 niveles puede escribirse como:

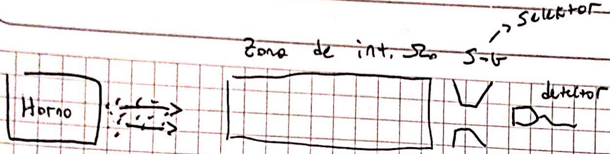
$$\boxed{|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|e\rangle + e^{i\phi}\,\mathrm{sen}\left(\frac{\theta}{2}\right)|g\rangle}$$

$\bar{a} = (\mathrm{sen}\,\theta\cos\phi,\ \mathrm{sen}\,\theta\,\mathrm{sen}\,\phi,\ \cos\theta)$

↳ vector de Bloch



Matriz densidad: $\rho = \frac{1}{2}(\mathbb{1} + \bar{a}\cdot\check{m})$


selector
Zona de int. Ω_0
S-G
Horno
detector