Chapter 13

Particle Physics

Chapter Outline

The goal of particle physics is to understand the tiniest objects of which the universe is made and the forces that govern them.

13.1 LEPTONS AND QUARKS

All matter is composed of leptons, quarks, and elementary particles called bosons which serve as the carriers of the force between particles. The lepton family includes the electron e^- which has an electric charge and interacts with other charged particles by means of the *electromagnetic force*. The electron also interacts by means of a force called the *weak force*, which is considerably weaker than the electromagnetic force. Associated with the electron is an illusive particle called the electron neutrino ν_e, which only interacts by means of the weak force. The other members of the lepton family are the muon μ^- with its neutrino ν_μ, and the tau τ^- with its neutrino ν_τ. The leptons are divided into distinct doublets or *generations* as follows:

$$\begin{bmatrix} \nu_e \\ e^- \end{bmatrix}, \quad \begin{bmatrix} \nu_\mu \\ \mu^- \end{bmatrix}, \quad \begin{bmatrix} \nu_\tau \\ \tau^- \end{bmatrix}. \tag{13.1}$$

Strongly interacting particles are composed of quarks. The particles composed of quarks and interacting by the strong interaction are known collectively as *hadrons* from the Greek word, hadros, meaning robust. Among the hadrons, the proton and neutron are members of a family of particles called *baryons*, which are made up of three quarks. Another family of strongly interacting particles is the *mesons*, which are made up of a quark/antiquark pair.

Quarks come in six types, called *flavors* denoted by *up* (u), *down* (d), *strange* (s), *charmed* (c), *bottom* (b), and *top* (t) quarks. The b and t quarks are also referred to by the more appealing names of *beauty* and *truth*. Like the leptons, the quarks are divided into three generations

$$\begin{bmatrix} u \\ d \end{bmatrix}, \quad \begin{bmatrix} c \\ s \end{bmatrix}, \quad \begin{bmatrix} t \\ b \end{bmatrix}. \tag{13.2}$$

An unusual property of quarks is that they have fractional charges (Q). For each quark doublet, the upper member (u, c, t) has electric charge $Q = +2/3$ times the charge of a proton and the lower member (d, s, b) has charge $Q = -1/3$ times the charge of a proton. The proton is made up of two up-quarks and one down-quark (uud), while the neutron is made up of one up-quark and two down-quarks (udd). Using this information, one can easily confirm that the proton has electric charge equal to one, while the neutron has an electric charge equal to zero.

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The properties of the leptons are summarized in Table 13.1.

The electron (e^-), muon (μ^-), and tau (τ^-) have negative charges. All of the particles shown in Table 13.1 have antiparticles. The antiparticle of the electron is the positron (e^+), and the antiparticles of muon and tau are the positive muon (μ^+) and tau (τ^+). While the electron is stable, the muon decays in 2.197 μs and the tau decays in 290.6 fm. A few of the decay modes of the muon and tau are shown in Table 13.1.

The determination of the mass of neutrinos is currently a subject of active research. Experiments designed to measure the flux of solar neutrinos upon the surface of the Earth have consistently measured a flux below the value predicted by the accepted model of the Sun. This discrepancy between measurements conducted in particle physics and the standard solar model can be understood if some of the electron neutrinos emitted by the Sun are converted into muon and tau neutrinos. Neutrinos are emitted and absorbed in states with a definite flavor but the flavor states are not mass eigenstates. The flavor of a neutrino can thus change as the neutrino travels through free space. However, neutrinos are detected by absorption and emission in states of definite flavor. The situation is analogs to the Stern-Gerlach experiment discussed in Chapter 4. While an electron can be in a state that is a superposition of states having a spin-up and spin-down character, the spin of the electron is always measured to be up or down. In the same way, the mass eigenstates of the neutrino are linear combinations of states with a definite flavor, but the neutrino always interacts as a particle with a definite flavor. The neutrino mass corresponding to the stationary neutrino states are generally denoted by m_1, m_2, and m_3. In Table 13.1, we have given only upper bounds to these three masses. While the current values of the neutrino masses are not very accurate, the experimental data do rule out the possibility that neutrinos have zero mass.

The charge and mass of the three generations of quarks are given in Table 13.2. An illustration of the three generations of leptons and quarks is given in Fig. 13.1.

Convincing evidence for the quark model can be obtained from scattering experiments in which high-energy electrons collide with protons. At energies of a few hundred megaelectron-volt, the proton target behaves as a particle with a

TABLE 13.1 The Family of Particles Called Leptons

Particle	Mc^2	Lifetime	Decay Mode
e^-	0.510998 MeV	$> 4.6 \times 10^{26}$ yr	
μ^-	105.658 MeV	2.197×10^{-6} s	$\mu^- \to e^- + \overline{\nu_e} + \nu_\mu$
τ^-	1776.99 MeV	290.6×10^{-15} s	$\tau^- \to \mu^- + \overline{\nu_\mu} + \nu_\tau$
			$\to e^- + \overline{\nu_e} + \nu_\tau$
ν_1	< 200 meV	Stable	
ν_2	< 200 meV	Stable	
ν_3	< 200 meV	Stable	

Leptons participate in the electromagnetic and weak interaction, but do not participate in the strong interaction. The masses and lifetimes are those given by the Particle Data Group in 2008.

TABLE 13.2 Properties of the Quarks

Flavor	Symbol	Q	Mc^2 (MeV)
Down	d	−1/3	3.5–6.0
Up	u	+2/3	1.5–3.3
Strange	s	−1/3	104
Charmed	c	+2/3	1270
Bottom	b	−1/3	4200
Top	t	+2/3	171,200

The masses of the quarks are those given by the Particle Data Group in 2008.

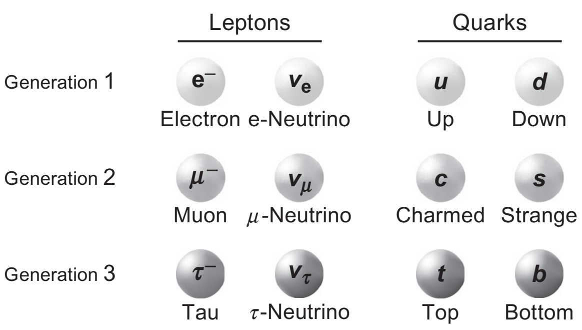

FIGURE 13.1 The three generations of quarks and leptons.

continuous distribution of matter. However, the collisions that occur at much higher energies, for which the electron has more than 20 GeV of energy, can only be explained by supposing that protons are composed of three spin $1/2$ particles. The short wavelength associated with such high-energy electrons probes the finer details of the structure of protons. High-energy scattering experiments showing that the proton has an internal structure are analogs to the scattering experiments of Rutherford, which showed that atoms have nuclei.

The forces between elementary particles are transmitted by means of a class of particles called bosons. The carrier of the electromagnetic force is the *photon*, while the *gluon* is the carrier of the strong force and the *graviton* has been postulated as the carrier of the gravitational force. All of these bosons have zero mass, and the force associated with each of these particles has infinite range. In contrast, the carriers of the weak force are the massive W^+, W^-, and Z bosons. The weak force has a very short range.

The quark compositions and several of the properties of a few of the lightest mesons are given in Table 13.3. In Table 13.3, the name of each particle and the symbol used to denote the particle are given in the first two columns. Columns three and four give the quark composition and rest-mass energy of the particles. All mesons are composed of a quark/antiquark pair. The π^-, for instance, is composed of a d quark and a $\bar{u}$ antiquark, while the π^+ is composed of a u quark and a $\bar{d}$ antiquark. The π^0 and ρ^0 are each linear combinations of $u\bar{u}$ and $d\bar{d}$. From the quark compositions in the table, one can see that the π^+ is the antiparticle of the π^-, the ρ^+ is the antiparticle of the ρ^-, the K^+ is the antiparticle of the K^-, and the $\overline{K}^0$ is the antiparticle of the K^0. All of the particles described in Table 13.3 are unstable. Column five of the table gives one of the possible decay processes of the particle, while column six gives information about the decay time.

TABLE 13.3 Properties of a Few of the Lightest Mesons

Name	Symbol	Composition	Mc^2 (MeV)	Decay Mode	Lifetime/Width
Pion	π^-	$d\bar{u}$	139.6	$\pi^- \to \mu^- + \overline{\nu_\mu}$	2.603×10^{-8} s
	π^+	$u\bar{d}$	139.6	$\pi^+ \to \mu^+ + \nu_\mu$	2.603×10^{-8} s
	π^0	$u\bar{u}, d\bar{d}$	135.0	$\pi^0 \to 2\gamma$	8.4×10^{-17} s
Kaon	K^+	$u\bar{s}$	493.7	$K^+ \to \mu^+ + \nu_\mu$	1.238×10^{-8} s
	K^-	$s\bar{u}$	493.7	$K^- \to \mu^- + \overline{\nu_\mu}$	1.238×10^{-8} s
	K^0	$d\bar{s}$	497.6		
	$\overline{K}^0$	$\bar{d}s$	497.6		
Eta	η	$u\bar{u}, d\bar{d}, s\bar{s}$	547.8	$\eta \to 2\gamma$	1.30 keV
Rho	ρ^+	$u\bar{d}$	775.5	$\rho^+ \to \pi^+ + \pi^0$	149.4 MeV
	ρ^-	$d\bar{u}$	775.5	$\rho^- \to \pi^- + \pi^0$	149.4 MeV
	ρ^0	$u\bar{u}, d\bar{d}$	775.5	$\rho^0 \to 2\pi^0$	149.4 MeV
Omega	ω	$u\bar{u}, d\bar{d}$	782.7	$\omega \to \pi^+ + \pi^-$	8.49 MeV

Data are that given by the Particle Data Group in 2008.

In the Introduction, we said that processes involving the strong force take place within 10^{-22} s, while processes involving the electromagnetic force typically take place in 10^{-14} to 10^{-20} s. Processes due to the weak interaction occur generally within 10^{-8} to 10^{-13} s. Using this information, one can see that the decay of the π^+, π^-, K^+, and K^- occurs by the weak interaction. This possibility is confirmed by the fact that the particles produced by the decay of these particles include the neutrino, which only participates in the weak interaction. Similarly, the decay of π^0, producing two photons, occurs by the electromagnetic interaction.

While the K-mesons are produced by the strong interactions, they decay by means of the weak interaction. One can see that the decay modes and the decay times of K^0 and $\overline{K}^0$ are not given in Table 13.3. These particles are not approximate eigenstates of the weak interaction and do not have decay modes and times associated with them. The neutral kaons, which are approximate eigenstates of the weak interaction, are generally denoted by K_S^0 (for K^0 short) and K_L^0 (for K^0 long). The states of K_S^0 and K_L^0 are linear combinations of the states of K^0 and $\overline{K}^0$. The K_S^0 meson generally decays in 8.958×10^{-11} s into $\pi^0 + \pi^0$ or into $\pi^+ + \pi^-$, while the K_L^0 decays in 5.116×10^{-8} s most often into $\pi^+ + e^- + \overline{\nu}_e$ or $\pi^+ + \mu^- + \overline{\nu}_\mu$. The properties of neutral kaons are similar to the properties of neutrinos, which, as we have seen earlier, are emitted or absorbed in states of definite flavor, which are not eigenstates of the mass.

We saw earlier in connection with the Heisenberg uncertainty principle that states that exist for a long time have well-defined values of the energy, while the energy of shorter-lived resonance states is more poorly defined. The rho meson decays by the strong interaction within 10^{-22} s and thus corresponds to defuse states having a poorly defined value of the energy. The lifetime of particles that decay by the strong interaction is most commonly reported by giving the width of the resonance state. As can be seen in Table 13.3, the width of the rho resonance is 149.4 MeV.

A number of mesons and baryons were discovered using the bubble chamber technology invented by Donald Glaser in 1952 and developed as a scientific instrument by Luis Averez and his coworkers at Berkeley. The bubble chamber used a superheated fluid, which boiled into tiny bubbles of vapor along the tracks of particles. A chamber filled with liquid hydrogen provided a dense concentration of hydrogen nuclei which served as the targets for nuclear scattering events, while a chamber filled with deuterium was used to study collision processes involving neutrons. Figure 13.2 shows a picture of the Big European Bubble Chamber at CERN in Switzerland. When the large piston on the bottom of the chamber was lowered, the liquid within the chamber became super-heated and bubbles would form along the tracks of charged particles. The tracks

FIGURE 13.2 Big European Bubble Chamber used at CERN in Switzerland. (From http://commons.wikimedia.org/wiki.)

of the particles curved due to the presence of a strong magnetic field with the tracks of positively charged particles curved in one way and the tracks of negatively charged particles curved in the other. After being used for many years in high-energy scattering experiments, the bubble chamber has been replaced by quicker electronic counters. The Big European Bubble Chamber now stands on a lawn outside the laboratory in CERN as a scientific exhibit.

Example 13.1

Find the length of the tracks produced in a bubble chamber by a particle traveling with a speed equal to 0.96c that decays by the weak interaction in 10^{-10} s. What would the length of the track be if the particle was to decay by the electromagnetic interaction in 10^{-16} s, or the strong interaction in 10^{-24} s?

Solution

Using Eq. (11.26), the lifetime of the particle in the laboratory frame of reference would be

$$\Delta t = \frac{10^{-10}\,\text{s}}{\sqrt{1-(0.96)^2}} = 3.57 \times 10^{-10}\,\text{s}.$$

Hence, the length of the track in the bubble chamber would be

$$\Delta x = 0.96\,\text{c} \times 3.57 \times 10^{-10}\,\text{s}.$$

Using the value for the velocity of light given in Appendix A, we obtain

$$\Delta x = 0.96 \times 2.998 \times 10^{8}\,\text{m/s} \times 3.57 \times 10^{-10}\,\text{s} = 0.1027\,\text{m}.$$

The length of the track is thus about 10.3 cm long and could be easily observed.

Using the same approach, the length of the track left by a particle that decayed by the electromagnetic interaction would be 0.1027×10^{-6} m and the length of the track of a particle decaying by the strong interaction would be $0.1027 \times 10^{-14}\,\text{m} = 1.027\,\text{fm}$. While it might be possible to observe the track of a rapidly moving particle that decays electromagnetically, the length of the path of a particle that decays by the strong interaction would be about equal to the radius of an atomic nucleus and would not be observable.

An example of a collision between two particles is illustrated in Fig. 13.3(a) and (b). The figures depict a scattering event in which a negative pion collides with a proton producing a neutron and positive and negative pions as follows:

$$\pi^- + p \rightarrow n + \pi^+ + \pi^-.$$

The process can occur as shown in Fig. 13.3(a) with the three particles in the final state being created independently, or it can occur as shown in Fig. 13.3(b) with a neutron and a neutral rho being produced. The neutral rho then decays into two pions. Since the neutron and the rho are neutral particles, they will not leave tracks in a bubble chamber. In order to decide

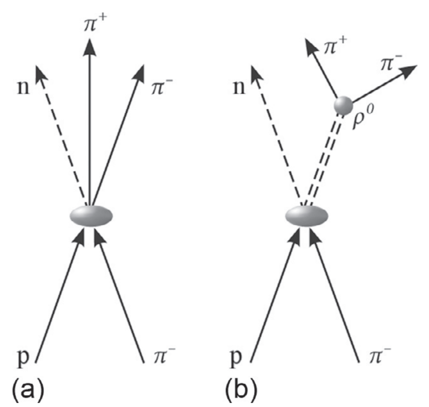

FIGURE 13.3 The reaction $\pi^- + p \rightarrow n + \pi^+ + \pi^-$ can proceed in the following two ways. (a) The three particles in the final state are produced all at once. (b) Two particles, n and ρ^0, are produced, and the ρ^0 subsequently decays into π^- and π^+.

which of these two processes actually occurs, we define the *invariant mass* of the two outgoing pions by the following equation:

$$m_{12} = \frac{1}{c^2}\left[(E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 c^2\right]^{1/2}, \tag{13.3}$$

where E_1 and $\mathbf{p}_1$ being the energy and momentum of the π^+ and E_2 and $\mathbf{p}_2$ being the energy and the momentum of the π^-.

The momenta of the two pions can be determined by measuring the curvature of their tracks in a magnetic field and the energy of the pions can be determined by the amount of ionization they produce. Equation (13.3) can then be used to calculate the invariant mass for every observed pair. If the reaction proceeds as shown in Fig. 13.3(a) with no correlation occurring between the two pions, the pions will share energy and momentum statistically. Plotting the number of events with a particular value of the invariant mass versus the invariant mass will then lead to the distribution having the form shown in Fig. 13.4(a). If, on the other hand, the reaction proceeds as shown in Fig. 13.3(b) with the production of a ρ, energy and momentum conservation lead to the following equations:

$$E_\rho = E_1 + E_2, \quad \mathbf{p}_\rho = \mathbf{p}_1 + \mathbf{p}_2.$$

According to Eq. (13.3), the invariant mass is then given by the equation

$$m_{12} = \frac{1}{c^2}\left[E_\rho^2 - \mathbf{p}_\rho^2 c^2\right]^{1/2}.$$

Using the relation Eq. (12.14), we may then identify the right-hand side of this last equation as being equal to the mass of the ρ. We thus have

$$m_{12} = m_\rho.$$

If the reaction proceeds as shown in Fig. 13.3(b), a plot of the number of events versus the invariant mass will thus lead to the distribution shown in Fig. 13.4(b). The invariant mass of the two pions in this case will be equal to the mass of the decaying particle.

Distribution curves such as those shown in Fig. 13.4(a) and (b) are known as *phase-space spectra*. Since the scattering of a π^- and a proton can occur in either of the two ways described, the distribution curve for an actual experiment will be some combination of these two distribution curves. Figure 13.5 shows the invariant mass spectrum obtained in an early experiment. A broad peak at an invariant mass of 765 MeV is clearly visible. Even though the rho lives for only about 6×10^{-24} s, its existence is well established and its mass has been determined.

Notice that we have given a single mass for the rho even though it has three distinct charge states (ρ^+, ρ^-, and ρ^0). The rho resonance is so broad that it is not possible to resolve experimentally the three charge states. This is consistent with the uncertainty principle. Since the rho decays in such a short time, its energy and, hence, its mass are more poorly determined than they are for particles that decay by the weak and electromagnetic interactions. The quark compositions and several of the properties of a few of the lightest baryons are given in Table 13.4.

The mesons shown in Table 13.3 and the baryons in Table 13.4 occur in groups with the same generic name. According to Table 13.3, the mass of π^- and its antiparticle π^+ is equal to 139.6 MeV, while the mass of π^0 is equal to 135.0 MeV. These particles all have about the same mass and may be regarded as members of a charge multiplet. The eta in Table 13.3 is a charge singlet. The masses of the baryons shown in Table 13.4 also cluster about certain common values. The proton and neutron have very similar masses and together form the first baryon multiplet. Other multiplets correspond to the lambda, sigma, delta, xi, and omega. Notice that the rho mesons given in Table 13.3 have the same quark composition as the pi

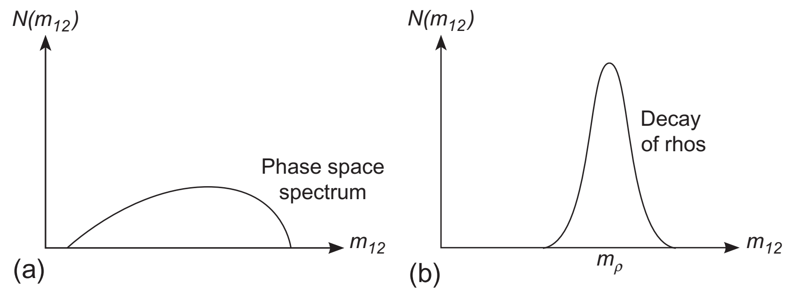

FIGURE 13.4 The number of events with a particular invariant mass versus the invariant mass: (a) when no correlation occurs between the two pions and (b) when a rho is produced which decays into two pions.

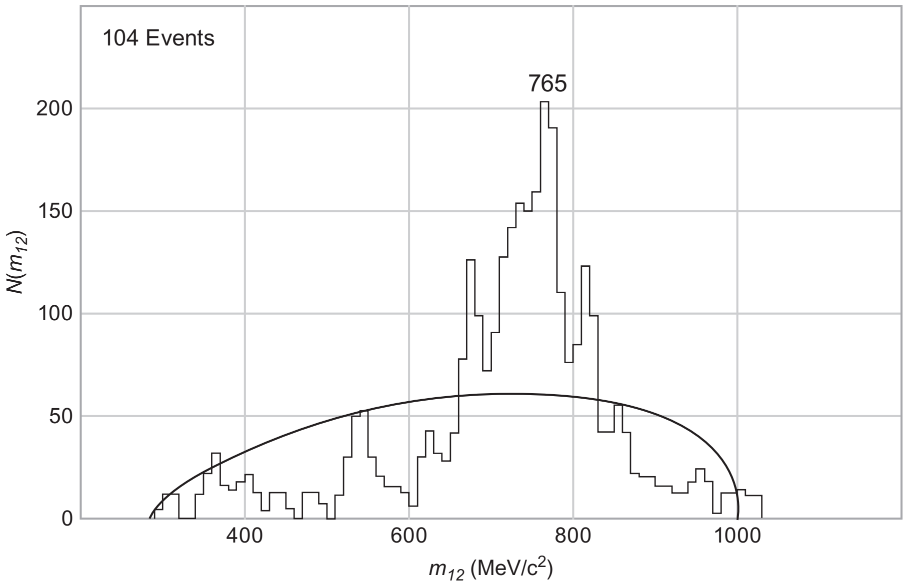

FIGURE 13.5 The invariant mass spectrum for $\pi^- + p \rightarrow n + \pi^+ + \pi^-$ in an early experiment.

TABLE 13.4 Properties of a Few of the Lightest Baryons

Name	Symbol	Composition	Mc^2 (MeV)	Decay Mode	Lifetime/Width
Nucleon	p	uud	938.3		$> 10^{31}$ yr
	n	udd	939.6	$n \rightarrow p + e^- + \overline{\nu_e}$	885.7 s
Lambda	Λ	uds	1115.7	$\Lambda \rightarrow p + \pi^-$	2.63×10^{-10} s
Sigma	Σ^+	uus	1189.4	$\Sigma^+ \rightarrow p + \pi^0$	8.02×10^{-11} s
	Σ^0	uds	1192.6	$\Sigma^0 \rightarrow \Lambda + \gamma$	7.4×10^{-20} s
	Σ^-	dds	1197.5	$\Sigma^- \rightarrow n + \pi^-$	1.48×10^{-10} s
Delta	Δ^{++}	uuu	1232	$\Delta^{++} \rightarrow p + \pi^+$	118 MeV
	Δ^+	uud	1232	$\Delta^+ \rightarrow n + \pi^+$	118 MeV
	Δ^0	udd	1232	$\Delta^0 \rightarrow p + \pi^-$	118 MeV
	Δ^-	ddd	1232	$\Delta^- \rightarrow n + \pi^-$	118 MeV
Xi	Ξ^0	uss	1314.9	$\Xi^0 \rightarrow \Lambda + \pi^0$	2.90×10^{-10} s
	Ξ^-	dss	1321.7	$\Xi^- \rightarrow \Lambda + \pi^-$	1.639×10^{-10} s
Omega	Ω^-	sss	1672.5	$\Omega^- \rightarrow \Lambda + K^-$	8.21×10^{-11} s

Data are that given by the Particle Data Group in 2008.

mesons, and the Δ^+ and Δ^0 given in Table 13.4 have the same quark composition as the proton and neutron. The rho mesons and the delta baryons can be thought of as resonances or excited states of lower lying pi and nucleon states. By comparing the mass of the proton to the masses of the up and down quark given in Table 13.2, one can see that the mass of the proton is very much greater than the combined mass of two up quarks and a down quark. Most of the mass of the proton is due to the energy (mass) of the powerful gluon field holding the quarks together.

Another example of an early bubble-chamber experiment is illustrated in Fig. 13.6(a). Here again a negative pion having a few gigaelectron-volt of energy collides with a proton in a bubble chamber experiment. In Fig. 13.6(a), the negative pion disappears at the point where it hits the proton and farther downstream two V-like events appear. By measuring the energy and momentum of the four particles forming the Vs, one V was shown to consist of a proton and a pion and the other V

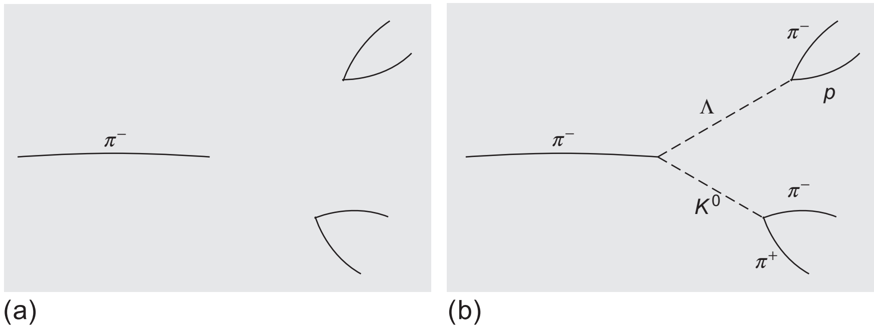

FIGURE 13.6 For an early experiment in which a π^- collides with a proton, (a) an illustration of the bubble chamber tracks and (b) an illustration of the tracks with the various particles identified.

shown to consist of two pions. As indicated in Fig. 13.6(b), the experimental data are consistent with two neutral particles being produced in the collision of the negative pion and the proton, and these neutral particles decaying to produce the two Vs. The identity of the neutral particles can be found by finding the invariant mass associated with the two Vs. Using Eq. (13.3), the invariant mass of the proton-pion pair was found to be 1116 MeV, while the invariant mass of the pion pair was found to be about 500 MeV. Scattering experiments of the kind we have just described enabled physicists to identify the neutral lambda having a mass of 1115.7 MeV and the neutral kaon having a mass of 497.7 MeV. The initial collision of the negative pion and the proton is described by the reaction formula

$$\pi^- + \mathrm{p} \rightarrow \Lambda + K^0, \tag{13.4}$$

while the decays of the particles forming the Vs are described by the formulas

$$\Lambda \rightarrow \mathrm{p} + \pi^- \tag{13.5}$$

and

$$K^0 \rightarrow \pi^+ + \pi^-. \tag{13.6}$$

The momentum and energy of the particles forming the Vs could be determined from their tracks, and the momentum and energy of the neutral particles could be calculated using the conservation of momentum and energy. The lifetime of each neutral particles could then be determined using the distance it traveled in the bubble chamber.

13.2 CONSERVATION LAWS

Particles accelerated to high-energy in modern accelerators collide to produce an astounding variety of new particles. Conservation laws provide a means of characterizing the possible outcomes of scattering events and describing what can and cannot occur.

13.2.1 Energy, Momentum, and Charge

We have already found that energy and momentum are conserved in scattering processes. As an example of the conservation of energy, we consider again the process in which a negative pion collides with a proton to produce a lambda and a neutral kaon. This process is described by Eq. (13.4) and illustrated in Fig. 13.6(b). Using the data given in Tables 13.3 and 13.4, we find that the rest energy of the incoming particles is 1077.9 MeV and the rest energy of the outgoing particles is 1613.3 MeV. The reaction can only occur if the incoming particles have sufficient kinetic energy to make up this mass difference.

The total electric charge of colliding particles is also conserved in collision processes. Since the charge of an assembly of particles is the sum of the charges of the individual particles and is always a multiple of the basic unit e, the charge is referred to as an *additive quantum number*. We shall soon define other quantum numbers of this kind. Additive quantum numbers always have opposite values for the members of a particle-antiparticle pair. To see this, we consider the pair creation process in which an incoming photon produces a particle-antiparticle pair

$$\gamma \rightarrow \mathrm{e}^- + \mathrm{e}^+, \tag{13.7}$$

and the corresponding annihilation process in which an electron and a positron collide to produce a pair of photons

$$e^- + e^+ \rightarrow \gamma + \gamma. \tag{13.8}$$

Additive quantum numbers always have the value zero for photons, which are entirely characterized by their energy and polarization. Since the additive quantum numbers are conserved in pair creation and annihilation processes, the additive quantum numbers of the antiparticle must cancel the quantum numbers of the particle in each case.

13.2.2 Lepton Number

We now turn our attention to scattering processes involving leptons. We begin by considering how neutrinos and antineutrinos appear in reaction formulas. While the π^+ decays into a positron and a neutrino,

$$\pi^+ \rightarrow e^+ + \nu_e, \tag{13.9}$$

the π^- decays into an electron and an antineutrino,

$$\pi^- \rightarrow e^- + \overline{\nu_e}. \tag{13.10}$$

Notice that a positron appears together with a neutrino on the right-hand side of Eq. (13.9), while an electron appears together with an antineutrino on the right-hand side of Eq. (13.10).

Further information about neutrinos can be obtained from neutrino reactions. We consider the following process in which an electron neutrino is captured by a neutron:

$$\nu_e + n \rightarrow e^- + p. \tag{13.11}$$

The corresponding capture process for an antineutrino is

$$\overline{\nu_e} + p \rightarrow e^+ + n. \tag{13.12}$$

A neutrino and an electron appear on opposite sides of Eq. (13.11), while an antineutrino and a positron appear on opposite sides of Eq. (13.12).

In order to explain which processes can and cannot occur, Konopinski and Mahmoud introduced the idea of lepton number L and lepton conservation. They assigned the value $L = 1$ to e^-, μ^-, ν_e, and ν_μ and the value $L = -1$ to the antileptons e^+, μ^+, $\overline{\nu_e}$, and $\overline{\nu_\mu}$. We may readily confirm that the lepton number is conserved in the reactions (13.9) and (13.10) and in the reactions (13.11) and (13.12). Taking the lepton numbers of the π^+ and π^- to be zero, the lepton numbers of the left-hand sides of Eqs. (13.9) and (13.10) are equal to zero. The lepton numbers of the right-hand sides of these equations are also zero since the right-hand side of each equation contains one lepton with $L = 1$ and another lepton with $L = -1$. Each side of Eq. (13.11) has a lepton with $L = 1$, while each side of Eq. (13.12) has a lepton with $L = -1$.

The rule of lepton conservation allows the reactions that we have found to occur and it prohibits some reactions that have been found not to occur. For instance, the reaction (13.12) is consistent with lepton conservation and has been found to occur, while the reaction

$$\overline{\nu_e} + n \rightarrow e^- + p \tag{13.13}$$

is inconsistent with lepton conservation and has been found not to occur. (The lepton number of the left-hand side of this last equation is -1, while the lepton number of the right-hand side is $+1$.) However, some reactions are consistent with all of the conservation laws discussed so far and still do not occur. An example of such a forbidden reaction is

$$\nu_\mu + n \rightarrow e^- + p. \tag{13.14}$$

The collision of a muon neutrino with a neutron leads to a muon and a proton, but not to an electron and a proton. The law of lepton conservation does not distinguish between electrons and muons or their corresponding neutrinos. Another example of the weakness of the lepton conservation law involves the decay of the muon. The μ^- decays according to the following equation:

$$\mu^- \rightarrow e^- + \overline{\nu_e} + \nu_\mu. \tag{13.15}$$

Another possible decay of the muon is

$$\mu^- \rightarrow e^- + \gamma. \tag{13.16}$$

This last reaction, which is allowed by lepton conservation, has been found not to occur. All attempts to find the gamma decay of the muon have been unsuccessful.

The simplest way to explain why the reactions (13.14) and (13.16) do not occur is to assign to the muon and its neutrino a muon lepton number (L_μ). L_μ has the value $+1$ for the μ^- and ν_μ, is equal to -1 for μ^+ and $\overline{\nu_\mu}$, and is zero for all other particles. Similarly, we define an electron lepton number L_e which is equal to $+1$ for the electron and the electron neutrino, -1 for the positron and the electron antineutrino, and is zero for all other particles. One can then see that the processes we have considered, which do occur, conserve both electron and muon lepton numbers, while a number of processes that do not conserve electron and muon numbers do not occur. To see whether Eq. (13.11) conserves electron and muon numbers, we write the equation with the appropriate value of the electron lepton number (L_e) and the muon lepton number (L_μ) under each term

$$\begin{array}{lcccccc} & \nu_e & + & n \rightarrow & e^- & + & p \\ L_e: & 1 & & 0 & 1 & & 0 \\ L_\mu: & 0 & & 0 & 0 & & 0. \end{array} \tag{13.17}$$

The sum of L_e for the left- and right-hand sides of Eq. (13.17) is equal to one, while the corresponding sums for L_μ are equal to zero. One may also see that Eq. (13.14) does not conserve muon and electron lepton numbers. Writing L_e and L_μ under each term of Eq. (13.14) as before, we obtain

$$\begin{array}{lcccccc} & \nu_\mu & + & n \rightarrow & e^- & + & p \\ L_e: & 0 & & 0 & 1 & & 0 \\ L_\mu: & 1 & & 0 & 0 & & 0. \end{array} \tag{13.18}$$

The lepton numbers, L_e and L_μ, are clearly not conserved in this reaction. One may also show that L_e and L_μ are not conserved for the process (13.16).

The discovery of the tau lepton has led to the introduction of yet another quantum number, the tau lepton number (Ł$_\tau$). We assign the value $L_\tau = +1$ for τ^- and ν_τ and the value $L_\tau = -1$ for τ^+ and $\overline{\nu_\tau}$. L_τ is zero for all other particles. The tau decays in a number of different ways including

$$\begin{aligned} \tau^- &\rightarrow e^- + \overline{\nu_e} + \nu_\tau \\ &\rightarrow \mu^- + \overline{\nu_\mu} + \nu_\tau \\ &\rightarrow \pi^- + \nu_\tau. \end{aligned} \tag{13.19}$$

Since L_τ has the value $+1$ for both τ^- and ν_τ and zero for the other particles involved in Eq. (13.19), one can readily see that the tau lepton number is conserved in all of these decay processes.

The values of L_e, L_μ, and L_τ for the leptons are shown in Table 13.5. Since the lepton numbers for an assembly of particles are equal to the algebraic sums of the lepton numbers for the individual particles, the lepton numbers are additive quantum numbers. As for the electric charge, the values of the lepton numbers for antiparticles are the negative of the values for the corresponding particles.

13.2.3 Baryon Number

Another quantity that is conserved in particle reactions is the baryon number, which can be expressed in terms of the number of quarks $N(q)$ and the number of antiquarks $N(\overline{q})$ by the following formula:

$$B = \frac{1}{3}\left[N(q) - N(\overline{q})\right]. \tag{13.20}$$

Baryons are composed of three quarks and thus have baryon number B equal to $+1$, while antibaryons are composed of three antiquarks and have baryon number equal to -1. Mesons, which are made up of a quark/antiquark pair, have baryon number equal to zero.

Baryon number is conserved in strong and electromagnetic interactions because in these interactions quarks and antiquarks are only created or destroyed in particle/antiparticle pairs. Consider, for example, the strong interaction process

$$p + p \rightarrow p + n + \pi^+.$$

The quark description of the particle involved in this interaction is

$$(uud) + (uud) \rightarrow (uud) + (udd) + (u\overline{d}).$$

TABLE 13.5 The Values of the Electron Lepton Number (L_e), Muon Lepton Number (L_μ), and Tau Lepton Number (L_τ) for the Leptons

Particle	L_e	L_μ	L_τ
electron (e^-)	+1	0	0
positron (e^+)	−1	0	0
electron neutrino (ν_e)	+1	0	0
electron antineutrino ($\overline{\nu_e}$)	−1	0	0
negative muon (μ^-)	0	+1	0
positive muon (μ^+)	0	−1	0
muon neutrino (ν_μ)	0	+1	0
muon antineutrino ($\overline{\nu_\mu}$)	0	−1	0
negative tau (τ^-)	0	0	+1
positive tau (τ^+)	0	0	−1
tau neutrino (ν_τ)	0	0	+1
tau antineutrino ($\overline{\nu_\tau}$)	0	0	−1

These quantum numbers are equal to zero for all other particles.

Comparing the quarks of each flavor in the initial and final states, we see that the final state contains the same number of quarks of each flavor as the initial state plus an additional $d\overline{d}$ pair. The baryon number is equal to two for each side of the equation. Similarly, the π^0, which is a linear combination of $u\overline{u}$ and $d\overline{d}$, decays electromagnetically

$$\pi^0 \rightarrow \gamma + \gamma.$$

The baryon number is zero for each side of the equation. For each of these two interaction processes, the number of quarks minus the number of antiquarks remains the same.

A quark/antiquark pair can also be created in a weak interaction; however, a weak interaction may also change the flavor of a quark. Consider the following decay of the neutron:

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \overline{\nu}_\mathrm{e},$$

having the quark description

$$(udd) \rightarrow (uud) + \mathrm{e}^- + \overline{\nu}_\mathrm{e}.$$

A d quark is changed into an u quark in this interaction. While the number of quarks and antiquarks of any flavor is not conserved by the weak interaction, the number of baryons is still conserved since quarks are not changed into antiquarks or vice versa.

For a particle reaction to occur, the sum of the baryon numbers of the incoming particles must be equal to the sum of the baryon numbers of the outgoing particles. Consider, for example, the following scattering process:

$$\begin{array}{rccccccc} & \pi^- & + & \mathrm{p} \rightarrow & K^+ & + & \Sigma^- \\ B: & 0 & & 1 & 0 & & 1. \end{array} \tag{13.21}$$

The proton and the sigma minus have baryon number one, while the pi minus and the K plus are mesons with baryon number zero. The sum of the baryon numbers for the incoming particles and the sum of the baryon numbers for the outgoing are both equal to one. The process thus conserves baryon number and is allowed. As a second example, we consider the decay process

$$\begin{array}{rcccc} & n \rightarrow & \pi^{+} & + & \pi^{-} \\ B: & 1 & 0 & & 0. \end{array} \tag{13.22}$$

The baryon number of the initial state in this reaction is equal to one, while the baryon number of the final state is equal to zero. Hence, the baryon number is not conserved and this process does not occur.

The decay modes of baryons provide many examples of baryon conservation. Consider, for example, the following observed decay modes of Σ^{+}:

$$\begin{aligned} \Sigma^{+} &\rightarrow \mathrm{p} + \pi^{0} \\ &\rightarrow \mathrm{n} + \pi^{+} \\ &\rightarrow \Lambda + \mathrm{e}^{+} + \nu_{\mathrm{e}}. \end{aligned} \tag{13.23}$$

The baryon number is conserved in each of these processes. Since the baryon number is conserved in the decay processes of baryons and since the proton is the only stable baryon, all baryons decay ultimately into a proton.

13.2.4 Strangeness

Soon after the discovery of the pion, other mesons and baryons were observed which were produced in the strong interaction but decayed by the weak interaction. These particles had the unlikely or strange property that they were produced in 10^{-22} s and yet lived long enough to produce considerable tracks in a bubble chamber. Figure 13.7 shows one of the first observed weak decays of a strongly interacting particle, in which the K^{+} meson decays into a μ^{+} and a ν_{μ}. Another example of the weak decay of a strongly interacting particle has been discussed previously in conjunction with Fig. 13.6(a) and (b). The neutral Λ and K^{0} particles depicted in Fig. 13.6(b) are produced by strong interactions but decay over a longer span of time producing characteristic V-like patterns. In 1952, Pais made the first step in explaining this paradox by observing that strange particles are always produced in pairs. The solution to this problem came the following year when Gell-Mann and Nishijima both introduced a new quantum number. Gell-Mann called the quantum number *strangeness*, and this name has been adopted. The strangeness quantum number S is conserved by the strong and electromagnetic interactions, but may be violated by weak interactions.

Using the idea of strangeness, the production of strange particles can be easily explained. One of the two strange particles produced by the strong interaction has a positive value of the strangeness quantum number and the other has a negative value. The total amount of strangeness produced by the strong interaction is equal to zero. Following its production, each strange particle decays by the weak interaction, which may involve a change in the strangeness quantum number. Consider the reaction

$$\pi^{-} + \mathrm{p} \rightarrow \Lambda + K^{0}. \tag{13.24}$$

The strangeness quantum number S is taken to be zero for pions and nucleons. Since strangeness is conserved by the strong interaction, the total strangeness for both sides of Eq. (13.24) must be equal to zero. The strangeness of the K^{0} must therefore

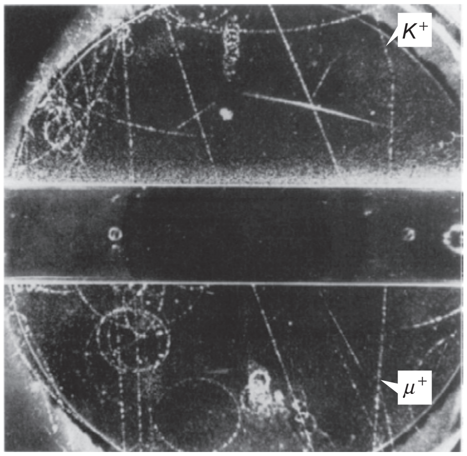

FIGURE 13.7 An early observed weak decays of a strongly interacting particle, in which the K^{+} meson decays into a μ^{+} and a ν_{μ}. (From http://commons.wikimedia.org/wiki.)

be the negative of the strangeness of the Λ. This explains the rule of Pais. If the incoming particles all have $S = 0$, strange particles must occur in conjunction with other particles. Also, the strange particles produced in the reaction can only decay to nonstrange particles by the weak interaction. This explains the rapid creation and slow decay of the Λ and K^0.

The assignment of strangeness to hadrons is based on reactions that are observed to occur by the strong interaction. If the K^+ is assigned a strangeness $S = +1$, the reaction

$$\pi^- + \mathrm{p} \rightarrow \mathrm{n} + K^+ + K^-, \tag{13.25}$$

which is observed to proceed by the strong interaction, may then be used to assign a strangeness $S = -1$ to K^-. We have seen previously that particle/antiparticle pairs have opposite values of additive quantum numbers. Since the positive and negative kaons have opposite strangeness, it is natural to suppose that K^- is the antiparticle of K^+.

The strangeness quantum number associated with the particles involved in a reaction can be understood in terms of the quark model. Consider, for example, the reaction

$$\Lambda \rightarrow \pi^- + \mathrm{p}.$$

The quark description of the particles appearing in this reaction is

$$\begin{array}{rcccc} & (uds) & \rightarrow & (d\overline{u}) + & (uud) \\ S\!: & -1 & & 0 & 0. \end{array}$$

In the reaction, a s quark changes into an u quark and an additional $d/\overline{u}$ pair of quarks is produced. Only the weak interaction can change the flavor of a quark.

A strange particle can either be described as a particle with a non-zero value of the strangeness quantum number S or as a particle which contains one or more strange or antistrange quarks. The strangeness quantum number is related to the number of strange and antistrange quarks by the equation

$$S = -[N(s) - N(\overline{s})], \tag{13.26}$$

where $N(s)$ is the number of strange quarks and $N(\overline{s})$ is the number of antistrange quarks. The lightest strange mesons and baryons have a single strange quark or a single antistrange quark. Since only the weak interaction can change the flavor of a quark, the lightest strange mesons and baryons can only decay by the weak interaction. Of the mesons shown in Table 13.3 and the baryons shown in Table 13.4, the K-mesons and the Lambda- and Σ-baryons all have a single strange quark or a single antistrange quark.

Example 13.2

State whether each of the following processes can occur. If the reaction cannot occur or if it can only occur by the weak interaction, state which conservation law is violated.

$$\begin{array}{lrl} \text{(a)} & \pi^- + \mathrm{p} & \rightarrow \Lambda + \overline{\Sigma}^0 \\ \text{(b)} & \mathrm{p} + \mathrm{p} & \rightarrow \Sigma^+ + \mathrm{n} + K^0 + \pi^+ + \pi^0 \\ \text{(c)} & \mu^- & \rightarrow \mathrm{e}^- + \nu_{\mathrm{e}} + \nu_\mu \\ \text{(d)} & \Lambda & \rightarrow \mathrm{p} + \mathrm{e}^- + \overline{\nu_{\mathrm{e}}} \\ \text{(e)} & \Xi^- & \rightarrow \Sigma^0 + K^- \end{array}$$

Solution

(a) This reaction does not occur because the baryon number is not conserved. The proton (p) and lambda (Λ) are baryons having baryon number equal to $+1$, while the $\overline{\Sigma}^0$ is the antiparticle of a baryon and has baryon number equal to -1. Thus, the total baryon number of the initial state is $+1$, while the total baryon number of the final state is 0.

(b) This process is allowed. Since the baryon number of p, Σ^+, and n are all equal to $+1$, the total baryon number of the initial and final states are equal to $+2$. Since the strangeness quantum number S is equal to -1 for the Σ^+ and $+1$ for K^0, the total strangeness is equal to zero for both the initial and final states. No leptons are involved in this reaction.

(c) This process is not allowed because electron lepton number L_e is not conserved. The total electron lepton number of the initial state is zero; however, the electron lepton number of both e^- and ν_{e} are equal to $+1$, and, hence, the total electron number of the final state is equal to $+2$.

(d) Since Λ has a strangeness quantum number S equal to -1 and all the particles in the final state have $S = 0$, strangeness is not conserved in this reaction, and the reaction can only occur by the weak interaction.

(e) While the lepton numbers, baryon number, and strangeness are all conserved for this process, the process will not occur because energy is not conserved. According to the data given in Tables 13.3 and 13.4, the rest energies of Ξ^-, Σ^0, and K^- are 1321.3, 1192.6, and 493.7 MeV, respectively. Hence, the energy of the initial state is 1321.7 MeV, while the minimum energy of the final state is 1686.3 MeV.

13.2.5 Charm, Beauty, and Truth

All hadrons discovered during the early years of particle physics can be described as bound states of the u, d, and s quarks. Then in 1974, a heavy particle was discovered at the Brookhaven National Laboratory and at the Stanford Linear Accelerator Center. The new particle, which the Brookhaven group named J and the Stanford group named ψ, has come to be known as J/ψ. The properties of this particle show that it is the lightest of a family of particles which are bound states of the charmed quark and anticharmed quark

$$J/\psi(3097) = c\bar{c}.$$

Just as the strangeness quantum number S can be defined in terms of the number of strange quarks and the antiparticle of the strange quark, the charm quantum number can be defined in terms of the number of charmed quarks and the antiparticle of the charmed quark

$$C = N(c) - N(\bar{c}). \tag{13.27}$$

The J/ψ particle is made up of a $c/\bar{c}$ pair of particles. Other particles have since been detected, which have a single c-quark or antiquark. The lightest charmed mesons are the D-mesons with the quark structure

$$\begin{aligned} D^+(1869) &= c\bar{d},\ D^0(1865) = c\bar{u} \quad (C = +1), \\ D^-(1869) &= d\bar{c},\ \overline{D}^0(1865) = u\bar{c} \quad (C = -1), \end{aligned}$$

and D_s mesons with quark structures

$$\begin{aligned} D_s^+(1969) &= c\bar{s} \quad (C = +1, S = +1), \\ D_s^-(1969) &= s\bar{c} \quad (C = -1, S = -1). \end{aligned}$$

The lightest charmed baryon is

$$\Lambda_c^+(2285) = udc \quad (C = +1).$$

The mesons and baryons with a single charmed quark or antiquark all decay in about 10^{-13} s, which is to be expected of particles decaying by means of the weak interaction. Just as for the strange hadrons, charmed hadrons are produced together with other charmed hadrons.

The prevailing theory of elementary particles requires that the number of leptons and quarks be the same, implying that there be six quarks to match the six known leptons. Evidence for the fifth quark—the bottom quark b with the associated quantum number beauty $\tilde{B}$—came with the discovery in 1977 of one of the lightest particles consisting of a $b/\bar{b}$ pair

$$\Upsilon(9460) = b\bar{b} \quad (\tilde{B} = 0).$$

The B-mesons with a single b-quark or antiquark have the quark structure

$$\begin{aligned} B^+(5279) &= u\bar{b},\ D^0(5279) = d\bar{b} \quad (\tilde{B} = +1), \\ B^-(5279) &= b\bar{u},\ \overline{D}^0(5279) = b\bar{d} \quad (\tilde{B} = -1). \end{aligned}$$

The lightest baryon with the b-quark is

$$\Lambda_b^0(5461) = udb \quad (\tilde{B} = -1).$$

The hadrons containing the top quark have a much higher rest mass energy.

Like the strangeness quantum number S, the charm, bottom (beauty), and top (truth) quantum numbers, which we denote by C, $\tilde{B}$, T, are conserved by the electromagnetic and strong interactions but violated by the weak interaction. The additive quantum numbers of all six quarks are summarized in Table 13.6.

TABLE 13.6 Additive Quantum Numbers of the Quarks

Quark	Q	B	S	C	$\tilde{B}$	T
d	$-1/3$	1/3	0	0	0	0
u	2/3	1/3	0	0	0	0
s	$-1/3$	1/3	-1	0	0	0
c	2/3	1/3	0	1	0	0
b	$-1/3$	1/3	0	0	-1	0
t	2/3	1/3	0	0	0	1

13.3 SPATIAL SYMMETRIES

We shall now consider the effect of spatial symmetries upon the states of leptons and upon the states of composite systems made up of leptons and quarks. Issues involving spatial symmetry were discussed before when we considered the hydrogen atom which is made up of a proton and a single electron. Because the electrostatic potential of the proton is spherically symmetric, the wave functions of the electron can have well-defined values of the orbital angular momentum. The importance of angular momentum for many-electron atoms depends upon the fact that to a good approximation atomic electrons move independently of each other in a spherically symmetric field due to the nucleus and other electrons.

13.3.1 Angular Momentum of Composite Systems

For elementary particles with no internal structure and for composite particles made up of quarks, the spin of the particle is defined as the angular momentum of the particle in its own rest frame. The angular momentum of a composite particle can be determined using the rule given in Section 4.3.4 for combining two angular momenta. For given values j_1 and j_2 of the angular momenta of the two parts of a composite system, the quantum number J of the total angular momentum can have the values $J = j_1 + j_2, j_1 + j_2 - 1, \ldots, |j_1 - j_2|$, and for each value of J, the azimuthal quantum number can be $M = -J, -J + 1, \ldots, J$. We used this rule in Chapter 4 to combine the spin and orbital angular momentum of a single electron and in Chapter 5 to combine the spins and orbital angular momenta of electrons to form the total spin S, the total orbital angular momentum L, and the total angular momentum J.

The first example of a composite system we will consider here is positronium, which is a hydrogen-like bound system consisting of an electron and a positron. An important difference between the hydrogen atom and positronium is that the nucleus of hydrogen is the proton, which is very much more massive than the electron, while the positron and electron have the same mass. Both systems rotate about their center of mass. However, while the center of mass of hydrogen is very close to the center of the proton, the center of mass of positronium is at the mid-point of a line from the positron to the electron. One can show that the level spacings for positronium are approximately one half of the level spacings for hydrogen. When a hydrogen atom makes the transition 2p $\rightarrow$ 1s, a photon with energy 10.2 eV is emitted. The corresponding transition for positronium leads to the emission of a 5.1 eV photon.

The states of positronium with the principal quantum numbers $n = 1$ and $n = 2$ are illustrated in Fig. 13.8(a). Each state is denoted using the spectroscopic notation introduced in Chapter 5 with the upper-case letters $S, P, D, \ldots$ standing for $L = 0, 1, 2, \ldots$. The superscript in each case gives the value of $2S + 1$ and the subscript gives the total angular momentum J for the state. The two lowest states have principal quantum number $n = 1$ with the lowest state $^1S_0$ having spin $S = 0$, orbital angular momentum $L = 0$, and total angular momentum $J = 0$, and the second state $^3S_1$ having $S = 1$, $L = 0$, and $J = 1$. A point to notice here is that the electron and the positron are different particles, and so they can be in a triplet S state while two electrons with $n = 1$ cannot. All of the higher-lying states shown in the figure have principal quantum number $n = 2$.

Mesons are bound states of a quark/antiquark pair. In the rest frame of the quark/antiquark system, there is a single orbital angular momentum and two spins. While the orbital angular momentum quantum number L can have several values, we expect the lightest mesons to have orbital angular momentum $L = 0$. As was shown in Example 4.6, the possible values of the total spin of two spin one-half particles are $S = 0$ and $S = 1$. Using the spectrographic notation as before, we thus expect the two least massive mesons to have the spectroscopic designation

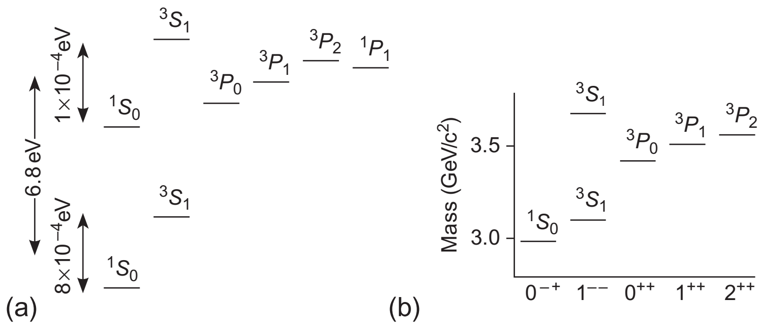

FIGURE 13.8 (a) The lowest states of positronium with principal quantum numbers $n = 1$ and $n = 2$ and (b) the lowest states of charmonium.

$$^1S_0, \quad ^3S_1.$$

The total orbital angular momentum L is equal to zero for both states, while $S = J = 0$ for the first state and $S = J = 1$ for the second state. The lowest observed states of charmonium, which consists of a $c\bar{c}$ pair, is shown in Fig. 13.8(b). The energy level structure of charmonium is very similar to the energy level structure of positronium shown in Fig. 13.8(a).

Baryons are bound states of three quarks. As for mesons, the possible values of the spin quantum number of two quarks are $S = 0$ and $S = 1$. Combining the third quark to these angular momenta gives $S = 1/2$ and $S = 3/2$. Expecting the lowest states again to have $L = 0$, the states of the lightest baryons should be

$$^2S_{1/2}, \quad ^4S_{3/2},$$

where the state with $S = 1/2$ has $2S + 1 = 2$ and the state with $S = 3/2$ has $2S + 1 = 4$. Of the baryons we have considered thus far, the proton, neutron, lambda, and sigma have spin J equal to one half, while the delta baryons have spin J equal to three halves.

As we have said, the *spin* of a composite particle made up of quarks is the total angular momentum J in the rest frame of the particle.

13.3.2 Parity

We now consider the parity transformation in which the coordinates of particles are inverted through the origin

$$\mathbf{x} \to -\mathbf{x}. \tag{13.28}$$

Under a parity transformation, the velocity $\mathbf{v}$ and the momentum $\mathbf{p}$ change sign. The orbital angular momentum $\mathbf{l} = \mathbf{r} \times \mathbf{p}$ and the spin of a particle are unaffected by a parity transformation.

A parity transformation can be achieved by a mirror reflection followed by a rotation of 180° about an axis perpendicular to the mirror. Since the laws of nature are invariant under rotations, the question of whether parity is conserved depends upon whether an event and its mirror image occur with the same probability. While the strong and electromagnetic interactions are invariant with respect to parity transformations, we shall find that the weak interactions do not always have this property.

To study the effect of spatial inversions upon single-particle states, we introduce a parity operator $\hat{P}$ that acts on the wave function describing a single particle

$$\hat{P}\psi(\mathbf{x}, t) = P_a\psi(-\mathbf{x}, t), \tag{13.29}$$

where a identifies a particular type of particle such as an electron e$^-$ or a quark. We thus suppose that each particle has an intrinsic parity in addition to the parity of its wave function. Since two successive parity operations leave the system unchanged, we require that

$$\hat{P}^2\psi(\mathbf{x}, t) = \psi(\mathbf{x}, t),$$

implying that the intrinsic parityP_a be equal to $+1$ or -1.

In addition to its intrinsic parity, the wave function of a particle has a parity associated with its orbital angular momentum. We found while studying selection rules for atomic transitions in Chapter 4 that a wave function is even or odd with respect to spatial inversion depending upon whether the orbital angular momentum quantum number is an even or an odd number. The parity associated with orbital angular momentum of a state is thus $(-1)^L$.

The free-particle states of electrons and positrons are represented by four-component wave functions in the relativistic Dirac theory. A careful study of the electron and positron shows that these particles have opposite parity. This is shown, for instance, in the book by Perkins which is cited at the end of this chapter. The parity of electrons and positrons cannot be determined in an absolute sense because they are always created or destroyed in pairs. We shall follow the ordinary conventions and assign to the electron, muon, and tau a positive parity

$$P_{e^-} = P_{\mu^-} = P_{\tau^-} = 1$$

and assign to the corresponding antiparticles a negative parity

$$P_{e^+} = P_{\mu^+} = P_{\tau^+} = -1.$$

Quarks like electrons are only created and destroyed in pairs. The usual convention for quarks is

$$P_u = P_d = P_s = P_c = P_b = P_t = 1,$$

and for antiquarks

$$P_{\overline{u}} = P_{\overline{d}} = P_{\overline{s}} = P_{\overline{c}} = P_{\overline{b}} = P_{\overline{t}} = -1.$$

The (intrinsic) parity of mesons and baryons can be predicted from the parity of the quarks. We recall that the rest frame of a meson corresponds to the center-of mass frame of a quark/antiquark pair. The spin states of the quark and antiquark are unaffected by an inversion of the spatial coordinates. Denoting the quark by a and the antiquark by $\overline{b}$ and denoting the orbital angular momentum of the quark/antiquark pair by L, the parity of a meson M is

$$P_M = P_a P_{\overline{b}} (-1)^L = (-1)^{L+1}. \tag{13.30}$$

The quark labels a and b can each be u, d, s, c, b, or t. The least massive mesons with $L = 0$ are expected to have negative parity. This is consistent with the observed parities of pi- and K-mesons which have been found to have spin zero and negative parity.

Baryons are composed of three quarks. The orbital angular momentum of two quarks can be combined to form total orbital angular momentum L_{12} and this angular momentum can then be combined with the orbital angular momentum of the third quark, which we denote by L_3. Denoting the quarks by a, b, and c, the parity of a baryon is

$$P_B = P_a P_b P_c (-1)^{L_{12}} (-1)^{L_3} = (-1)^{L_{12}+L_3}, \tag{13.31}$$

and the corresponding antibaryon has parity

$$P_{\overline{B}} = P_{\overline{a}} P_{\overline{b}} P_{\overline{c}} (-1)^{L_{12}} (-1)^{L_3} = -(-1)^{L_{12}+L_3}. \tag{13.32}$$

Low-lying baryons with $L_{12} = L_3 = 0$ are predicted to have positive parity, while the low-lying antibaryons are predicted to have negative parity. These predictions are consistent with the observed parities of the p, n, and Λ.

A thorough study of the experimental evidence for parity conservation was conducted by Yang and Lee in 1956. They showed that while there was strong evidence for parity conservation in electromagnetic and strong interactions, there was no evidence for parity conservation in the weak interactions. In 1957, following suggestions by Yang and Lee, Wu and her coworkers at Columbia University placed a sample of cobalt-60 inside a solenoid and cooled it to a temperature of 0.01 K. At such low temperatures, the cobalt nuclei align parallel to the direction of the magnetic field. Polarized cobalt-60 nuclei decay to an excited state of nickel-60 by the process

$$^{60}\mathrm{Co} \rightarrow {}^{60}\,\mathrm{Ni}^* + \mathrm{e}^- + \overline{\nu_e}.$$

Parity violation was established by the observation that more electrons were emitted in the direction of the nuclear spins than the backward directions.

An illustration of an electron being emitted from a ^{60}Co nucleus is shown in Fig. 13.9(a). The spin of the cobalt nucleus is illustrated by an arrow indicating the rotational motion of the nucleus, and by an arrow beside the nucleus pointing upward because a right-hand screw would move up if it were to rotate in the way the cobalt nucleus is spinning. As shown in Fig. 13.9(b), a parity transformation reverses the direction of the emitted electron but leaves the direction of the nuclear

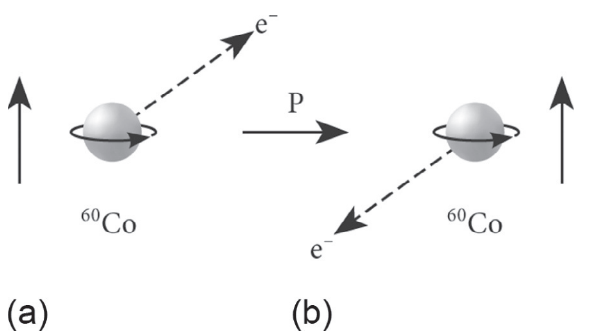

FIGURE 13.9 (a) An electron being emitted from a ^{60}Co nucleus and (b) the result of a parity transformation upon the ^{60}Co nucleus.

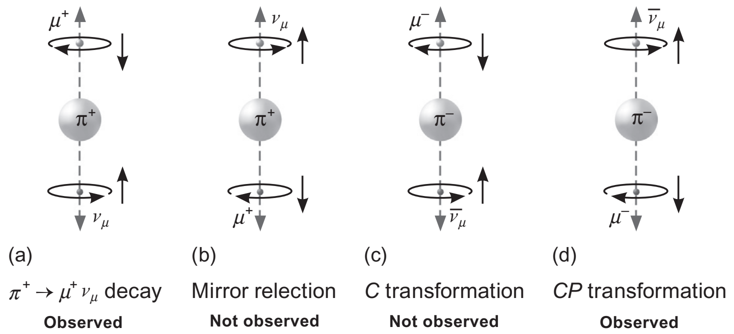

FIGURE 13.10 (a) A μ^+ emitted from π^+ in the decay process (13.33), (b) the result of a parity transformation upon the π^+ decay, (c) the result of a charge conjugation transformation upon the π^+ decay, and (d) the result of both a parity and charge conjugation transformation upon the π^+ decay.

spin unchanged. Parity is violated since a beta decay in the forward direction of the spin of the cobalt nucleus such as that shown in Fig. 13.9(a) occurs more often than a beta decay in the backward direction as shown in Fig. 13.9(b).

Another example of parity violation is provided by the dominant decay mode of π^+ described by the formula,

$$\pi^+ \to \mu^+ + \nu_\mu. \tag{13.33}$$

In this decay process illustrated in Fig. 13.10(a), the spin of the μ^+ is indicated by the downward arrow next to the particle. The emitted muon has negative *helicity* which means that its spin points in the direction opposite to its motion. In the parity transformed process shown in Fig. 13.10(b), the μ^+ is rotating as before and the spin still points down. The process shown in Fig. 13.10(b), for which the muon has positive helicity, does not occur.

A number of interesting examples of parity violation are provided by weak decays of mesons.

13.3.3 Charge Conjugation

The transformation of charge conjugation replaces all particles by their antiparticles without changing the position or the variables that describe the motion of the particle. In studying the effects of charge conjugation, we must distinguish between particles that have antiparticles and those that do not. While all charged particles have antiparticles with opposite charge, some neutral particles can be considered to be their own antiparticle. The photon and the π^0, for instance, do not have antiparticles. As before, we shall denote a particle by the letter a and the corresponding antiparticle by $\overline{a}$. The states of a and $\overline{a}$ will be denoted by $\psi(a)$ and $\psi(\overline{a})$. The effect of the charge conjugation operator $\hat{C}$ upon the state $\psi(a)$ is

$$\hat{C}\psi(a) = C_a \psi(\overline{a}).$$

For a particle that is its own antiparticle, this equation is

$$\hat{C}\psi(a) = C_a \psi(a).$$

Since a second transformation turns antiparticles back into particles, $(C_a)^2$ must be equal to one, and hence

$$C_a = \pm 1.$$

We thus see that the particles without distinct antiparticles are eigenstates of the charge conjugation operator $\hat{C}$ with eigenvalues $C_a = \pm 1$. The eigenvalue of $\hat{C}$ for a particular particle is called its C-parity. As we shall soon find, the C-parity of the π^0 is $+1$. The C-parity of the photon is -1.

We consider now a particle/antiparticle pair for which the particle and the antiparticle are both fermions with spin equal to one half. An example of such a pair is positronium, which consists of an electron and a positron, and mesons, which are made up of a quark and an antiquark. A state of the pair is described by the product function $\psi(a)\psi(\overline{a})$. The effect of charge conjugation upon such a product state will be to exchange the particle and antiparticle. If the particle/antiparticle pair have total angular momentum L with respect to the center of mass of the pair, interchanging the particle and antiparticle will have the effect of reversing the relative position vector in the spatial wave function and give rise to a phase factor $(-1)^L$.

We now consider the possible spin states of the fermion/antifermion pair. Two spin one-half particles can have a total spin angular momentum S equal to one or zero. Denoting the spin-up state of a single particle by α and the spin-down state by β, the states with $S = 1$ are

$$\begin{array}{ll} \alpha_1\alpha_2, & S=1, M_S=1 \\ \frac{1}{\sqrt{2}}(\alpha_1\beta_2+\beta_1\alpha_2), & S=1, M_S=0 \\ \beta_1\beta_2, & S=1, M_S=1, \end{array}$$

and the state with $S = 0$ is

$$\frac{1}{\sqrt{2}}(\alpha_1\beta_2-\beta_1\alpha_2),\;\; S=0, M_S=0.$$

A discussion of the states obtained when one combines two angular momenta can be found, for instance, in the book by Griffiths cited at the end of this chapter. The point we want to note here is that the $S = 1$ states are symmetric with respect to the exchange of the two particles while the state with $S = 0$ is antisymmetric. The symmetry of the states of total spin is thus equal to $(-1)^{S+1}$.

Drawing our description of the spatial and spin wave functions together, suppose that a state with definite value of the total orbital and spin angular momentum is formed from the product of fermion and antifermion states. We will denote the state

$$\Psi(f\overline{f}, SM_SLM_L).$$

The effect of the charge conjugation transformation will be to introduce a phase factor of $(-1)^L$ due to the orbital angular momentum and a factor of $(-1)^{S+1}$ due to the spin angular momentum. As shown in the book by Gottfried and Weisskopf cited at the end of this chapter, an additional factor (-1) must be added due to interchanging a fermion and antifermion. We thus have

$$\hat{C}\Psi(f\overline{f}, SM_SLM_L) = (-1)^{L+S}\Psi(f\overline{f}, SM_SLM_L). \tag{13.34}$$

For example, the π^0 with both S and L equal to zero must have C-parity $C_{\pi^0} = 1$.

The states of positronium provide good examples of both parity and charge conjugation. Like the mesons, which are composed of quark/antiquark pairs, positronium is made up of an electron/positron pair. Electrons and quarks are both fermions described by states that are antisymmetric with respect to an interchange of particles. The parity of the states of positronium may be obtained using Eq. (13.30) and is given by the formula

$$P = P_{e^-}P_{e^+}(-1)^L = (-1)^{L+1}. \tag{13.35}$$

The C-parity of positronium states is given by Eq. (13.34). We thus have

$$C = (-1)^{S+L}. \tag{13.36}$$

The parity and C-parity of the lowest states of positronium are given in the last two columns of Table 13.7.

Positronium, which is a fundamental system made up of a fermion/antifermion pair, serves as a useful model for understanding the properties of mesons.

We conclude this section by considering again the decay of π^+ discussed earlier. The decay process is described by Eq. (13.33) and illustrated in Fig. 13.10(a). A C transformation of the particles converts the π^+ to π^-, the μ^+ to μ^-, and the ν_μ to $\overline{\nu}_\mu$ to give the process

$$\pi^- \rightarrow \mu^- + \overline{\nu}_\mu, \tag{13.37}$$

TABLE 13.7 The Lowest States of Positronium

	State	J	P	$\hat{C}$
$n=1$	$^1S_0$	0	−1	1
	$^3S_1$	1	−1	−1
$n=2$	$^1S_0$	0	−1	1
	$^3S_1$	1	−1	−1
	$^1P_1$	1	1	−1
	$^3P_2$	2	1	1
	$^3P_1$	1	1	1
	$^3P_0$	0	1	1

which is illustrated in Fig. 13.10(c). The μ^- produced by a C-parity transformation of the μ^+ has negative helicity as does the μ^+ produced by the decay of π^+; however, experiment shows the μ^- emitted in π^- decay actually has positive helicity. So, C is violated. The result of both P and C transformations upon the decay process illustrated in Fig. 13.10(a) yields the decay of the negative pion shown in Fig. 13.10(d) with correct helicities. Thus, although both P and C are violated in pion decay, CP, which is the simultaneous transformation of both P and C, is conserved.

Although CP is conserved to a very good approximation in most circumstances, a few examples can be found of CP violation. The example, which is most widely known, concerns the decay of the neutral meson K_L^0 discussed following Table 13.3. As we said, the K_L^0 usually decays into three particles; however, in 1964 Christenson, Cronin, Fitch, and Turlay discovered that in one decay for every thousand K_L^0 decays into two pions

$$K_L^0 \to \pi^+ + \pi^-.$$

This result is clear evidence of CP violation since the two pion state transforms differently under CP than the three particle decay modes.

13.4 ISOSPIN AND COLOR

The purpose of this section is to describe two fundamental properties of strongly interacting particles, *isospin* and *color*. We shall find that isospin is an approximate symmetry that provides a framework for understanding a wealth of experimental data, while color is related to an exact symmetry of the strong interactions.

13.4.1 Isospin

One of the most striking properties of hadrons is that they occur in families of particles with approximately equal masses. Within a given family, all particles have the same spin, parity, baryon number, strangeness, charm, beauty, and truth but differ in their electric charge. Of the mesons shown in Table 13.3, the π^- with quark composition $d\bar{u}$, the π^+ with quark composition $u\bar{d}$, and the π^0, which is a combination of $u\bar{u}$ and $d\bar{d}$, have approximately the same mass. This is also true of the K^+, K^-, K^0, and $\overline{K}^0$, with quark composition $u\bar{s}$, $s\bar{u}$, $d\bar{s}$, and $s\bar{d}$, respectively. Of the baryons shown in Table 13.4, the proton with quark composition uud and the neutron with composition udd have about the same mass. For all of these examples, the members of a charge multiplet can be distinguished by the varying number of u and d quarks. The similarity of the masses within a charge multiplet is due to the fact that the u and d quarks have approximately the same mass and interact by means of the same strong interaction.

To describe the isospin symmetry, we introduce three quantum numbers that are conserved by the strong interactions. Two of these quantum numbers are combinations of quantum numbers introduced previously. The first new quantum number is the *hypercharge* defined by the following equation:

$$Y = B + S + C + \tilde{B} + T, \tag{13.38}$$

where B, S, C, $\tilde{B}$, and T are the baryon number, strangeness, charm, beauty, and truth, respectively. Since the quantum numbers appearing on the right-hand side of this last equation have the same values for all members of an isospin multiplet, so does the hypercharge. The second combination of quantum numbers is the azimuthal isospin quantum number I_3 defined by the equation

$$I_3 = Q - Y/2, \tag{13.39}$$

where Q is the charge. The different members of an isospin multiplet have different charges and hence different values of I_3. We define the isospin I to be the maximum value of I_3 within a multiplet.

Quarks

The hypercharge quantum number Y can be assigned to the quarks using Eq. (13.38), and the isospin quantum number I_3 can be assigned using (13.39). These quantum numbers together with the baryon quantum number B, the charge Q, and the isospin I are given in Table 13.8.

Only the u and d quarks have isospin quantum numbers, I_3 and I, different from zero. The isospin quantum number I is equal to $1/2$ for both the up and down quarks, while I_3 is equal to $+1/2$ and $-1/2$, for u and d quarks, respectively. The hypercharge and isospin quantum numbers for the u, d, and s quarks and for $\overline{u}$, $\overline{d}$, and $\overline{s}$ antiquarks are shown in Fig. 13.11.

TABLE 13.8 Values of the Baryon Number B, Hypercharge Y, Charge Q, and Isospin Quantum Numbers I_3 and I for quarks

Quark	B	Y	Q	I_3	I
d	1/3	1/3	$-1/3$	$-1/2$	1/2
u	1/3	1/3	2/3	1/2	1/2
s	1/3	$-2/3$	$-1/3$	0	0
c	1/3	4/3	2/3	0	0
b	1/3	$-2/3$	$-1/3$	0	0
t	1/3	4/3	2/3	0	0

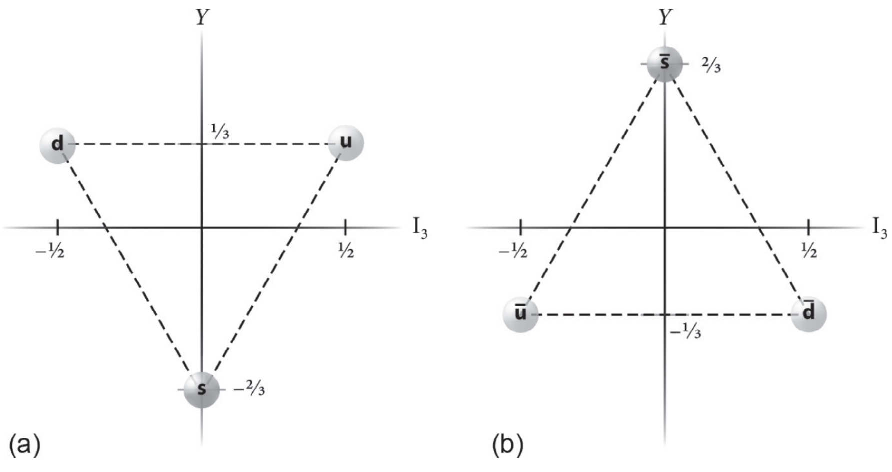

FIGURE 13.11 The hypercharge and isospin quantum numbers for (a) u, d, and s quarks and (b) $\overline{u}$, $\overline{d}$, and $\overline{s}$ antiquarks.

The Light Mesons

The states of the lightest mesons have $L = 0$ and according to Eq. (13.30) have negative parity. The quark and antiquark, of which each meson is composed, have an intrinsic spin equal to one half. Hence, mesons must have total spin S equal to 0 or 1. Since the total orbital angular momentum L is equal to zero, the total angular momentum J must like the spin S be equal to 0 or 1. The lightest mesons are observed experimentally to consist of a family of nine mesons with spin-parity $J = 0^-$ and a family of nine mesons with spin-parity $J = 1^-$. Using the Greek word *nonet* to describe nine objects, the states are said to belong to scalar and vector nonets. These two families of mesons with their quark assignments are given in Table 13.9. While all the charged particles shown in Table 13.9 correspond to a specific quark/antiquark pair, the neutral particles correspond to a linear combination of quark states. The π^0 and ρ^0 are a linear combination of $u\bar{u}$ and $d\bar{d}$, while the η, η', ω, and ϕ correspond to linear combinations of $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$. The scalar and vector meson nonets are illustrated in Fig. 13.12.

The Light Baryons

Like the mesons, baryons occur in families of particles with the same baryon number, spin, and parity. These families are called *supermultiplets*. While the supermultiplets of mesons have nine members and are called nonets, the supermultiplets of baryons can have 1, 8, or 10 members and are called *singlets*, *octets*, and *decuplets*. The lightest baryons observed experimentally are the octet with $J^P = \frac{1}{2}^+$ shown in Table 13.10 and the decuplet with $J^P = \frac{3}{2}^+$ shown in Table 13.11

TABLE 13.9 Light Mesons with Spin Equal to Zero and One

Quarks	0^- meson	1^- meson	I_3	I	Y
$u\bar{s}$	$K^+(494)$	$K^{*+}(892)$	1/2	1/2	1
$d\bar{s}$	$K^0(498)$	$K^{*0}(896)$	−1/2	1/2	1
$u\bar{d}$	$\pi^+(140)$	$\rho^+(776)$	1	1	0
$u\bar{u}, d\bar{d}$	$\pi^0(135)$	$\rho^0(776)$	0	1	0
$d\bar{u}$	$\pi^-(140)$	$\rho^-(776)$	−1	1	0
$s\bar{d}$	$\bar{K}^0(498)$	$\bar{K}^{*0}(896)$	1/2	1/2	−1
$s\bar{u}$	$K^-(494)$	$K^{*-}(892)$	−1/2	1/2	−1
$u\bar{u}, d\bar{d}, s\bar{s}$	$\eta(548)$	$\omega(783)$	0	0	0
$u\bar{u}, d\bar{d}, s\bar{s}$	$\eta'(958)$	$\phi(1019)$	0	0	0

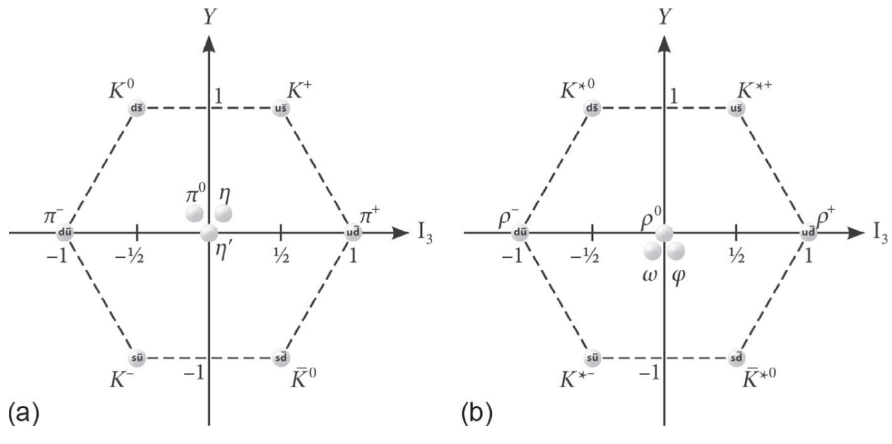

FIGURE 13.12 (a) The scalar meson nonet and (b) the vector meson nonet.

TABLE 13.10 States of the $\frac{1}{2}^+$ Octet of Light Baryons

Quarks	Baryon	I_3	I	Y
uud	$p(938)$	1/2	1/2	1
udd	$n(940)$	$-1/2$	1/2	1
uds	$\Lambda(1116)$	0	0	0
uus	$\Sigma^+(1189)$	1	1	0
uds	$\Sigma^0(1193)$	0	1	0
dds	$\Sigma^-(1197)$	-1	1	0
uss	$\Xi^0(1315)$	1/2	1/2	-1
dss	$\Xi^-(1322)$	$-1/2$	1/2	-1

TABLE 13.11 States of the $\frac{3}{2}^+$ Decuplet of Light Baryons

Quarks	Baryon	I_3	I	Y
uuu	$\Delta^{++}(1232)$	3/2	3/2	1
uud	$\Delta^+(1232)$	1/2	3/2	1
udd	$\Delta^0(1232)$	$-1/2$	3/2	1
ddd	$\Delta^-(1232)$	$-3/2$	3/2	1
uus	$\Sigma^+(1383)$	1	1	0
uds	$\Sigma^0(1384)$	0	1	0
dds	$\Sigma^-(1387)$	-1	1	0
uss	$\Xi^0(1532)$	1/2	1/2	-1
dss	$\Xi^-(1535)$	$-1/2$	1/2	-1
sss	$\Omega^-(1672)$	0	0	-2

One can see that Δ^+ and Δ^0 shown in Table 13.11 have the same quark composition as the proton and neutron shown in Table 13.10. The Δ's in Table 13.11 may be thought of as excited or resonance states of nucleons. The baryon octet and decuplet are illustrated in Fig. 13.13.

Pion-Nucleon Scattering

We now consider the scattering processes that occur when a beam of pions is incident upon a proton or neutron. The isospin symmetry, which we have just discussed, provides information about which scattering processes can occur and about the relative amplitude of different scattering processes.

The isospin quantum numbers of pions are given in Table 13.9, and the isospin quantum numbers of protons and neutrons are given in Table 13.10. The quantum number I is equal to one for the pions with $I_3 = 1, 0, -1$ for π^+, π^0, and π^-, respectively. For the proton, the isospin quantum numbers are $I = 1/2$ and $I_3 = +1/2$, while the isospin quantum numbers are $I = 1/2$ and $I_3 = -1/2$ for the neutron. Using the rule for addition of angular momentum, we immediately see that a pion-nucleon system can have a total isospin $I = 3/2$ or $I = 1/2$. The different possible combinations of pions and nucleons that can occur in a pion-nucleon scattering experiment and the isospin quantum number I_3, to which they correspond, are given in Table 13.12.

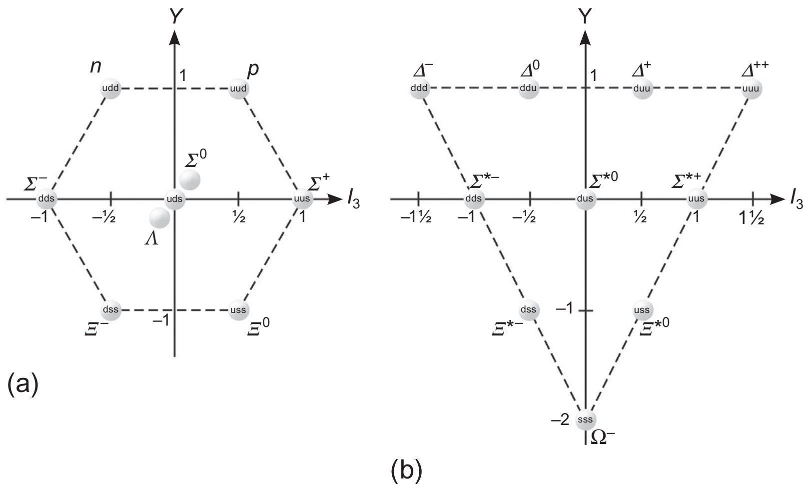

FIGURE 13.13 (a) The baryon octet and (b) the baryon decuplet.

TABLE 13.12 Possible Combinations of Pions and Nucleons

Pion and Nucleon	I_3
$\pi^+ p$	3/2
$\pi^+ n, \pi^0 p$	1/2
$\pi^0 n, \pi^- p$	−1/2
$\pi^- n$	−3/2

The I_3 values shown are equal to the sum of the I_3 quantum numbers of the pion and nucleon.

To a good approximation, the isospin quantum numbers, I and I_3, are conserved in a collision process. Since the combination of a π^+ and a p is the only combination of a pion and a nucleon having $I_3 = 3/2$, collisions of a π^+ and a p are elastic scattering processes described by the formula

$$\pi^+ + \mathrm{p} \to \pi^+ + \mathrm{p}. \tag{13.40}$$

Similarly, the combination of a π^- and a n is the only combination of a pion and a nucleon having $I_3 = -3/2$. Hence, the collisions of a π^- and an n are elastic scattering processes described by the formula

$$\pi^- + \mathrm{n} \to \pi^- + \mathrm{n}. \tag{13.41}$$

The two processes, (13.40) and (13.41), differ only in the sign of the azimuthal quantum number I_3. The cross sections for these two scattering processes have approximately the same dependence upon the energy of the incoming pion.

As we saw in Section 13.1 when we considered the collision process depicted in Fig. 13.3, the scattering of a pion and a nucleon can occur independently or by forming a resonance state. Since the isospin quantum number is conserved, a baryon resonance corresponding to either π^+p or π^-n scattering should have isospin $I = 3/2$. The cross sections for π^+p and π^-p scattering are shown in Fig. 13.14. We note that a sharp peak occurs for π^+p scattering for an effective mass around 1232 MeV, which is the mass of the Δ^{++} resonance. The Δ^{++} resonance has isospin equal to 3/2. Closer inspection of the scattering data shows that a shoulder corresponding to a smaller peak occurs in the range between 1600 and 1700 MeV. A more well-defined peak in the cross section for π^+p scattering occurs at 1900 MeV. A review of the data at the Web site of the Particle Data Group in 2008 shows that Δ resonances occur at 1600, 1700, and 1905 MeV.

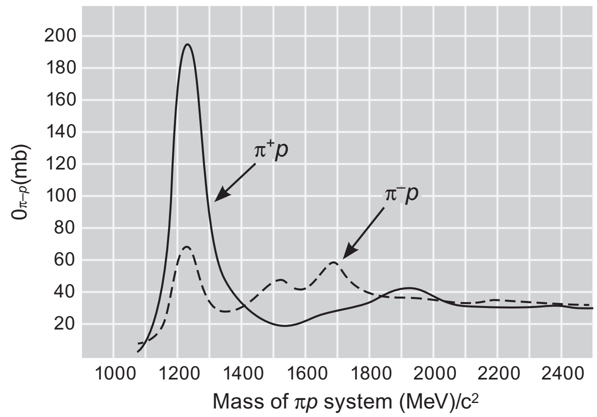

FIGURE 13.14 The cross sections for π^+p and π^-p scattering.

The collision of a π^- and a p can lead to the elastic scattering process

$$\pi^- + \mathrm{p} \rightarrow \pi^- + \mathrm{p} \tag{13.42}$$

or to the exchange process

$$\pi^- + \mathrm{p} \rightarrow \pi^0 + \mathrm{n}. \tag{13.43}$$

The π^-p and π^0n states are included in Table 13.12. These two states both have $I_3 = -1/2$ and are the only possible outcome of a collision of π^- and a proton. The state π^-p state having $I_3 = -1/2$ is a linear combination of isospin states with $I = 3/2$ and $I = 1/2$. A careful analysis of the role of isospin in pion-nucleon scattering can be found in the book by Perkins, which is cited at the end of this chapter. We shall use only a few features of this analysis here to describe the scattering curve of π^-p scattering shown in Fig. 13.14. While the states of π^+p have $I = 3/2$, one can show that one third of the states of the π^-p system have a $I = 3/2$ character and two thirds have a $I = 1/2$ character. This is the reason the peak of the π^-p cross section corresponding to the Δ^{++} resonance is approximately one third of the corresponding peak for π^+p scattering.

Reviewing the cross section for π^-p scattering shown in Fig. 13.14, one can see that the π^-p cross section has two additional peaks. One of these peaks occurs in the range between 1600 and 1700 MeV lying above the shoulder of the π^+p cross section we have referred to previously. This peak may be due to the Δ^{++} resonance considered earlier. Another peak occurs slightly above 1500 MeV where there is no corresponding peak in the π^+p cross section. A review of the data of the Particle Data Group in 2008 shows that a nucleon resonance with $J = 1/2$ occurs at 1535 MeV. The isolated peak of the cross section for π^-p scattering occurring above 1500 MeV may well correspond to this resonance.

Resonances formed in pion-proton scattering can be classified as baryon resonances with $B = 1$ or meson resonances with $B = 0$. An illustration of a scattering process in which a baryon resonance is formed is shown in Fig. 13.15. Using the conservation of baryon number B, one can readily see that the resonance state R formed in the collision has baryon number $B = 1$. One can say that baryon resonances are formed in a direct path. The pion and nucleons form a resonant state which then typically decays by the strong interaction in less than 10^{-22}.

The formation of a meson resonance is illustrated in Fig. 13.16. Using the conservation of baryon number B, one can readily see that the resonance state R^0 formed in this collision process has baryon number $B = 0$. Meson resonances can be expected to occur, for instance, in the reaction

$$\pi^- + \mathrm{p} \rightarrow \pi^+ + \pi^- + \mathrm{n}.$$

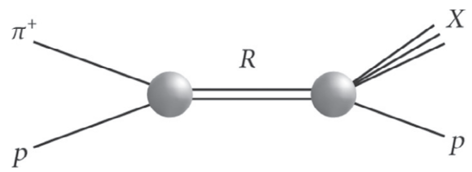

FIGURE 13.15 A baryon resonance formed in a pion-proton scattering experiment.

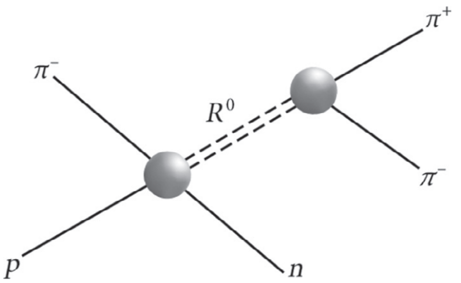

FIGURE 13.16 A meson resonance formed in a pion-proton scattering experiment.

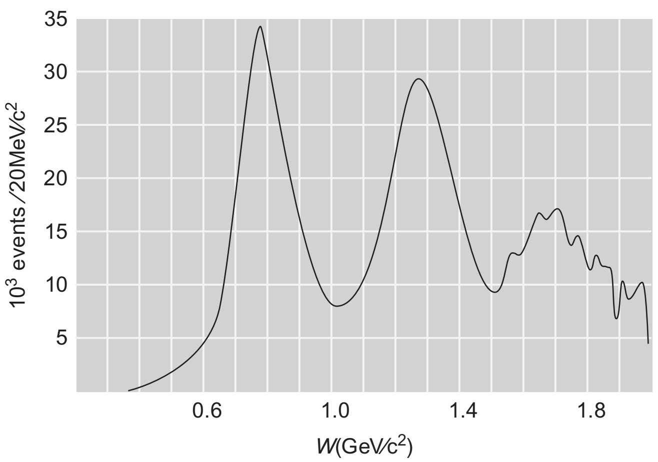

FIGURE 13.17 The invariant mass distribution of pion pairs produced in the reaction $\pi^- + \mathrm{p} \rightarrow \pi^+ + \pi^- + \mathrm{n}$. (After Grayer et al., Nuclear Physics B75, 1974.)

Figure 13.17 shows the invariant mass distribution of pion pairs produced in this reaction with a beam of 17 GeV pions. Three peaks corresponding to meson resonances occur for invariant masses of 770, 1270, and 1700 MeV.

13.4.2 Color

The spatial wave function of the lowest-lying state of physical systems are generally symmetric with respect to the interchange of like particle. We shall find that the number of hadron states in Tables 13.10 and 13.11 can be successfully accounted for if one also supposes that the spin wave functions of these baryons are symmetric with respect to the interchange of two quarks. This simple hypothesis shall allow us to account for the number of states that can be formed with up, down, and strange quarks.

We consider first the following six combinations of three quarks, which consist of two identical quarks and an additional quark of another kind:

$$uud, uus, ddu, dds, ssu, ssd. \tag{13.44}$$

In Section 13.2.3, we found that the spin-wave function of two fermions with spin $S = 1$ is symmetric with respect to an interchange of the two spins while the spin wave function with $S = 0$ is antisymmetric. The requirement that the spin wave functions of the particles (13.44) be symmetric with respect to the interchange of two spins then implies that the two identical quarks are in an $S = 1$ state. Coupling the additional quark with spin one half to the spin-one pair then results in spin one half or spin three halves. Since orbital angular momentum L is zero for these low-lying states, the three quarks will have total angular momentum J equal to one half and three halves. The quark states (13.44) thus contribute six states to the baryon octet shown in Fig. 13.13(a) and six states to the baryon decuplet shown in Fig. 13.13(b).

We next consider the states of three identical quarks,

$$uuu, ddd, sss. \tag{13.45}$$

The states of three identical quarks for which each quark is in a spin-up state will have a total value of the azimuthal quantum number M_S equal to $3/2$. For such a state, the spin quantum number S must also have the value $3/2$ since only the $S = 3/2$ state can have $M_S = 3/2$. Other states with $S = 3/2$ and with $M_S = 1/2, -1/2, -3/2$ can be generated from the $M = 3/2$ states by replacing the spin-up functions of one of the three quarks with spin-down function. The states (13.45) contribute three states to the baryon decuplet.

The only remaining state of a particle consisting of three quarks is

$$uds, \tag{13.46}$$

in which all the quarks are different. The spin angular momenta of the ud pair can be combined to form spin states with total spin angular momentum S equal to 0 and 1. The spin-zero state of the ud-pair can be combined with the spin of the s quark giving S equal to $1/2$, while the spin-one state of the ud pair can be combined with the spin of the s quark to give S equal to $1/2$ or $3/2$. The states (13.46) thus contribute two baryons to the octet and one baryon to the decuplet.

The assumption that the spatial and spin wave functions of baryons is symmetric with respect to an interchange of identical quarks allows one to explain the mass spectra of the light baryons, and yet this assumption appears to contradict the basic assumption of quantum mechanics that the wave function of fermions be antisymmetric with an exchange of two particles. The requirement that the wave function of many-fermion systems be anti-symmetric insures that fermion states automatically satisfy the Pauli exclusion principle. One can see immediately that the state of the Δ^{++} resonance with all three u quarks having spin up appears to violate the Pauli exclusion principle.

The apparent contradiction between the quark model and the Pauli principle was resolved in 1964 when Oscar W. Greenberg suggested quarks possess another attribute, which he called color. The combined space and spin wave function can then be symmetric with respect to the interchange of two quarks of the same flavor—as required by experiment—provided that the color part of the wave function is antisymmetric. The basic assumption of the color theory proposed by Greenberg is that the quarks of any flavor can exist in three different color states, *red*, *green*, and *blue*, denoted by r, g, b.

Just as the electromagnetic and weak interactions depend upon the hypercharge Y and isospin I_3 of the particles, the strong interaction depends on the two color charges called *color hypercharge* Y^C and *color isospin* I_3^C. The values of these new quantum numbers for the color states r, g, and b are given in Table 13.13. The color charges Y^C and I_3^C are conserved by the strong interaction.

The quantum numbers I_3^C and Y^C are additive quantum numbers with values for antiparticles being the negative of the values for particles.

Notice that the sum of the columns of Table 13.13 giving the total values of the color charges for red, green, and blue quarks and for red, green, and blue antiquarks is equal to zero. Hadrons only exist in color singlet states with zero values of the color charges. All baryons are made of three quarks of different colors, while mesons consist of a quark and an antiquark of the same color. For such states, the total value of the additive quantum numbers, I_3^C and Y^C, are equal to zero. This is called *color confinement*. The idea that quarks are always found in nature in color-neutral states was part of the motivation for using the word "color." Just as white light can be obtained by combining the three primary colors, baryons combine red, green, and blue quarks into a color neutral state.

The modern theory of the strong interactions, which is called *quantum chromodynamics* has been successful in describing a broad range of experimental data. Here, we only give one success of the theory. The differential cross section for the electromagnetic interaction

$$e^+ + e^- \rightarrow q + \bar{q},$$

TABLE 13.13 Values of the Color Charges I_3^C and Y^C for the Color States of Quarks and Antiquarks

	Quarks			Antiquarks	
	I_3^C	Y^C		I_3^C	Y^C
r	1/2	1/3	$\bar{r}$	−1/2	−1/3
g	−1/2	1/3	$\bar{g}$	1/2	−1/3
b	0	−2/3	$\bar{b}$	0	2/3

in which a positron/electron pair annihilate to produce a quark/antiquark pair, can be obtained from the cross section for the electromagnetic interaction producing a μ^+/μ^- pair,

$$e^+ + e^- \to \mu^+ + \mu^-,$$

by replacing the square of the charge of the quark, $(e\, e_q)^2$ with the square of the charge of a muon, e^2, times a factor of three to take into account the fact that there are three colors.

13.5 FEYNMAN DIAGRAMS

We would now like to look more closely at the scattering processes that occur when beams of particles are directed upon a target. An incident beam of particles can be characterized by its flux, which is the total number of particles that would pass through a surface perpendicular to the beam per area and per time. If the density of particles in the beam is denoted by n_i and the velocity of the beam is denoted by v_i, then the flux of the beam is

$$\phi_i = n_i v_i. \tag{13.47}$$

One can easily see this is true by considering the number of particles that would pass through an area A, which is perpendicular to the beam, in an infinitesimal time $\mathrm{d}t$. In a time $\mathrm{d}t$, all of the particles within a box with volume $v_i\,\mathrm{d}tA$ would pass through the area A. The number of particles within this box would be $n_i v_i\,\mathrm{d}tA$, and the number of particles passing through A per area and per time would be $n_i v_i$.

The transition rate W is the total number of particles deflected from the beam by an interaction with a particle in the target, and the cross section σ is the transition rate divided by the incident flux. The relation between the transition rate and the cross section can be written as

$$W = \phi_i \sigma. \tag{13.48}$$

The transition rate itself is given by an equation which can be derived by quantum mechanics. This equation, which Enrico Fermi called the *Golden Rule*, is

$$W = \frac{2\pi}{\hbar}|M_{if}|^2 \rho_\mathrm{f}. \tag{13.49}$$

The quantity ρ_f, which appears on the right-hand side of this equation, is the density of final states, and M_{if} is the matrix element of the interacting potential V between the initial and final states

$$M_{ij} = \int \psi_\mathrm{f}^* V \psi_i\,\mathrm{d}V. \tag{13.50}$$

To obtain an appropriate expression for the density of states ρ_f, we require that the wave functions of a particle be periodic with respect to a large cube of volume V containing the scattering event. One may then show that the number of states with the momentum $\mathbf{p}$ in a volume element, $\mathrm{d}^3\mathbf{p} = \mathrm{d}p_x\,\mathrm{d}p_y\,\mathrm{d}p_z$, is

$$\mathrm{d}N = \frac{V}{(2\pi\hbar)^3}\mathrm{d}^3\mathbf{p}. \tag{13.51}$$

(See Problem 11.) Since experiments often record the number of particles leaving a scattering event in a particular direction, it is usually best to use a spherical volume element in momentum space. The volume element in spherical coordinates can be written as

$$\mathrm{d}^3\mathbf{p} = p^2\,\mathrm{d}p\,\mathrm{d}\Omega, \tag{13.52}$$

where

$$\mathrm{d}\Omega = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi.$$

The number of states within a particular range of angles can be obtained by substituting Eq. (13.52) into Eq. (13.51) to obtain

$$\Delta N = \frac{V}{(2\pi\hbar)^3}p^2\,\mathrm{d}p\,\mathrm{d}\Omega, \tag{13.53}$$

and the density of states is

$$\frac{\mathrm{d}N}{\mathrm{d}E} = \frac{V}{(2\pi\hbar)^3}p^2\frac{\mathrm{d}p}{\mathrm{d}E}d\Omega. \tag{13.54}$$

An expression for the number of particles per second scattered into the angular region corresponding to dΩ can now be obtained by substituting Eq. (13.54) for ρ_{f} into Eq. (13.49) to obtain

$$W = \frac{2\pi}{\hbar}|M_{if}|^2 \frac{V}{(2\pi\hbar)^3} p_f^2 \frac{\mathrm{d}p_f}{\mathrm{d}E} \mathrm{d}\Omega. \tag{13.55}$$

An equation for the *differential cross section* can now be obtained by applying Eq. (13.48) to scattering events producing final state particles within a particular solid angle dΩ to obtain

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{2\pi}{\hbar} \frac{|M_{if}|^2}{\phi_i} \frac{V}{(2\pi\hbar)^3} p_f^2 \frac{\mathrm{d}p_f}{\mathrm{d}E}. \tag{13.56}$$

We found in Chapter 12 that scattering matrix elements M_{if} can be obtained using Feynman diagrams. The simplest diagrams correspond to a simple rearrangements of quarks. The Feynman diagram shown in Fig. 13.18 corresponds to the scattering process

$$\pi^+ + \mathrm{p} \rightarrow \Delta^{++} + \pi^0.$$

In this scattering process, the u quark from the π^+ joins the two u quarks from the proton to produce the Δ^{++}, while the d and $\bar{d}$ quarks form the π^0.

13.5.1 Electromagnetic Interactions

The Feynman diagram corresponding to two electrons scattering by means of the electromagnetic interaction is shown in Fig. 12.4 and reproduced in this chapter as Fig. 13.19(a). The diagram corresponds to the mathematical expression

$$\mathcal{M} = (-\mathrm{e}^2)\delta_{p+q,p'+q'}\bar{u}^{(r')}(\mathbf{p}')\bar{u}^{(s')}(\mathbf{q}')\gamma^\mu \frac{\mathrm{i}}{k^2}\gamma^\nu u^{(r)}(\mathbf{p})u^{(s)}(\mathbf{q}). \tag{13.57}$$

Each term in the expression on the right-hand side of Eq. (13.57) corresponds to a part of the Feynman diagram. The in-coming and out-going lines of the diagram correspond to free-particle solutions of the Dirac equation, the factors, $\mathrm{i}e\gamma^\mu$

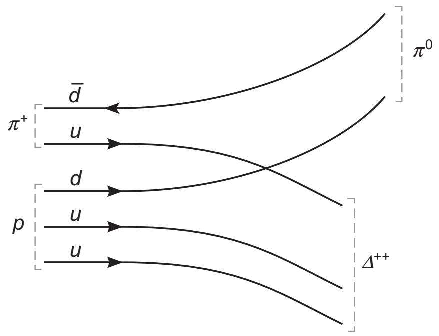

FIGURE 13.18 Feynman diagram for the reaction $\pi^+ + \mathrm{p} \rightarrow \Delta^{++} + \pi^0$.

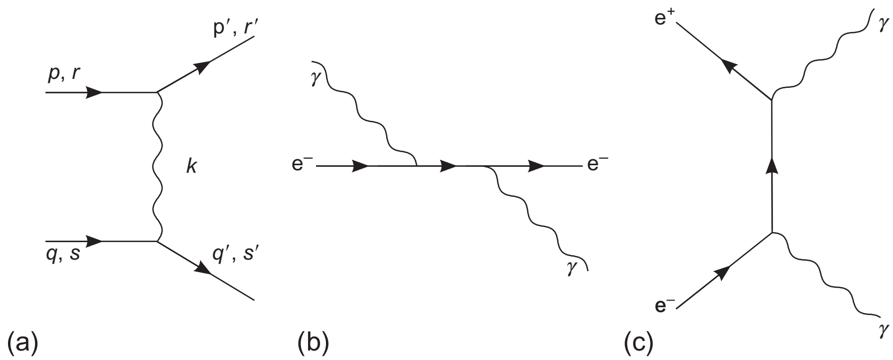

FIGURE 13.19 Feynman diagrams for (a) electron-electron scattering, (b) Compton scattering, and (c) positron-electron annihilation.

and $ie\gamma^\nu$, correspond to the two vertices of the diagram, and the wavy line corresponds to the *propagator*. The propagator of the electromagnetic interaction is represented by the term i/k^2 appearing in Eq. (13.57). The delta function $\delta_{p+q,p'+q'}$ ensures that the total four-momentum of the electrons is conserved in the interaction.

The Feynman diagram shown in Fig. 13.19(b) describes Compton scattering in which a photon scatters off a free electron producing a photon and an electron with different momentum and energy. The in-coming and out-going photons in this scattering process are represented by free wavy lines, while the solid line joining the two vertices is referred to as the *Fermion propagator*. Figure 13.19(c) describes a process in which an electron and positron annihilate producing two photons. Arrows directed to the right in Feynman diagrams correspond to particles, while lines directed to the left correspond to antiparticles. Notice that the incoming positron is represented by a free solid line with an arrow directed away from the vertex. This is consistent with the hole theory described in Chapter 12 in which an antiparticle was depicted as a hole. Absorbing a positron corresponds to filling a hole and is hence represented by an arrow directed away from the vertex. In a similar fashion, the creation of a positron correspond to an arrow directed toward a vertex.

13.5.2 Weak Interactions

The Feynman diagrams for electromagnetic processes have vertices with two electron or positron lines and one line of a photon. We would now like to consider weak interactions of leptons mediated by the vector bosons. As an example of weak interaction processes of this kind, we consider muon decay,

$$\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu,$$

which is represented by the Feynman diagram shown in Fig. 13.20(a), and inverse muon decay,

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e,$$

which is represented by the Feynman diagram shown in Fig. 13.20(b). Vector bosons (photon, W, and Z bosons) are represented in this book by wavy lines.

Processes mediated by W^+ or W^- have vertices such as those shown in Fig. 13.21(a) and (b), while processes mediated by the neutral Z^0 have vertices such as those shown in Fig. 13.21(c) and (d). In Fig. 13.21(a), a neutrino (ν_l) of one generation is absorbed at the vertex with a lepton of the same generation (l^-) being created. The interaction line corresponds to a W^- boson. Figure 13.21(b) shows the corresponding process with a lepton being absorbed and a neutrino being

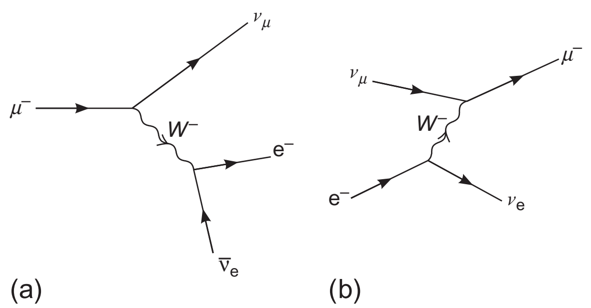

FIGURE 13.20 Feynman diagrams for (a) muon decay and (b) inverse muon decay.

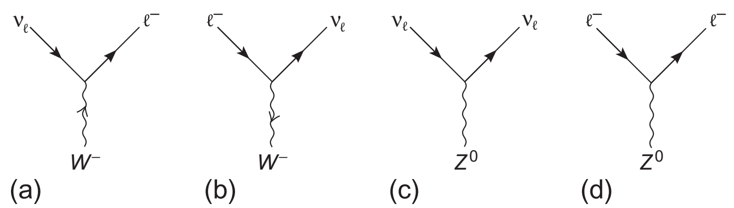

FIGURE 13.21 Vertices in which (a) a neutrino is absorbed and a lepton of the same generation is created due to an interaction mediated by W^-, (b) a lepton is absorbed and a neutrino of the same generation is created due to an interaction mediated by W^-, (c) a neutrino is absorbed and another created due to an interaction mediated by Z^0, and (d) a lepton is absorbed and another created due to an interaction mediated by Z^0.

produced. The fact that the leptons and neutrinos involved in these interactions belong to the same generation corresponds to conservation laws we have seen earlier which require that electron lepton number, muon lepton number, and tau lepton number are individually conserved.

The vertices shown in Fig. 13.21(c) and (d) for the neutral Z^0 are similar to the photon vertices seen earlier. At energies small compared to the Z^0 mass, the Z^0 exchange interactions can be neglected in comparison to the corresponding photon exchange interactions. However, at very high energies, Z^0 exchange interactions become comparable to photon exchange interactions.

Additional vertices can be generated from the four vertices shown in Fig. 13.21 by rotating the free lines about the point where the free lines meet the interaction line. The free lines may then be directed from right to left converting particles into antiparticles. Fig. 13.21(a) would then describe a process in which an antilepton is absorbed and an antineutrino is created, while Fig. 13.21(b) would describe a process in which an antineutrino is absorbed and an antilepton is created. Similarly, Fig. 13.21(c) would describe a process in which antineutrinos are absorbed and created, and Fig. 13.21(d) would describe a process in which antileptons are absorbed and created. Whether a line that has been rotated describes a particle or an antiparticle depends upon whether the arrow points to the right or to the left.

The weak interactions of quarks that can occur may be obtained by replacing leptons and neutrinos of allowed weak processes with quarks of the same generation and by allowing mixing to occur between different generations of quarks. We recall that the three generations of leptons are given by Eq. (13.1) and the three generations of quarks are given by Eq. (13.2). Allowed $W^{\pm}$ vertices obtained from the vertices shown in Fig. 13.21(a) and (b) by making the replacements $\nu_e \rightarrow u$ and $e^- \rightarrow d$ are shown in Fig. 13.22(a) and (b), while allowed $W^{\pm}$ vertices obtained from the vertices shown in Fig. 13.21(a) and (b) by making the replacements $\nu_\mu \rightarrow c$ and $\mu^- \rightarrow s$ are shown in Fig. 13.22(c) and (d). The mixing that occurs between the first- and second-generations of quarks are described by the equations

$$\begin{aligned} d' &= d\cos\theta_c + s\sin\theta_c \\ s' &= -d\sin\theta_c + s\cos\theta_c, \end{aligned} \tag{13.58}$$

where the parameter θ_c is called the *Cabibbo angle*. The experimental value of the Cabibbo angle is about 12.7°. Allowed weak processes for quarks can thus be obtained from allowed weak processes of leptons by making the replacements $\nu_e \rightarrow u$, $e^- \rightarrow d$, $\nu_\mu \rightarrow c$, and $\mu^- \rightarrow s$ and allowing the quarks to mix in prescribed ways.

We now give a number of examples of processes that occur by means of the weak interaction. The decay of the π^- by the reaction

$$\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$$

is described by the diagram shown in Fig. 13.23(a). The vertex on the left of this diagram is obtained from the vertex shown in Fig. 13.22(b) by rotating the line associated with the up-quark so that it is pointed to the left away from the vertex. According to conventions described previously, the line associated with the up-quark pointing to the left then corresponds to $\overline{u}$. A particle line approaching a vertex corresponds to particle absorption and a particle line leaving a vertex corresponds to particle creation, while an antiparticle line approaching a vertex corresponds to antiparticle creation and an antiparticle line leaving a vertex corresponds to antiparticle absorption. A $d/\overline{u}$ pair is absorbed at the left vertex of Fig. 13.23(a) producing a W^- which then gives a $\overline{\nu}_\mu/\mu_-$ pair. The decay of π^- we have considered would occur without Cabibbo mixing. Still, as shown in Fig. 13.23(a), the Cabibbo mixing contributes a factor $\cos\theta_c$ to the quark vertex.

The decay of the K^- is described by the Feynman diagram in Fig. 13.23(b). The vertex on the left of the diagram is again obtained from the vertex shown in Fig. 13.22(b) by rotating the line associated with the u-quark so that it points to the left. The decay process described by the diagram in Fig. 13.23(b) occurs because of Cabibbo mixing, which contributes a

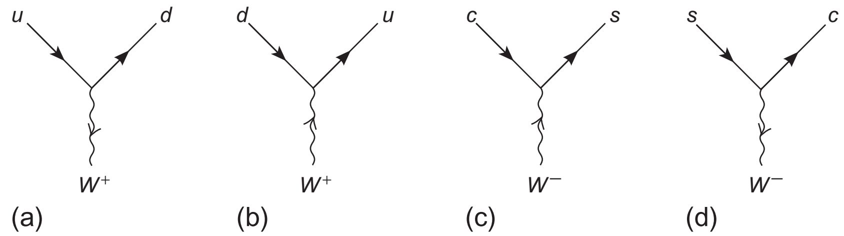

FIGURE 13.22 Vertices in which due to an interaction mediated by $W^{\pm}$ (a) an u quark is absorbed and a d quark is created, (b) a d quark is absorbed and a u quark is created, (c) a c quark is absorbed and a s quark is created, or (d) a s quark is absorbed and a c quark is created.

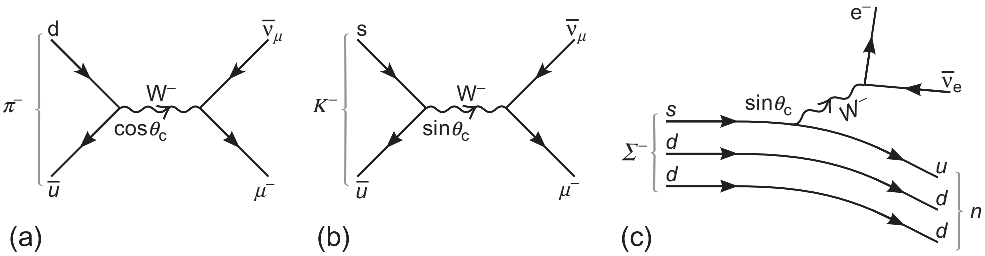

FIGURE 13.23 The decays (a) $\pi^- \to \mu^- + \overline{\nu}_\mu$, (b) $K^- \to \mu^- + \overline{\nu}_\mu$, and (c) $\Sigma^- \to n + e^- + \overline{\nu}_e$.

factor of $\sin\theta_c$ to the quark vertex. A $s/\overline{u}$ pair is absorbed at the left vertex producing a W^-, which then gives a $\overline{\nu}_\mu/\mu^-$ pair. Another example of a weak interaction process is the decay of the Σ^- shown in Fig. 13.23(c).

13.5.3 Strong Interactions

The electromagnetic and strong interactions are both mediated by massless spin-1 bosons. However, there is an important difference between these two interactions which affect the kinds of processes that can occur. While photons have no electric charge, gluons have nonzero values of the color charges. This is illustrated in Fig. 13.24(a), which shows the interaction of two quarks by gluon exchange. The gluon is represented in this diagram by a "quarkscrew" line to distinguish it from the electromagnetic and weak interactions.

The values of the color charges of the gluon exchanged can be calculated from the color charges of the in-coming and out-going quarks. Using the values of the color charges of the quarks given in Table 13.13, we obtain the following values of color charges of the gluon exchanged

$$I_3^C = I_3^C(r) - I_3^C(b) = 1/2,$$
$$Y^C = Y^C(r) - Y^C(b) = 1.$$

A gluon thus carries both I_3 and Y color charges and can interact with other gluons. The two types of gluon-gluon interaction are illustrated in Fig. 13.24(b) and (c). Figure 13.24(b) illustrates the exchange of a gluon between two other gluons, while Fig. 13.24(c) illustrates a zero range "contact" interaction between two gluons.

Of the many scattering processes involving quarks that occur by means of the strong interaction, we consider only the following example:

$$\pi^- + p \to K^0 + \Lambda,$$

which corresponds to the diagram shown in Fig. 13.25. In this scattering process, a $\overline{u}/u$ pair is absorbed giving a gluon which produces a $\overline{s}/s$ pair.

The strong interaction has the distinctive properties of color confinement and asymptotic freedom. The property of color confinement means that observed particles all have zero color charge. Mesons are made up of a quark/antiquark pair with the two constituents having opposite values of the color charge, while baryons are made up of three quarks with the total values of I_3^C and Y^C equal to zero. Asymptotic freedom means the interaction gets weaker at short distances. At distances less than about 0.1 fm, the lowest order diagrams dominate.

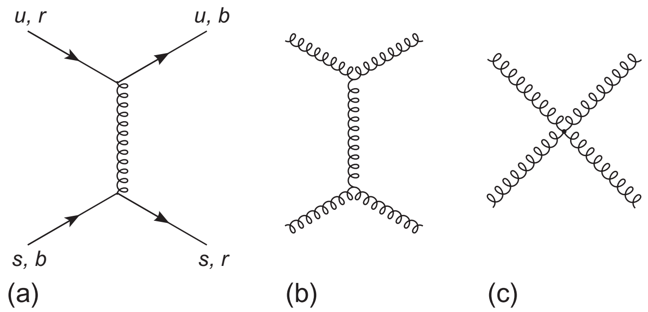

FIGURE 13.24 (a) Interaction of two quarks by gluon exchange, (b) interaction of two gluons by gluon exchange, and (c) a zero range "contact" interaction between two gluons.

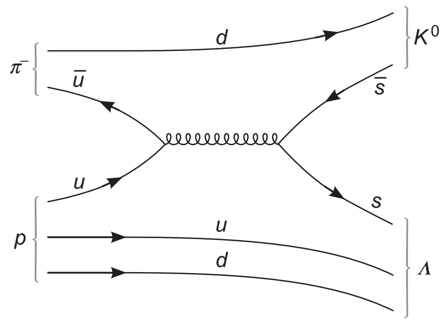

FIGURE 13.25 Feynman diagram for the process $\pi^- + \text{p} \to K^0 + \Lambda$.

13.6 * THE FLAVOR AND COLOR SU(3) SYMMETRIES

The isospin symmetry is due to the fact that the up- and down-quarks have approximately the same mass and interact by means of the same strong interaction. Consequences of the isospin symmetry can naturally be explored by representing the u- and d-quarks by vectors in a two-dimensional space. We shall denote a quark wave function by

$$q = \begin{bmatrix} u \\ d \end{bmatrix}.$$

A transformation due to the two-by-two matrices **U** operating on this space has the general form

$$\begin{bmatrix} u' \\ d' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix}. \tag{13.59}$$

A little nomenclature is necessary to describe the properties of these transformation matrices. The *adjoint* of a matrix is the matrix obtained by taking the complex conjugate of each element of the matrix and reflecting the matrix through its diagonal. A *Hermitian* matrix is a matrix which is equal to its adjoint, and a *unitary* matrix is a matrix whose inverse is equal to its adjoint. The transformation (13.59) will preserve the orthogonality and normalization of physical states if the two-by-two **U** matrices are unitary and have determinant equal to one. The collection of all two-by-two unitary matrices with determinant equal to one is said to form the special unitary group in two dimensions denoted by $SU(2)$. Saying the matrices form a *group* means that matrix multiplication is defined and each matrix has a unique inverse.

A considerable simplification of the $SU(2)$ theory is obtained by considering infinitesimal transformations, which are close to the identity matrix

$$\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The matrix for an infinitesimal transformation can be written as

$$\mathbf{U}_{\text{inf}} = \mathbf{I}_2 + \mathrm{i}\xi, \tag{13.60}$$

where $\mathbf{I}_2$ is the identity matrix and the matrix ξ has infinitesimal elements.

The properties of the transformation matrix $\mathbf{U}_{\text{inf}}$ depend upon the properties of the matrix ξ. One can show that $\mathbf{U}_{\text{inf}}$ is unitary if ξ is Hermitian and that the determinant of $\mathbf{U}_{\text{inf}}$ is equal to one if the sum of the diagonal elements of ξ is equal to zero. We shall make the following choice of ξ

$$\xi = \epsilon \cdot \tau/2 = (\epsilon_1\tau_1 + \epsilon_2\tau_2 + \epsilon_3\tau_3)/2, \tag{13.61}$$

where τ_1, τ_2, and τ_3 are the Pauli matrices introduced in Chapter 12 in conjunction with the Dirac equation. These matrices, which are denoted by "tau" here to distinguish them from the mathematically identical "spin" matrices associated with real spin degrees of freedom, are defined as

$$\tau_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \tau_2 = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}, \quad \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{13.62}$$

The τ-matrices are called the *generators* of the $SU(2)$ symmetry group.

13.6.1 The $SU(3)$ Symmetry Group

We have seen that isospin multiplets are parts of larger supermultiplets containing both strange and non-strange particles. Murry Gell-Mann and Yuval Ne'eman were the first to suggest that the $SU(3)$ group, which consists of unitary 3×3 matrices with determinant equal to one, was the appropriate generalization of the $SU(2)$ symmetry group. Gell-Mann showed that the lightest mesons and baryons could be described in a unified way using the $SU(3)$ symmetry, and he was able to make predictions of the possible decay modes of these particles.

As for the $SU(2)$ symmetry, the transformation of the symmetry group $SU(3)$ can be built up from infinitesimal transformations

$$\mathbf{W}_{\text{inf}} = \mathbf{I}_3 + \mathrm{i}\chi, \tag{13.63}$$

where $\mathbf{I}_3$ is the 3×3 identity matrix and the Hermitian matrix χ has infinitesimal elements. The χ matrix, which involves *eight* independent parameters, can be written as

$$\chi = \eta \cdot \mathbf{T} = \eta_1 \mathbf{T}_1 + \eta_2 \mathbf{T}_2 + \cdots + \eta_8 \mathbf{T}_8, \tag{13.64}$$

where $\eta = (\eta_1, \eta_2, \ldots, \eta_8)$ and the matrices $\mathbf{T}_i$ are defined by the following equations:

$$\begin{gathered}
\mathbf{T}_1 = \frac{1}{2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{T}_2 = \frac{1}{2}\begin{bmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{T}_3 = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
\mathbf{T}_4 = \frac{1}{2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{T}_5 = \frac{1}{2}\begin{bmatrix} 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 \end{bmatrix}, \quad \mathbf{T}_6 = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \\
\mathbf{T}_7 = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{bmatrix}, \quad \mathbf{T}_8 = \frac{1}{2\sqrt{3}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}.
\end{gathered} \tag{13.65}$$

These matrices are called the Gell-Mann matrices. The matrices $\mathbf{T}_1 - \mathbf{T}_7$ can be constructed by adding an additional row and column to the Pauli matrices (13.62) and multiplying the resulting matrix by an additional factor of one half, while the matrix, $\mathbf{T}_8$ is obtained by adding an extra row and column and also a multiplicative factor to the identity matrix. We shall begin our study of implications of the $SU(3)$ symmetry, by considering the commutation relations of the Gell-Mann matrices. The commutator, $[\mathbf{A}, \mathbf{B}]$, of two matrices, $\mathbf{A}$ and $\mathbf{B}$, is defined to be

$$[\mathbf{A}, \mathbf{B}] = \mathbf{AB} - \mathbf{BA}$$

and the matrices are said to commute if

$$\mathbf{AB} - \mathbf{BA} = 0$$

or equivalently if $\mathbf{AB} = \mathbf{BA}$. The matrices $\mathbf{T}_i$ satisfy commutation relations

$$[\mathbf{T}_i, \mathbf{T}_j] = \mathrm{i} f_{ijk} \mathbf{T}_k, \tag{13.66}$$

where the constants f_{ijk} are real.

Of the matrices defined by Eqs. (13.65), the matrices $\mathbf{T}_3$ and $\mathbf{T}_8$ will play a special role in our discussion because they commute with each other. We shall make the following associations with these matrices:

$$\mathbf{H}_1 = \mathbf{T}_3, \quad \mathbf{H}_2 = \mathbf{T}_8. \tag{13.67}$$

One can easily find linear combinations of the other $\mathbf{T}$-matrices which have single nonzero elements. For instance, the combination $\mathbf{T}_4 + \mathrm{i}\mathbf{T}_5$ gives

$$\mathbf{T}_4 + \mathrm{i}\mathbf{T}_5 = \frac{1}{2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \frac{\mathrm{i}}{2}\begin{bmatrix} 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The matrix $\mathbf{T}_4 + \mathrm{i}\mathbf{T}_5$ satisfies the following commutation relations with $\mathbf{H}_1$ and $\mathbf{H}_2$

$$[\mathbf{H}_1, (\mathbf{T}_4 + \mathrm{i}\mathbf{T}_5)] = \frac{1}{2}(\mathbf{T}_4 + \mathrm{i}\mathbf{T}_5),$$

$$[\mathbf{H}_2, (\mathbf{T}_4 + \mathrm{i}\mathbf{T}_5)] = \frac{\sqrt{3}}{2}(\mathbf{T}_4 + \mathrm{i}\mathbf{T}_5).$$

The commutation relations involving $\mathbf{T}_4$ and $\mathbf{T}_5$ can be written more concisely by defining a vector

$$\alpha_1 = \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix} \tag{13.68}$$

and the matrix

$$\mathbf{E}_{\alpha_1} = \mathbf{T}_4 + \mathrm{i}\mathbf{T}_5 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{13.69}$$

With this notation, the matrix $\mathbf{E}_{\alpha_1}$ satisfies the equations

$$[\mathbf{H}_1, \mathbf{E}_{\alpha_1}] = [\alpha_1]_1 \mathbf{E}_{\alpha_1},$$

$$[\mathbf{H}_2, \mathbf{E}_{\alpha_1}] = [\alpha_1]_2 \mathbf{E}_{\alpha_1}.$$

The vector α_1, which determines the commutation relations of the generators $\mathbf{H}_1$ and $\mathbf{H}_2$ with the matrix $\mathbf{E}_{\alpha_1}$, is called a *root vector* and the matrix $\mathbf{E}_{\alpha_1}$ is regarded as the generator corresponding to that root.

One may easily identify other roots. The matrix $\mathbf{E}_{-\alpha_1}$, defined by the equation

$$\mathbf{E}_{-\alpha_1} = \mathbf{T}_4 - \mathrm{i}\mathbf{T}_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \tag{13.70}$$

satisfies the equations

$$[\mathbf{H}_1, \mathbf{E}_{-\alpha_1}] = -1/2\mathbf{E}_{-\alpha_1},$$

$$[\mathbf{H}_2, \mathbf{E}_{-\alpha_1}] = -\sqrt{3}/2\mathbf{E}_{-\alpha_1},$$

and is thus associated with the root vector

$$-\alpha_1 = \begin{bmatrix} -1/2 \\ -\sqrt{3}/2 \end{bmatrix}. \tag{13.71}$$

Similarly, the matrix $\mathbf{E}_{\alpha_2}$, defined by the equation

$$\mathbf{E}_{\alpha_2} = \mathbf{T}_6 - \mathrm{i}\mathbf{T}_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \tag{13.72}$$

corresponds to the root

$$\alpha_2 = \begin{bmatrix} 1/2 \\ -\sqrt{3}/2 \end{bmatrix}, \tag{13.73}$$

and the matrix $\mathbf{E}_{-\alpha_2}$, defined by the equation

$$\mathbf{E}_{-\alpha_2} = T_6 + \mathrm{i}T_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \tag{13.74}$$

corresponds to the root $-\alpha_2$. The four matrices, $\mathbf{E}_{\pm\alpha_1}$ and $\mathbf{E}_{\pm\alpha_2}$, correspond to the roots $\pm\alpha_1$ and $\pm\alpha_2$. These matrices may be used to replace the matrices $\mathbf{T}_4$, $\mathbf{T}_5$, $\mathbf{T}_6$, and $\mathbf{T}_7$.

One may also show that the matrix $\mathbf{E}_{[\mathbf{1},\mathbf{0}]}$, defined by the equation

$$\mathbf{E}_{[1,0]} = \mathbf{T}_1 + i\mathbf{T}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{13.75}$$

corresponds to the root

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

and the matrix $\mathbf{E}_{[-\mathbf{1},\mathbf{0}]}$, defined by the equation

$$\mathbf{E}_{[-1,0]} = \mathbf{T}_1 - \mathrm{i}\mathbf{T}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{13.76}$$

corresponds to the root

$$\begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

The properties of the generators of $SU(3)$ are important for our further discussion. While the operators $\mathbf{H}_1$ and $\mathbf{H}_2$ commute with each other, the matrices $\mathbf{E}_\alpha$ satisfy the commutation relations

$$[\mathbf{H}_i, \mathbf{E}_\alpha] = \alpha_i \mathbf{E}_\alpha. \tag{13.77}$$

The matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ are like the angular momentum operator l_z whose eigenvalues can be used to label physical states, while the matrices $\mathbf{E}_\alpha$ are like the step-up and step-down operators, l_+ and l_-, which act upon angular momentum states to produce states with larger and smaller values of the quantum number m.

All of the roots of $SU(3)$ may be written in terms of the roots α_1 and α_2, which are given by Eqs. (13.68) and (13.73). Before further developing the theory, we summarize the equations defining the roots

$$\begin{aligned} \alpha_1 &= \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix}, \quad -\alpha_1 = \begin{bmatrix} -1/2 \\ -\sqrt{3}/2 \end{bmatrix}, \\ \alpha_2 &= \begin{bmatrix} 1/2 \\ -\sqrt{3}/2 \end{bmatrix}, \quad -\alpha_2 = \begin{bmatrix} -1/2 \\ \sqrt{3}/2 \end{bmatrix}, \\ \begin{bmatrix} 1 \\ 0 \end{bmatrix} &= \alpha_1 + \alpha_2, \quad \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -\alpha_1 - \alpha_2. \end{aligned} \tag{13.78}$$

The roots of $SU(3)$, which are given by these equations, are all illustrated in Fig. 13.26. A root is said to be *positive* if the first non-zero component is positive. In this sense, the two roots, α_1 and α_2, are positive. Since the first non-zero component of the vector $\alpha_1 - \alpha_2$ is positive, α_1 is called the highest root.

13.6.2 The Representations of $SU(3)$

A representation of a group consists of a set of operators or matrices acting upon a vector space with a unique operator or matrix of the representation corresponding to each member of the group. The simplest representation of the group $SU(3)$ is formed by the matrices of the group acting on the column vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{13.79}$$

The representation of $SU(3)$ formed by the matrices of the group acting on the column vectors (13.79) is known as a *fundamental representation*.

We may assign weights to each column vector, $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$, by seeing how it is affected by the diagonal generators of the group. The diagonal generators, $\mathbf{H}_1$ and $\mathbf{H}_2$, are defined by Eqs. (13.67) and (13.65). For instance, we may derive the following equations:

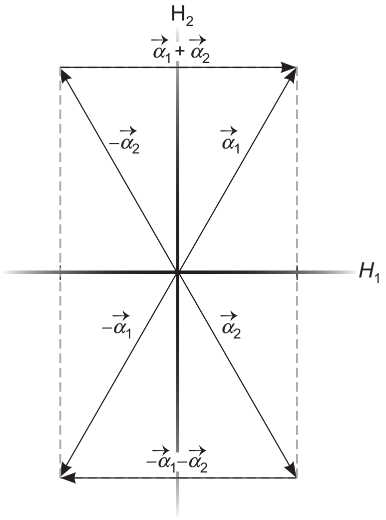

FIGURE 13.26 The roots of $SU(3)$.

$$\begin{aligned}\mathbf{H}_1\mathbf{v}_1 &= \mathbf{T}_3\mathbf{v}_1\\ &= (1/2)\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = (1/2)\begin{bmatrix}1\\0\\0\end{bmatrix} = (1/2)\mathbf{v}_1,\end{aligned} \tag{13.80}$$

$$\begin{aligned}\mathbf{H}_2\mathbf{v}_1 &= \mathbf{T}_8\mathbf{v}_1\\ &= (\sqrt{3}/6)\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = (\sqrt{3}/6)\begin{bmatrix}1\\0\\0\end{bmatrix} = (\sqrt{3}/6)\mathbf{v}_1.\end{aligned} \tag{13.81}$$

We may thus assign to the vector $\mathbf{v}_1$ a weight vector with components $1/2$ and $\sqrt{3}/6$. The components of the weight determine the result of operating with the generators $\mathbf{H}_1$ and $\mathbf{H}_2$ upon the corresponding vector. Constructing the weights for the vectors $\mathbf{v}_2$ and $\mathbf{v}_3$ in a similar manner, we obtain the following three weight vectors:

$$w(\mathbf{v}_1) = \begin{bmatrix}1/2\\ \sqrt{3}/6\end{bmatrix}, \quad w(\mathbf{v}_2) = \begin{bmatrix}-1/2\\ \sqrt{3}/6\end{bmatrix}, \quad w(\mathbf{v}_3) = \begin{bmatrix}0\\ -\sqrt{3}/3\end{bmatrix}. \tag{13.82}$$

Each weight corresponds to a particular column vector. The roots translate one weight into another. Using Eqs. (13.78) and (13.82), one can derive, for example, the following equation:

$$\alpha_1 + w(\mathbf{v}_3) = w(\mathbf{v}_1).$$

The three weights of the fundamental representation together, with the roots α_1, α_2, and $\alpha_1+\alpha_2$, are illustrated in Fig. 13.27. Each root translates one weight into another. The root α_1 translates the weight, $w(\mathbf{v}_3)$, into the weight, $w(\mathbf{v}_1)$. Similarly, the root α_2 translates the weight, $w(\mathbf{v}_2)$, into the weight, $w(\mathbf{v}_3)$, while the root $\alpha_1+\alpha_2$ translates the weight, $w(\mathbf{v}_2)$, into the weight,$w(\mathbf{v}_1)$. The root diagram can be thought of as a map which shows us how to get from one weight to another.

Another representation of $SU(3)$ may be obtained by taking the negative of the complex conjugate of the matrices of the fundamental representation. The new representation obtained in this way is called the *complex conjugate representation*. Taking the complex conjugate of Eq. (13.66) and using the fact that the constants f_{ijk} are real, we obtain

$$[\mathbf{T}_i^*, \mathbf{T}_j^*] = -\mathrm{i}f_{ijk}\mathbf{T}_k^*.$$

This equation can be written

$$[-\mathbf{T}_i^*, -\mathbf{T}_j^*] = \mathrm{i}f_{ijk}(-\mathbf{T}_k^*),$$

which shows that the matrices $-\mathbf{T}_i^*$ satisfy the same algebraic equations as the $\mathbf{T}_i$ matrices themselves.

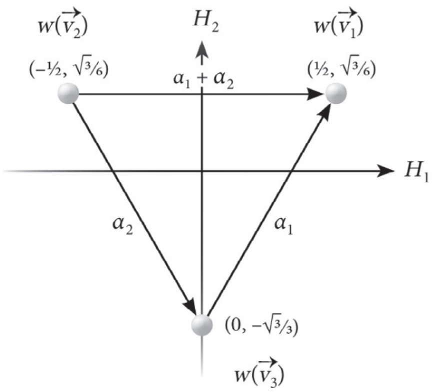

FIGURE 13.27 The three weights of the fundamental representation together with the roots α_1, α_2, and $\alpha_1 + \alpha_2$.

We assign weights to each vector of the complex conjugate representation by allowing the diagonal generators, $-\mathbf{H}_1^*$ and $-\mathbf{H}_2^*$, to act upon the vector. For the vector $\mathbf{v}_1$, we obtain

$$-\mathbf{H}_1{}^*\mathbf{v}_1 = \frac{1}{2}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = -\frac{1}{2}\mathbf{v}_1,$$

$$-\mathbf{H}_2{}^*\mathbf{v}_1 = \frac{\sqrt{3}}{6}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = -\frac{\sqrt{3}}{6}\mathbf{v}_1.$$

We may thus assign to the vector $\mathbf{v}_1$ a weight vector with components $-1/2$ and $-\sqrt{3}/6$. Since the matrices $\mathbf{H}_1^\star$ and $\mathbf{H}_2^\star$, are the negatives of $\mathbf{H}_1$ and $\mathbf{H}_2$, the weights of the complex conjugate representation can be obtained by reflecting the weights of the fundamental representation through the origin. Using the weights of the fundamental representation, which are given by Eq. (13.82), the weights of the complex conjugate representation are seen to be

$$w^*(\mathbf{v}_1) = \begin{bmatrix} -1/2 \\ -\sqrt{3}/6 \end{bmatrix}, \quad w^*(\mathbf{v}_2) = \begin{bmatrix} 1/2 \\ -\sqrt{3}/6 \end{bmatrix}, \quad w^*(\mathbf{v}_3) = \begin{bmatrix} 0 \\ \sqrt{3}/3 \end{bmatrix}. \tag{13.83}$$

The weights of the complex conjugate representation are shown in Fig. 13.28.

We recall that a root is said to be positive if the first non-zero component is positive and the highest root has the property that the difference between this root and any other root is positive. Applying these definitions to the weights, the highest weight of the fundamental representation is

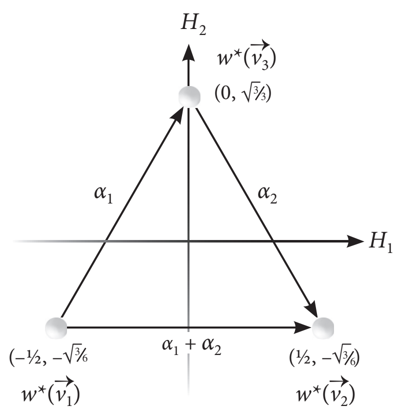

FIGURE 13.28 The weights of the complex conjugate representation of $SU(3)$.

$$\mu_1 = \begin{bmatrix} 1/2 \\ \sqrt{3}/6 \end{bmatrix}, \tag{13.84}$$

while the highest weight of the complex conjugate representation is

$$\mu_2 = \begin{bmatrix} 1/2 \\ -\sqrt{3}/6 \end{bmatrix}. \tag{13.85}$$

The representations of $SU(3)$ may be characterized by their highest weights.

As we have said, a representation of a group consists of a set of operators or matrices acting upon a vector space. An important representation of $SU(3)$ called the *adjoint representation* is obtained by allowing the matrices corresponding to the generators of the group to play the role of *both* the operators and the states of the vector space. In the adjoint representation, the product of two generators is equal to the commutator of the generators. Since the commutation relations are used to define the roots and the product of the generators $\mathbf{H}_i$ with the states of the representation, the weights of the adjoint representation are equal to the roots of the group. The root diagram shown in Fig. 13.26 thus serves also as the weight diagram of the adjoint representation.

Various notations are used by particle physicists to denote the different representations of $SU(3)$. The fundamental and the complex conjugate representations of $SU(3)$, which have the highest weights μ_1 and μ_2, respectively, are denoted by $(1,0)$ and $(0,1)$, respectively. Representations of $SU(3)$ can also be denoted by their dimension with the complex conjugate representation being distinguished by a bar. The two fundamental representations can thus be denoted by

$$(1,0) \equiv 3, \quad (0,1) \equiv \overline{3}. \tag{13.86}$$

The representation $(2,0)$ has the highest weight

$$2\mu_1 = \begin{bmatrix} 1 \\ \sqrt{3}/3 \end{bmatrix}.$$

The other weights of this representation may be obtained by shifting the highest weight by the root vectors, α_1 and α_2. The weights of the $(2,0)$ representation are shown in Fig. 13.29(a), and the weights of the $(3,0)$ representation with highest weight $3\mu_1$ are shown in Fig. 13.29(b). In his book, *Lie Algebras in Particle Physics*, Howard Georgi gives a beautiful pictorial scheme for finding all the weights of representations of $SU(3)$.

The Flavor $SU(3)$ Symmetry

One may notice that the figures of the weights of the representations of $SU(3)$ in the preceding section are very similar to the figures in Section 4.1 giving the isospins I_3 and hypercharge Y of supermultiplets of particles. We consider first the weights of the fundamental representation of $SU(3)$, which are given by Eq. (13.82) and shown in Fig. 13.27. Using Fig. 13.11, the vectors with components I_3 and Y, associated with the three quarks are

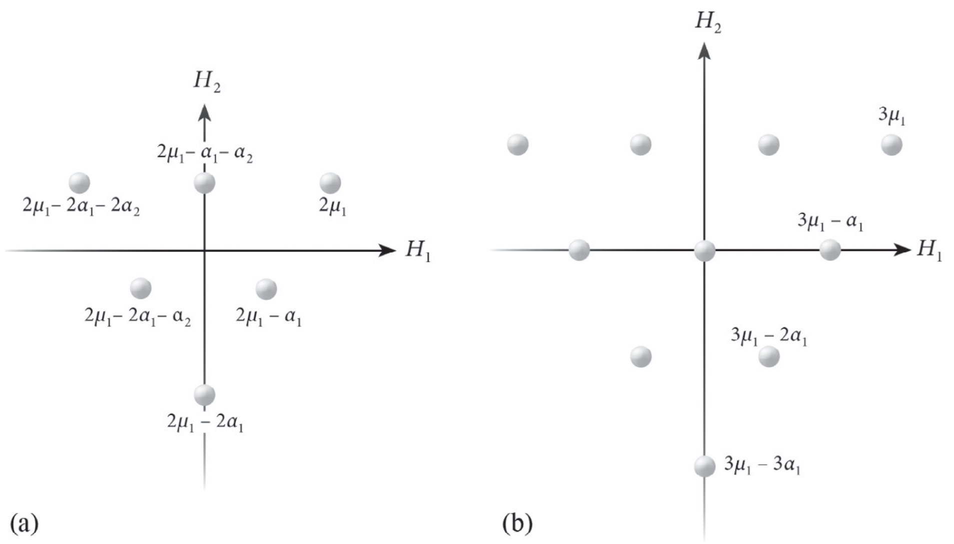

FIGURE 13.29 (a) The weights of the $(2,0)$ representation and (b) the weights of the $(3,0)$ representation.

$$u = \begin{bmatrix} 1/2 \\ 1/3 \end{bmatrix}, \quad d = \begin{bmatrix} -1/2 \\ 1/3 \end{bmatrix}, \quad s = \begin{bmatrix} 0 \\ -2/3 \end{bmatrix}. \tag{13.87}$$

The vectors appearing in Eq. (13.87) become identical to the vectors in Eq. (13.82) if we multiply the second component of each vector in Eq. (13.87) by $\sqrt{3}/2$. This amounts to labeling the second axis of Fig. 13.11(a) by $(\sqrt{3}/2Y)$ rather than Y, while still labeling the first axis by I_3. A similar argument may be used for the complex conjugate representation. Multiplying the values of hypercharge Y by $\sqrt{3}/2$ in Fig. 13.11(b) give the weights of the complex conjugate representation shown in Fig. 13.28.

Quarks thus transform according to the fundamental representation of $SU(3)$ and antiquarks transform according to the complex conjugate representation. Mesons, which are composed of a quark and an antiquark, transform according to the direct-product of these two representations of $SU(3)$. This nine-dimensional direct-product representation is reducible, which means that it is possible to find linear combinations of the quark/antiquark states that transform among themselves according to the $SU(3)$ symmetry. We have already encountered this effect in atomic physics. We have seen that angular momenta with quantum numbers j_1 and j_2 can be combined to form states with total angular momentum $J = j_1 + j_2, \ldots, |j_1 - j_2|$. Two p-electrons can combine to form states with $L = 2, 1, 0$. This result can be stated in the language of group theory. The three states of a p-electron with $l = 1$ and $m_l = 1, 0, -1$ transform among themselves under rotations and form the basis of a three-dimensional representation of the rotation group. Two p-electrons thus transform according to the direct product of two three-dimensional representations of the rotation group $R(3)$. However, this representation is reducible. It is possible to show that linear combinations of the np^2 functions transform among themselves with respect to rotations. These linear combinations correspond to states for which the electrons have total angular momentum L equal to 2, 1, or 0. The nine-dimensional representation of the rotation group due to two p-electrons decomposes into a five-dimensional, a three-dimensional, and a one-dimensional representation. In spectroscopic notation these states of p^2 are denoted by D, P, and S. In the same way, the nine-dimensional representation of $SU(3)$ corresponding to mesons decomposes into an octet corresponding to eight mesons and a singlet corresponding to a single meson.

The lightest mesons are described in Table 13.9 and Fig. 13.12. Of the spin-zero mesons in Table 13.9, the π^0, η, and η' have I_3 and Y equal to zero. The π^0 and η are members of the octet representation of $SU(3)$, while the η' with wave function

$$\psi = \frac{1}{3}\left[d\bar{d} + u\bar{u} + s\bar{s}\right] \tag{13.88}$$

is an $SU(3)$ singlet. For the spin-one mesons shown in Fig. 13.12(b), octet-singlet mixing occurs with the ω and ϕ both being mixtures of octet and singlet states.

The lightest baryons described in Tables 13.10 and 13.11 and illustrated in Fig. 13.13 all consist of three quarks and hence correspond to a reducible 27-dimensional representation of $SU(3)$. The 27-dimensional representation of $SU(3)$ corresponds to a decuplet with $J^P = \frac{3}{2}^+$, an octet with $J^P = \frac{1}{2}^+$, and one other octet and singlet; however, only the $J = 3/2$ decuplet and the $J = 1/2$ octet correspond to states whose spatial wave functions are symmetric with respect to an interchange of two quarks, and only these representations occur in nature.

The Color $SU(3)$ Symmetry

We would like now to consider the color component of the wave functions of quarks and gluons. The color component of wave functions is conveniently represented by the three spinors

$$\mathbf{r} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tag{13.89}$$

which transform according to the fundamental representation of $SU(3)$. As for the antiquarks, the anticolor states, $\bar{r}$, $\bar{g}$, and $\bar{b}$, transform according to the complex conjugate representation of $SU(3)$.

We have found that mesons, which are composed of quark/antiquark pairs, transform according to the nine-dimensional representation of $SU(3)$ which is the product of the fundamental and complex conjugate representations. This representation is reducible, being composed of an octet and a singlet representation of $SU(3)$. The singlet state of mesons is described by the wave function (13.88). Similarly, the states of gluons are made up of color/anticolor states that transform according to the octet representation of $SU(3)$. There are eight independent color states of the gluon. The linear combination of color/anticolor states which transforms according to the singlet representation of $SU(3)$ is

$$\psi = \frac{1}{3}\left[r\overline{r} + g\overline{g} + b\overline{b}\right]. \tag{13.90}$$

This state is analogs to the singlet state (13.88) of mesons.

In Section 4.2, we saw that the color component of baryon wave functions is antisymmetric with respect to an exchange of two quarks. An appropriate color wave function of this kind can be written as

$$\psi_B^C = \frac{1}{\sqrt{6}}\left[r_1 g_2 b_3 - g_1 r_2 b_3 + b_1 r_2 g_3 - b_1 g_2 r_3 + g_1 b_2 r_3 - r_1 b_2 g_3\right]. \tag{13.91}$$

Since the space-spin part of baryon wave functions is symmetric with respect to the interchange of two quarks and the color part of the wave function is antisymmetric, the total wave function of baryons is antisymmetric and thus satisfies the Pauli exclusion principle.

There is now a good deal of experimental evidence that the color $SU(3)$ symmetry is exact and differs in this way from the approximate flavor $SU(3)$ symmetry, which was found first.

13.7 * GAUGE INVARIANCE AND THE ELECTROWEAK THEORY

The most fundamental and enduring question in particle physics concerns the existence of higher symmetries. We have already considered the parity and charge conjugation symmetries and the isospin and $SU(3)$ symmetries. As we shall now show, gauge symmetries provide a general framework for understanding the interactions between elementary particles.

The concept of a gauge symmetry first arose in classical electromagnetic theory . The electric field and the magnetic field can be expressed in terms of a scalar potential V and a vector potential $\mathbf{A}$. Together the scalar potential and the vector potential form the components of a four-vector $A_\mu = (V, \mathbf{A})$.

The electric and magnetic fields can be shown to be unaffected by gauge transformations of the kind

$$A'_\mu(x) = A_\mu(x) - \frac{1}{\mathrm{e}}\partial_\mu \alpha(x), \tag{13.92}$$

where $\alpha(x)$ is a function of the spatial coordinates and ∂_μ has components

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right).$$

Maxwell's equations are invariant with respect to gauge transformations, and we might expect this symmetry to carry over to quantum mechanics.

The Dirac equation of a fermion with charge e moving in an electromagnetic field is

$$\left[\mathrm{i}\gamma^\mu(\partial_\mu + \mathrm{i}\mathrm{e}A_\mu) - k_0\right)\psi = 0, \tag{13.93}$$

where

$$k_0 = \frac{mc}{\hbar}. \tag{13.94}$$

In so-called natural units with $\hbar$ and c be equal to one, k_0 is equal to the mass of the particle. Equation (13.93) for a particle moving in an electromagnetic field can be obtained from the Dirac equation of a free particle (12.48) by making the replacement $\partial_\mu \rightarrow \partial_\mu + \mathrm{i}\mathrm{e}A_\mu$. The Dirac equation (13.93) is not invariant with respect to a gauge transformation because the wave function ψ cannot satisfy Eq. (13.93) and the analogs equation obtained by replacing A_μ by A'_μ. We must keep in mind, however, that the phase of the wave function ψ is not an observable quantity. Any transformation of the kind

$$\psi'(x) = \mathrm{e}^{\mathrm{i}\alpha(x)}\psi(x) \tag{13.95}$$

will not affect the probability density of a particle. For the Dirac equation to be invariant with respect to a local gauge transformation with an arbitary function $\alpha(x)$, the gauge transformations (13.92) must be accompanied by a transformation of the wave functions described by Eq. (13.95) so that the transformed wave function ψ' satisfies the equation

$$\left[\mathrm{i}\gamma^\mu(\partial_\mu + \mathrm{i}\mathrm{e}A'_\mu) - k_0\right)\psi' = 0, \tag{13.96}$$

To obtain this result, we first define the *covariant derivative*

$$D_\mu = \partial_\mu + \mathrm{i}\mathrm{e}A_\mu \tag{13.97}$$

and see how the product $D_\mu \psi(x)$ transforms with respect to a gauge transformation. Using Eqs. (13.92) and (13.95), we obtain

$$D'_\mu \psi' = (\partial_\mu + \mathrm{i}eA'_\mu)\mathrm{e}^{\mathrm{i}\alpha(x)}\psi(x)$$
$$= \mathrm{i}\frac{\partial\alpha(x)}{\partial x^\mu}\psi + \mathrm{e}^{\mathrm{i}\alpha(x)}\partial_\mu\psi + \mathrm{i}eA_\mu \mathrm{e}^{\mathrm{i}\alpha(x)}\psi - \mathrm{i}\frac{\partial\alpha(x)}{\partial x^\mu}\psi. \tag{13.98}$$

The first and last terms on the right-hand side of this last equation cancel, and we obtain

$$D'_\mu \psi' = \mathrm{e}^{\mathrm{i}\alpha(x)} D_\mu \psi. \tag{13.99}$$

Thus, although the space-time function, $\alpha(x)$, feels the effect of the derivatives, the terms due to the derivatives of the phase function cancel and the phase function passes through the operator D_μ allowing $D'_\mu\psi'$ to transform in the same way as the wave function ψ. The Dirac equation for a fermion in an electromagnetic field, which can be written

$$\left(\mathrm{i}\gamma^\mu D_\mu - k_0\right)\psi = 0,$$

is thus invariant with respect to a local gauge transformation.

We have thus far supposed that the wave function of a charged fermion in an electromagnetic field satisfies the Dirac equation (13.93) and then showed that the Dirac equation is invariant with respect to the gauge transformation defined by Eqs. (13.92) and (13.95). This line of argument can be reversed. One can start by demanding that the theory be invariant with respect to the phase transformation

$$\psi(x) \to \psi'(x) = \mathrm{e}^{\mathrm{i}\alpha(x)}\psi(x).$$

Such a gauge invariance is not possible for a free particle, but rather requires the existence of a vector field A^μ, which we have seen in Chapter 12 is coupled to the Dirac current j^μ. The demand of phase invariance has thus lead to the introduction of a vector field that interacts with a conserved current. Because the gauge transformation of the Dirac equation corresponds to a unitary transformation depending upon a single parameter α, this gauge symmetry is referred to as a $U(1)$ symmetry.

The standard model of the electroweak interactions is bases on the more complicated symmetry group $SU(2) \otimes U(1)$. The model contains fields, W^1_μ, W^2_μ, W^3_μ, associated with the gauge group $SU(2)$ and a field B_μ associated with the group $U(1)$. As we shall find in the next section, the electromagnetic field, A_μ, and the field associated with the neutral boson, Z^0, are linear combinations of the fields, W^3 and B_μ. Just as isospin and hypercharge can be used to describe multiplet of particles that interact by the strong interaction, weak isospin and weak hypercharge can be used to describe the quanta of the weak interaction. We shall denote the weak isospin quantum number by I_W, the azimuthal quantum number of the weak isospin by I^3_W, and weak hypercharge by Y_W. The charge of the bosons that are the carriers of the weak force are related to I^3_W and Y_W by the equation

$$Q = I^3_W + \frac{Y_W}{2}. \tag{13.100}$$

This last equation is entirely analogs to Eq. (13.39).

The quantum fields associated with the charged W^+ and W^- bosons are

$$W^+_\mu = \sqrt{\frac{1}{2}}\left(W^1_\mu - \mathrm{i}W^2_\mu\right),$$

$$W^-_\mu = \sqrt{\frac{1}{2}}\left(W^1_\mu + W^2_\mu\right).$$

The isospin quantum numbers of W^+_μ are $I_W = 1$ and $I^3_W = 1$, while the isospin quantum numbers of W^-_μ are $I_W = 1$ and $I^3_W = -1$. The field W^3_μ has $I_W = 1$ and $I^3_W = 0$ and the neutral field B_μ has $I_W = 0$ and $I^3_W = 0$.

Just as we have achieved gauge invariance for the Dirac theory, we introduce the following covariant derivative for the electroweak theory:

$$D_\mu = \partial_\mu + \mathrm{i}g_W \mathbf{W}_\mu \cdot \mathbf{T}_\mu + \frac{1}{2}\mathrm{i}Y_W g'_W B_\mu, \tag{13.101}$$

where g_W is the constant associated with the weak $SU(2)$ coupling and g'_W is the coupling constant associated with the $U(1)$ coupling. The inner product $\mathbf{T} \cdot \mathbf{W}_\mu$ that occurs in this last equation can be expanded in terms of the individual isospin matrices

$$\mathbf{W}_\mu \cdot \mathbf{T} = T^3 W_\mu^3 + \frac{1}{\sqrt{2}} W_\mu^+ T^+ + \frac{1}{\sqrt{2}} W_\mu^- T^-. \tag{13.102}$$

The form of the isospin T matrices depends upon the spin of the states on which they act. For example, in the doublet representation of $SU(2)$ we have

$$T^3 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}, \quad T^+ = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad T^- = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}. \tag{13.103}$$

While the electroweak theory with the covariant derivative (13.101) describes the electromagnetic and weak interactions in a unified way, the theory can be shown to be gauge invariant only if the vector bosons have zero mass. The theory cannot account for the observed experimental data because the photon is the only carrier of the electroweak force with zero mass. This difficulty will be overcome in the next section in which we describe the Higgs mechanism of spontaneous symmetry breaking which maintains the gauge invariance of the theory while giving mass to all but one of the vector bosons.

13.8 SPONTANEOUS SYMMETRY BREAKING AND THE DISCOVERY OF THE HIGGS

We begin this section by considering a possibility. Suppose that a field has a gauge symmetry but the ground state of the system—the vacuum state—does not have the symmetry of the field. Like a pencil standing on its end, the state of the system will eventually fall into another state for which the potential energy of the system has a local minimum. To be concrete, we consider the potential energy function, $V(\phi_1, \phi_2)$, illustrated in Fig. 13.30. This function—popularly described as a "Mexican Sombrero"—has a maximum in the center and a circular trough around the center. A particle moving in this potential could fall arbitrarily in any direction and find itself in this circular trough—a distance v from the origin. Since a motion of the particle around the trough would cost no energy, this direction would be associated with a quantum of the field with zero mass, while motion in the other directions would be associated with massive field quanta.

One can imagine that in the first instant after the big bang—the first 10^{-35} or 10^{-45} s—the Universe was in a pure state in which the vector bosons which are the carriers of the weak interaction had zero mass. And then the universe fell from grace. The symmetry of the Universe was spontaneously broken. The photon and the Higgs boson were born and the W^+, W^-, and Z^0 bosons, which are the carriers of the weak force, acquired a mass. We now want to find the mass of the vector bosons after this primordial symmetry was broken.

In the previous section, we found that the equations of the quantum fields associated with the weak interaction could be made gauge invariant if we replaced ordinary partial derivatives in the field equations by the covariant derivative (13.101). In the following, we shall write the expression for the appropriate covariant derivative

$$D_\mu = \partial_\mu + G, \tag{13.104}$$

where G is equal to

$$G = \mathrm{i} g_W \mathbf{W}_\mu \cdot \mathbf{T} + \frac{1}{2} \mathrm{i} Y_W g'_W B_\mu. \tag{13.105}$$

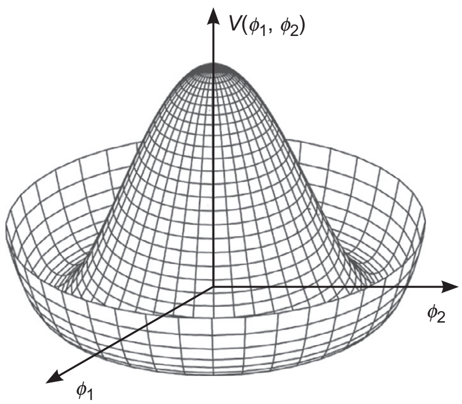

FIGURE 13.30 The potential energy function associated with the Higgs boson.

After the gauge symmetry is spontaneously broken, the state of the Higgs field ϕ will have fallen into a local minimum in the trough surrounding the central bulge in Fig. 13.30. Which direction the state vector falls to is entirely arbitrary. The state of the Higgs field can then be represented by the vector

$$\phi_0 = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\v\end{bmatrix}. \tag{13.106}$$

The zero that occurs in the first component of this vector may be identified as the charge of the vacuum state while v in the second component is the distance from the local minimum to the origin shown in Fig. 13.30.

The kinetic energy in quantum field theory is associated with products of partial derivatives just as in nonrelativistic quantum mechanics, while the mass of particles in quantum field theory is associated with quadratic expressions of the fields. We shall now form the product of G and ϕ_0. The masses of the vector bosons can be obtained by taking the square of the norm of $G\phi_0$ and identify the coefficients of squares of quantum fields in the resulting expression as the masses of the corresponding particles. The masses of the vector bosons being non-zero depends upon the fact that the vacuum expectation value of the Higgs field is not equal to zero.

We first use Eqs. (13.105) and (13.106) to form the product of G and the Higgs field

$$G\phi_0 = \frac{1}{\sqrt{2}}\left(ig_W\mathbf{T}\cdot\mathbf{W}_\mu + ig'_W B_\mu\right)\begin{bmatrix}0\\v\end{bmatrix}, \tag{13.107}$$

where we have taken the weak hypercharge $Y_W = 1$. The right-hand side of this last equation can be simplified using Eqs. (13.102) and (13.103) to obtain

$$G\phi_0 = -\frac{ig_W W_\mu^3}{2\sqrt{2}}\begin{bmatrix}0\\v\end{bmatrix} + \frac{ig_W W_\mu^+}{2}\begin{bmatrix}v\\0\end{bmatrix} + \frac{ig'_W B_\mu}{2\sqrt{2}}\begin{bmatrix}0\\v\end{bmatrix} \tag{13.108}$$

The square of the norm of a vector can be obtained by taking the adjoint of the vector and then multiplying the adjoint times the vector itself. The adjoint of $G\phi_0$ is

$$(G\phi_0)^\dagger = \frac{ig_W W_\mu^3}{2\sqrt{2}}\begin{bmatrix}0 & v\end{bmatrix} - \frac{ig_W W_\mu^-}{2}\begin{bmatrix}v & 0\end{bmatrix} - \frac{ig'_W B_\mu}{2\sqrt{2}}\begin{bmatrix}0 & v\end{bmatrix} \tag{13.109}$$

Multiplying Eqs. (13.109) and (13.108), we obtain

$$||G\phi_0||^2 = \frac{v^2}{8}\left[(g_W W_\mu^3 - g'_W B_\mu)(g_W W_\mu^3 - g'_W B_\mu) + 2g_W^2 W_\mu^- W_\mu^+\right]. \tag{13.110}$$

The above expression for the square of the norm of $G\phi_0$ contains products of different fields; however, the quadratic form may transformed into a quadratic expression containing the sum of squares of individual fields by making the transformations

$$W_\mu^3 = \cos\theta_W Z_\mu + \sin\theta_W A_\mu \tag{13.111}$$

$$B_\mu = -\sin\theta_W Z_\mu + \cos\theta_W A_\mu, \tag{13.112}$$

where the angle θ_W (the Weinberg or electroweak mixing angle) is defined by the relative strengths of the coupling constants

$$\sin^2\theta_W = \frac{g'^2_W}{g_W^2 + g'^2_W}. \tag{13.113}$$

The transformation defined by Eqs. (13.111) and (13.112) expresses the fields, W_μ^3 and B_μ, having definite values of the isospin quantum numbers in terms of the physical Z_μ and A_μ fields. After these transformations, the quadratic terms in the expression for $||G\phi_0||^2$ becomes

$$||G\phi_0||_S^2 = \frac{g_W^2 v^2}{4} W_\mu^+ W_\mu^- + \frac{(g_W^2 + g'^2_W)v^2}{4} Z_\mu Z_\mu \tag{13.114}$$

We therefore find that the W and Z bosons have acquired masses given by the formulas

$$m_W = \frac{1}{2} v g_W \tag{13.115}$$

$$m_Z = \frac{1}{2} v\sqrt{g_W^2 + g'^2_W} = \frac{M_W}{\cos\theta_W}. \tag{13.116}$$

The photon remains massless because there are no terms quadratic in the field A_μ in Eq. (13.114).

The Higgs field ϕ thus acquired a non-zero vacuum expectation value at a particular point on a circle of minima away from $\phi = 0$ and the gauge symmetry was broken. As a result, the three vector mesons, W^+, W^-, and Z, acquired a mass but the photon remained massless. The Higgs boson itself acquired a mass but the gauge theory does not provide any information about the mass of the Higgs. The interaction of the Higgs boson with other particles depends upon the mass of the particle with which the Higgs interacts. Its interaction is greatest with the massive top quark and with the W^+, W^-, and Z^0 bosons.

Robert Brout, Peter Higgs, and Francois Englert presented their theory of spontaneous symmetry breaking in 1964 and the particle physics community has devoted much of its resources in the intervening years trying to discover the Higgs boson. The existence of the Higgs would confirm the theory of spontaneous symmetry breaking that accounts for the masses of the W and Z. Other theories have been formulated to describe the masses of the vector bosons, but none of the other theories are as simple or as beautiful as the Higgs mechanism.

The first attempts to find the Higgs boson were carried out at the large electron-positron (LEP) collider. As described in Section 13.1 of this chapter, a short-lived resonance state such as the Higgs can be identified as a local maxima of an invariant mass distribution curve. While early experiments at LEP and LEP II were not able to identify a Higgs resonance, these experiments did establish a lower limit of 114 GeV for the Higgs mass. A very massive Higgs boson should not produce a narrow, easily identifiable resonance state.

The opening of the Large Hadron Collider (LHC) at the CERN laboratory near Geneva in Switzerland in 2010 offered the possibility of performing experiment at very high energies. The CERN collider, which is shown in Fig. 13.31, produces a colliding beam of protons. The collider should be able to identify a Higgs between the lower limit of 114 GeV provided by the LEP II experiments extending up to around 800 Mev.

A central difficulty in the search for the Higgs boson has been that the possible mechanisms for producing the Higgs and the possible decay mechanisms all depend upon the mass of the Higgs which was entirely unknown. At an energy of a few hundred gigaelectron-volt, the most important production mechanism is the so-called gluon-gluon fusion, which is described by the Feynman diagram in Fig. 13.32(a). In this process, two gluons produced by the proton beams interact with top quarks which then produce a Higgs. The next most important processes are the boson fusion processes depicted in Fig. 13.32(b) and (c). In both of these processes, top quarks produced by the interacting protons produce W or Z bosons which then produce the Higgs. On July 4, 2012, the CERN laboratory announce that it had identified a resonance state at

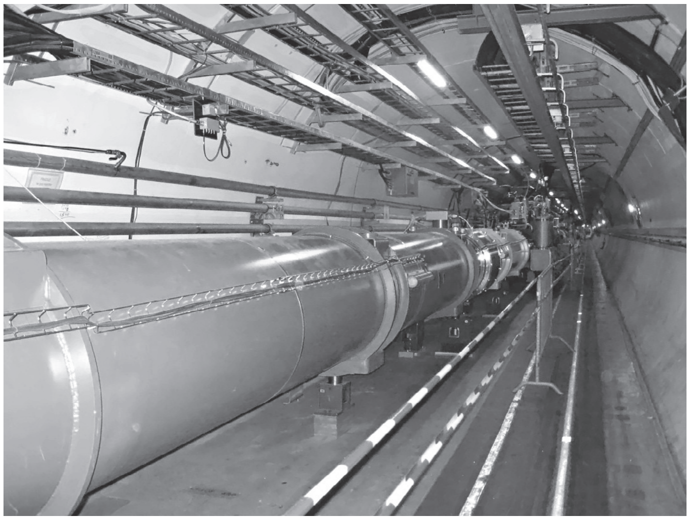

Photo credit:
"CERN LHC Tunnel1" by Julian Herzog | Wikimedia Commons

Description:
Tunnel of the Large Hadron Collider (LHC) of the European Organization for Nuclear Research (French: Organisation européenne pour la recherche nucléaire, known as CERN) with all the Magnets and Instruments. The shown part of the tunnel is located under the LHC P8, near the LHCb.

FIGURE 13.31 The tunnel of the Large Hadron Collider (LHC) of the European Organization for Nuclear Research (CERN) near Geneva, Switzerland. The photo is due to Julian Herzog, Wikipedia Commons.

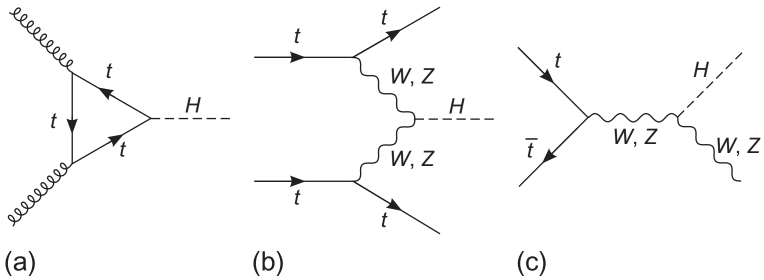

FIGURE 13.32 The most important processes for the production of the Higgs.

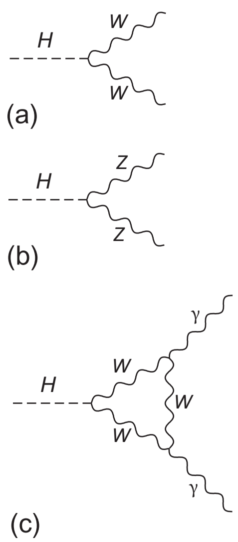

FIGURE 13.33 The most important processes for the decay of the Higgs.

125 GeV which could be the Higgs. Further analysis since performed at CERN has verified the presence of a resonance at this energy and have studied the decay channels of the resonance.

As shown by the Feyman diagrams in Fig. 13.33(a) and (b), the Higgs can decay into W^+ and W^- bosons or into two Z bosons. The decay into W^+ and W^- should occur more often; however, all of the decay modes of W^+ and W^- bosons involve neutrinos which are difficult to detect. The decay of the Higgs by producing two Z bosons is more promising because Z bosons decay into two leptons that can be easily detected. Another important decay process is described by the Feynman diagram shown in Fig. 13.32(c). In higher-order processes of this kind, which can include the top quark, the decay of the Higgs produces a pair of gamma rays.

Since the discovery of a Higgs-like resonance was announced by CERN in July of 2012, the ATLAS and CMS collaborations at CERN have succeeded in establishing that the resonance has a spin zero and even parity—as the Higgs should. They have also found that the resonance decays producing four leptons and two gamma rays. There can be little doubt that the resonance found at CERN is the particle postulated by Higgs and his collaborators so long ago. Peter Higgs and Francois Englert received the Nobel Prize in 2013 for conceiving of spontaneous symmetry breaking and the Higgs boson. We should note though that the breaking of the gauge symmetry by a single scalar field is the simplest possible way the gauge symmetry could be broken. It is unlikely that the discovery of the Higgs resonance on July 4, 2012 will be the last act in the Higgs drama.

SUGGESTION FOR FURTHER READING

Martin, B.R., Shaw, G., 1997. Particle Physics, second ed. Wiley: Chichester, England.

Perkins, D.H., 1995. Introduction to High Energy Physics, fourth ed. Benjamin/Cummings: Menlo Park, California.

Halzen, F., Martin, A.D., 1984. Quarks and Leptons: An Introductory Course in Modern Particle Physics. Wiley: Chichester, England.

Ellis, R.K., Stirling, W.J., Webber, B.R., 2003. QCD and Collider Physics. Cambridge, England.

Georgi, H., Lie Algebras in Particle Physics: From Isospin to Unified Theories, second ed. Westview Press: Boulder, Colorado.

BASIC EQUATIONS

Leptons and Quarks

Generations of Leptons

$$\begin{bmatrix} \nu_{\mathrm{e}} \\ \mathrm{e}^- \end{bmatrix}, \quad \begin{bmatrix} \nu_\mu \\ \mu^- \end{bmatrix}, \quad \begin{bmatrix} \nu_\tau \\ \tau^- \end{bmatrix}$$

Generations of Quarks

$$\begin{bmatrix} u \\ d \end{bmatrix}, \quad \begin{bmatrix} c \\ s \end{bmatrix}, \quad \begin{bmatrix} t \\ b \end{bmatrix}$$

Definition of Hypercharge and Isospin

Hypercharge

$$Y = B + S + C + \tilde{B} + T,$$

where B, S, C, $\tilde{B}$, and T are the baryon number, strangeness, charm, beauty, and truth, respectively.

Isospin

$$I_3 = Q - Y/2$$

For every isospin multiplet, I is the maximum value of I_3.

Feynman Diagrams

Definition of Flux

$$\phi_i = n_i v_i.$$

Golden Rule

$$W = \frac{2\pi}{\hbar} |M_{if}|^2 \rho_{\mathrm{f}}$$

Scattering Cross Section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{2\pi}{\hbar} \frac{|M_{if}|^2}{\phi_i} \frac{V}{(2\pi\hbar)^3} p_f^2 \frac{\mathrm{d}p_f}{\mathrm{d}E},$$

where $p_f = |\mathbf{p}_f|$ is the momentum of the final-state particle.

$SU(3)$ Symmetry

Diagonal Generators

$$[H_1, H_2] = 0.$$

Equations Defining Roots

$$[\mathbf{H}_1, \mathbf{E}_{\alpha_1}] = [\alpha_1]_1 \, \mathbf{E}_{\alpha_1},$$

$$[\mathbf{H}_2, \mathbf{E}_{\alpha_1}] = [\alpha_1]_2 \, \mathbf{E}_{\alpha_1}$$

Equations Defining Weights

$$H_i \mathbf{v}_i = w(v_i) \mathbf{v}_i$$

SUMMARY

All matter is composed of leptons, quarks, and elementary particles called bosons, which serve as the carriers of the force between particles. The lepton family includes the electron, muon, tau, and their neutrinos. Quarks come in six types, called flavors, denoted by up (u), down (d), strange (s), charmed (c), bottom (b), and top (t) quarks. All interactions conserve the lepton numbers and the number of baryons (B). The weak interaction can change the flavor of a quark and thus violate the conservation laws associated with the quark quantum numbers (strangeness, charm, beauty, and truth). There are two general kinds of strongly interacting particles: mesons, which are composed of quark/antiquark pairs, and baryons, which consist of three quarks.

The strongly interacting particles occur in isospin multiplets with all members of a multiplet having approximately the same mass. The members of an isospin multiplet all have the same value of the hypercharge Y, which is the sum of B, S, C, $\tilde{B}$, and T. The members of each multiplet are distinguished by the charge Q and I_3. Particles can also be grouped in larger families of particles called supermultiplets. The lightest mesons consist of two nonets with each consisting of nine mesons, while the lightest baryons consist of an octet with eight baryons and a decuplet with ten baryons.

The interactions between leptons and quarks are described by Feynman diagrams. These diagrams describe processes by which particles interact by exchanging quanta of the interaction fields. Recent work on gauge symmetries provide a general framework for understanding the interactions between elementary particles. There are gauge symmetries associated with the electroweak and strong interactions. Associated with each gauge symmetry are gauge fields and gauge bosons which serve as carriers of the interactions. The photon and the W^+, W^-, and Z^0 are the gauge particles associated with the electroweak interaction, while gluons serve as the gauge particles of the strong interaction.

QUESTIONS

1. Give the names of the particles corresponding to the first and second generations of leptons.
2. Give the names of the particles corresponding to the first and second generations of quarks.
3. Which family of particles interacts by means of the weak and electromagnetic forces but not by means of the strong force?
4. What evidence is there for the quark model?
5. Which particles serve as the carriers of the weak force?
6. Suppose that two particles produced in a scattering experiment have energy and momentum, E_1, $\mathbf{p}_1$ and E_2, $\mathbf{p}_2$, respectively. Write down a formula for the invariant mass of the two particles.
7. Which of the mesons we have encountered have a single strange quark or a single strange antiquark?
8. Which of the baryons we have encountered have two strange quarks?
9. The μ^- decays into e^- and two other particles. Using the conservation of electron and muon lepton numbers determine the identity of the other particles.
10. The μ^+ decays into e^+ and two other particles. Using the conservation of electron and muon lepton numbers determine the identity of the other particles.
11. Which conservation law is violated by the decay process $n \rightarrow \pi^+ + \pi^-$?
12. What quantum number can be assigned to particles that are produced by the strong interaction but decay by the weak interaction?
13. How is the spin of a composite particle defined?
14. What property of the decay of cobalt-60 nuclei by the reaction $^{60}\text{Co} \rightarrow {}^{60}\text{Ni}^* + e^- + \overline{\nu_e}$ showed that parity was violated?
15. What does it mean to say that the μ^+ has negative helicity?
16. How is the hypercharge Y related to the baryon number (B), strangeness (S), charm (C), beauty ($\tilde{B}$), and truth (T)?
17. Write down an equation relating I_3 to Q and Y.
18. What are the possible values of the isospin I of the baryon resonances produced in pion-nucleon scattering?
19. What are the possible values of the isospin I of the meson resonances produced in pion-nucleon scattering?
20. Draw a Feynman diagram for Compton scattering.
21. Draw a Feynman diagram for μ^- decay.
22. Draw an acceptable Feynman vertex involving a lepton l^- and the Z^0.
23. Draw an acceptable Feynman vertex involving quarks of the second generation and the W^-.
24. Draw a Feynman diagram in which a $\overline{u}/u$ pair is absorbed giving a gluon that produces a $\overline{s}/s$ pair.
25. Draw the root diagram for $SU(3)$.

26. Draw a diagram showing the weights of the fundamental representation of $SU(3)$.
27. Make a drawing showing how the root α_2 and the weights $w(\mathbf{v}_2)$ and $w(\mathbf{v}_3)$ of the fundamental representation of $SU(3)$ are related.
28. Which gauge symmetry is associated with the electroweak theory?
29. What causes the symmetry the Higgs field to be spontaneously broken?

PROBLEMS

1. State whether each of the following processes can occur. If the reaction cannot occur or if it can only occur by the weak interaction, state which conservation law is violated.

$$\begin{array}{lrl}
\text{(a)} & p & \to \mathrm{e}^+ + \gamma \\
\text{(b)} & \Sigma^0 & \to \Lambda + \pi^0 \\
\text{(c)} & \Delta^+ & \to \mathrm{p} + \pi^0 \\
\text{(d)} & \pi^+ & \to \mu^+ + \gamma \\
\text{(e)} & K^- & \to \pi^- + \pi^0 \\
\text{(f)} & \rho^0 & \to \pi^0 + \pi^0 \\
\text{(g)} & K^+ & \to \pi^+ + \gamma
\end{array}$$

2. State whether each of the following processes can occur. If the reaction cannot occur or if it can only occur by the weak interaction, state which conservation law is violated.

$$\begin{array}{lrl}
\text{(a)} & \mathrm{p} + K^- & \to \Sigma^+ + \pi^- + \pi^+ + \pi^- + \pi^0 \\
\text{(b)} & \mathrm{e}^+ + \mathrm{e}^- & \to \nu + \overline{\nu} \quad \text{(give neutrino type)} \\
\text{(c)} & \nu_\mu + \mathrm{p} & \to \mu^+ + \mathrm{n} \\
\text{(d)} & \nu_\mu + \mathrm{p} & \to \mu^- + \mathrm{n} + \pi^+ \\
\text{(e)} & \nu_\mu + \mathrm{e}^- & \to \mu^- + \nu_\mathrm{e} \\
\text{(f)} & \nu_\mathrm{e} + \mathrm{p} & \to \mathrm{e}^+ + \Sigma^0 + K^0 \\
\text{(g)} & K^0 + \mathrm{p} & \to \Lambda + \pi^+ + K^- + \pi^+
\end{array}$$

3. Find the total rest mass energy of the initial and final states of each of the following reactions. For which of the following reactions must the kinetic energy of the initial state be greater than the kinetic energy of the final state?

$$\begin{array}{lrl}
\text{(a)} & \pi^- + \mathrm{p} & \to \mathrm{n} + \pi^0 \\
\text{(b)} & \Lambda & \to \mathrm{p} + \pi^0 \\
\text{(c)} & \pi^+ + \mathrm{p} & \to K^+ + \Sigma^+
\end{array}$$

4. For which of the following reactions is the conservation law of electron lepton number violated? Explain any violation of the laws that occur.

$$\begin{array}{lrl}
\text{(a)} & \mu^- & \to \mathrm{e}^- + \overline{\nu_\mathrm{e}} + \nu_\mu \\
\text{(b)} & \nu_\mathrm{e} + \mathrm{p} & \to \mathrm{n} + \mathrm{e}^+ \\
\text{(c)} & \mathrm{n} & \to \mathrm{p} + \mathrm{e}^- + \nu_\mathrm{e} \\
\text{(d)} & \mathrm{p} & \to \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}
\end{array}$$

5. Determine the change in strangeness for each of the following reactions, and state whether the reaction can occur by the strong, electromagnetic, or weak interactions. Give reasons for your answers.

$$\begin{array}{lrl}
\text{(a)} & \Omega^- & \to \Lambda + K^- \\
\text{(b)} & \Sigma^0 & \to \Lambda + \gamma \\
\text{(c)} & \Lambda & \to \mathrm{p} + \pi^-
\end{array}$$

6. Determine the change in strangeness for each of the following reactions, and state whether the reaction can occur by the strong, electromagnetic, or weak interactions. Give reasons for your answers.

$$\begin{array}{lrl}
\text{(a)} & \Xi^0 & \to \Lambda + \pi^0 \\
\text{(b)} & \Sigma^- & \to \Lambda + \mathrm{e}^- + \overline{\nu_\mathrm{e}} \\
\text{(c)} & K^- & \to \pi^- + \pi^0
\end{array}$$

7. For each of the following decay processes, state which are strictly forbidden, which are weak, electromagnetic, or strong. Give reasons for your answers.

$$\begin{array}{lrl} \text{(a)} & p+\overline{p} & \rightarrow \pi^{+}+\pi^{-}+\pi^{0}+\pi^{+}+\pi^{-} \\ \text{(b)} & \pi^{-}+p & \rightarrow p+K^{-} \\ \text{(c)} & \pi^{-}+p & \rightarrow \Lambda+\overline{\Sigma^{0}} \\ \text{(d)} & \overline{\nu_{\mu}}+p & \rightarrow e^{+}+n \\ \text{(e)} & \nu_{e}+p & \rightarrow e^{+}+\Lambda+K^{0} \end{array}$$

8. Give the quark composition of the particles in the following processes and describe how the quark composition of the final state differs from the quark composition of the initial state.

$$\begin{array}{lrl} \text{(a)} & \Lambda & \rightarrow p+\pi^{0} \\ \text{(b)} & \pi^{-}+p & \rightarrow n+\pi^{0} \\ \text{(c)} & \pi^{+}+p & \rightarrow K^{+}+\Sigma^{+} \end{array}$$

9. Give the quark composition of the particles in the following processes and describe how the quark composition of the final state differs from the quark composition of the initial state.

$$\begin{array}{lrl} \text{(a)} & \Omega^{-} & \rightarrow \Lambda+K^{-} \\ \text{(b)} & \pi^{-}+p & \rightarrow \Lambda+K^{0} \\ \text{(c)} & p+K^{-} & \rightarrow \Xi^{-}+K^{+} \end{array}$$

10. For each of the following processes, give the isospin quantum numbers of the particles involved in the collision and the quantum number I_3 of the total isospin. Give the possible values of I and I_3 for the final state and give an example of what the final state might be.

$$\begin{array}{lr} \text{(a)} & \pi^{-}+p \\ \text{(b)} & \pi^{-}+n \\ \text{(c)} & \pi^{+}+p \\ \text{(d)} & n+n \\ \text{(e)} & n+p \end{array}$$

11. Suppose that a wave with wavelength λ satisfies periodic boundary conditions at the two ends of a large region of length L. Using the deBroglie relation, show that the momentum of the particle has the possible values, $n2\pi\hbar/L$, where n is an integer, and the number of states with momentum in the range between p and $p+dp$ is $(L/2\pi\hbar)dp$. Confirm the validity of Eq. (13.51) by generalizing this result to three dimensions.

12. Give the quark composition of the particles in the following processes and draw the Feynman diagram giving the cosine or sine of the Cabibbo angle where appropriate.

$$\begin{array}{lrl} \text{(a)} & \Lambda & \rightarrow \pi^{-}+p \\ \text{(b)} & n & \rightarrow p+e^{-}+\overline{\nu}_{e} \\ \text{(c)} & \Xi^{0}- & \rightarrow \Sigma^{-}+e^{+}+\nu_{e} \\ \text{(d)} & \Omega^{-} & \rightarrow \Xi^{0}+e^{-}+\overline{\nu}_{e} \end{array}$$

13. Give the weights of the $(3,0)$ representation of $SU(3)$ showing how each of the weights can be obtained from the highest weight by a translation by one or more of the root vectors.