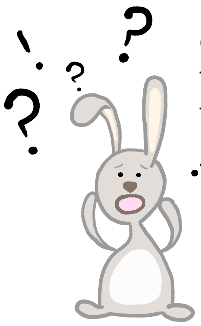
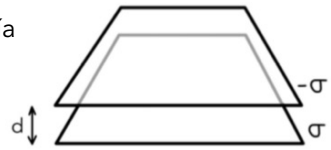
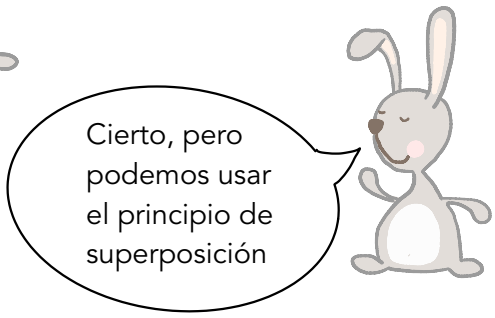
## Superposición de campos



Esta distribución de cargas-fuente no tiene una geometría tan sencilla!




**Ejercicio 7:** Se disponen dos planos infinitos, paralelos, separados por una distancia $d$, con distribuciones de carga superficial uniformes $\sigma$ y $-\sigma$, respectivamente.

i. Dibuje las líneas de campo eléctrico generadas por cada plano separadamente, y por el conjunto, en todo el espacio.
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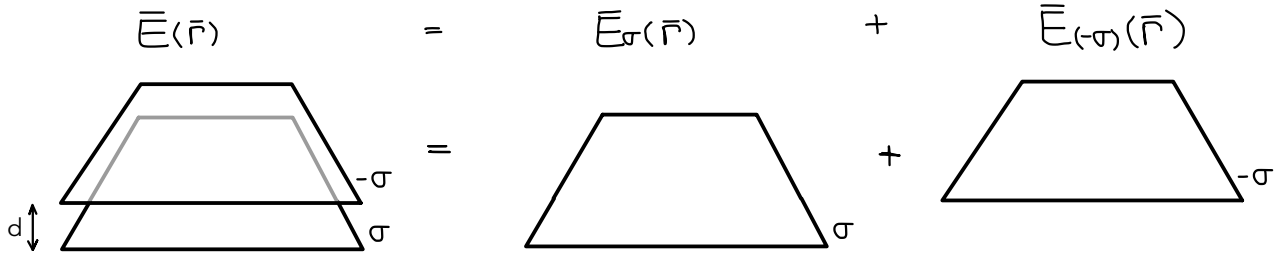
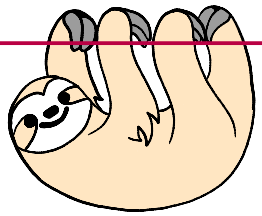
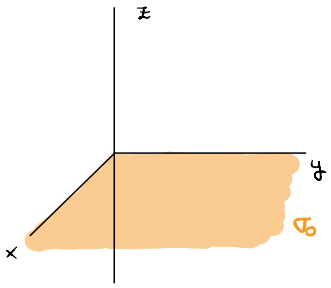


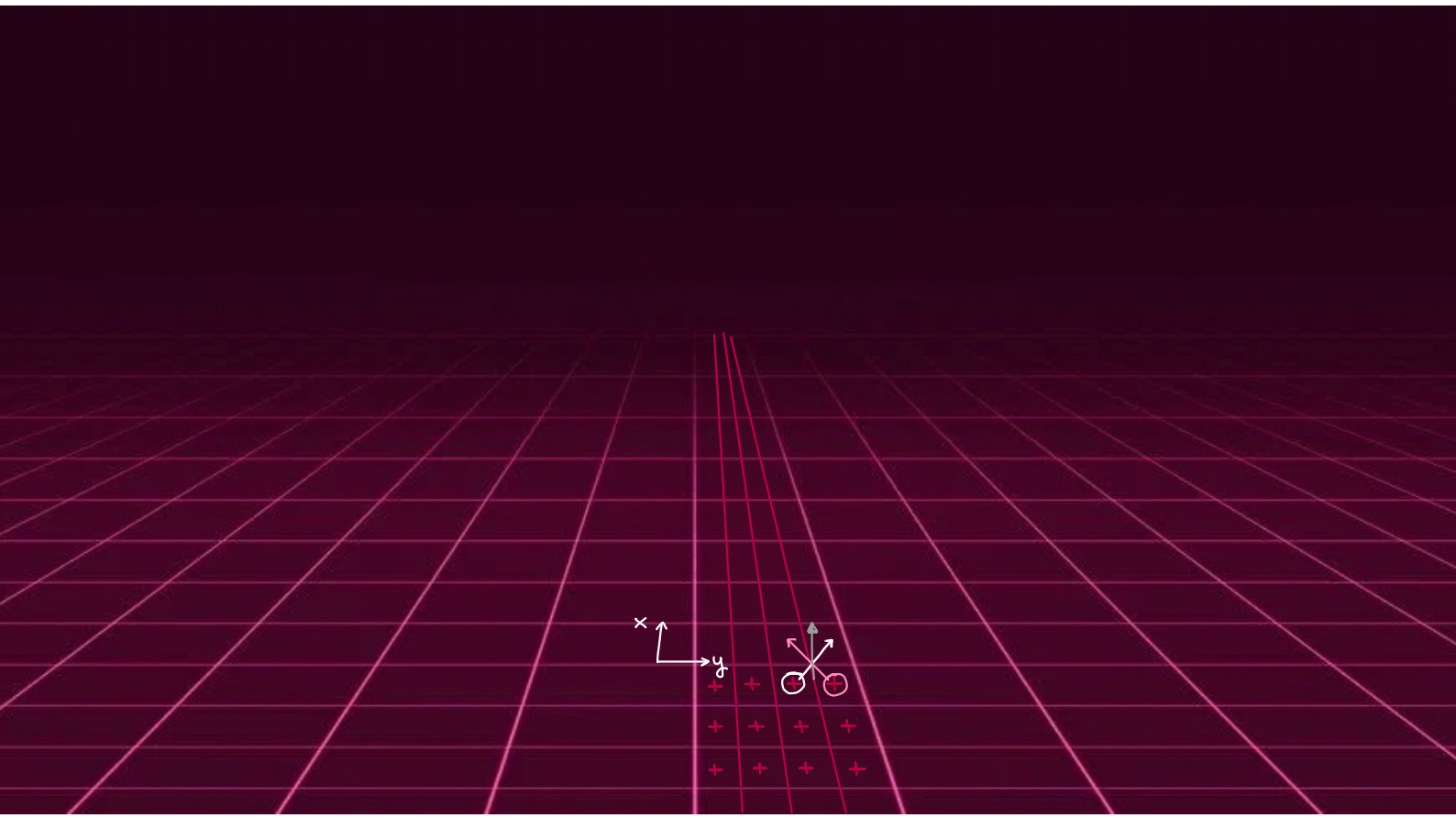

**AUXILIAR:** Campo generado por un plano infinito cargado ubicado en z=0





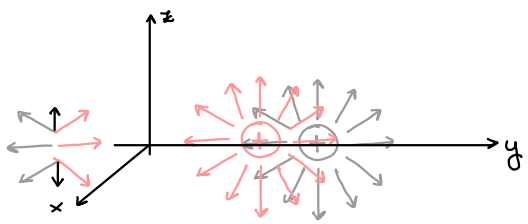

$$\bar{E}(\bar{r}) = E_x(x,y,z)\,\hat{x} + E_y(x,y,z)\,\hat{y} + E_z(x,y,z)\,\hat{z}$$

$$\bar{E}(\bar{r}) = E_x(\cancel{x,y},z)\,\hat{x} + E_y(\cancel{x,y},z)\,\hat{y} + E_z(\cancel{x,y},z)\,\hat{z}$$



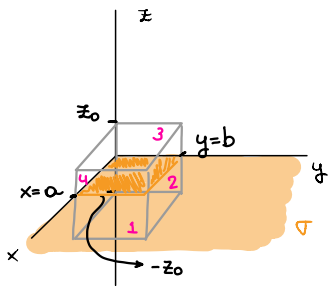

$$\bar{E}(\bar{r}) = \cancel{E_x(z)\,\hat{x}} + \cancel{E_y(z)\,\hat{y}} + E_z(z)\,\hat{z}$$

$$\bar{E}(\bar{r}) = E_z(z)\,\hat{z}$$



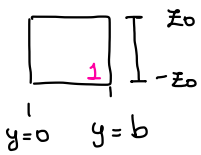
## Ley deGauss

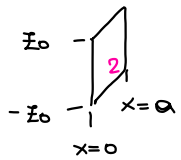
$$\oint_{\partial V} \bar{E}(\bar{r}) \cdot d\vec{S} = 4\pi k\, Q_{enc} \quad ; \quad \bar{E}(\bar{r}) = E_z(z)\,\hat{z}$$



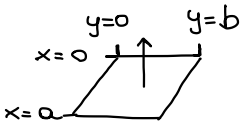

Se anulan porque la superficie es paralela al campo
$\hat{z}\cdot\hat{x}=0$ ; $\hat{z}\cdot\hat{y}=0$

$$\oint_{\partial V} \bar{E}(\bar{r})\, d\vec{S} = \cancel{\int_{-z_0}^{z_0}\int_0^b E_z(z)\,\hat{z}\cdot\left(dy\,dz\,\hat{x}\right)} + \cancel{\int_{-z_0}^{z_0}\int_0^a E_z(z)\,\hat{z}\cdot\left(dx\,dz\,\hat{y}\right)} + \int_{S_3} \bar{E}(\bar{r})\cdot d\vec{S}_3$$




$$+ \int_{S_4} \bar{E}(\bar{r})\cdot d\vec{S}_4 + \int_0^a\int_0^b E_z(z_0)\,\hat{z}\,\left(dx\,dy\,\hat{z}\right) + \int_0^a\int_0^b E_z(-z_0)\,\hat{z}\,\left(dx\,dy\,(-\hat{z})\right)$$

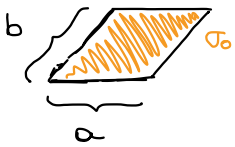



$$\oint_{\partial V} \bar{E}(\bar{r})\, d\vec{S} = E_z(z_0)\int_0^a\int_0^b dx\,dy\,\underbrace{\left(\hat{z}\cdot\hat{z}\right)}_{=1} + E_z(-z_0)\int_0^a\int_0^b dx\,dy\,\underbrace{\left(\hat{z}\cdot(-\hat{z})\right)}_{=-1}$$

$$= E_z(z_0)\left(\int_0^a dx\right)\left(\int_0^b dy\right) - E_z(-z_0)\left(\int_0^a dx\right)\left(\int_0^b dy\right)$$

$$= E_z(z_0)\, a\cdot b - E_z(-z_0)\, ab \qquad E_z(-z_0) = -E_z(z_0)$$

$$= 2ab\, E_z(z_0)$$




$$Q_{enc} = \sigma_0 \cdot \underset{\text{encerrada}}{\text{Sup plano}} = \sigma_0\, ab$$

## Ley deGauss

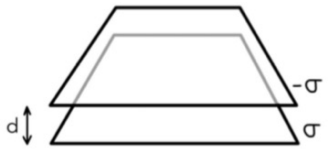
$$\oint_{\partial V} \bar{E}(\bar{r})\cdot d\vec{S} = 4\pi k\, Q_{enc}$$

$$2\cancel{ab}\, E_z(z_0) = 4\pi k\, \sigma_0\, \cancel{ab}$$

$$E_z(z_0) = \frac{4\pi k\, \sigma_0}{2}$$

$$\bar{E}(\bar{r}) = \begin{cases} 2\pi k\sigma_0\,\hat{z} & z>0 \\ -2\pi k\sigma_0\,\hat{z} & z<0 \end{cases}$$

## Superposición de campos

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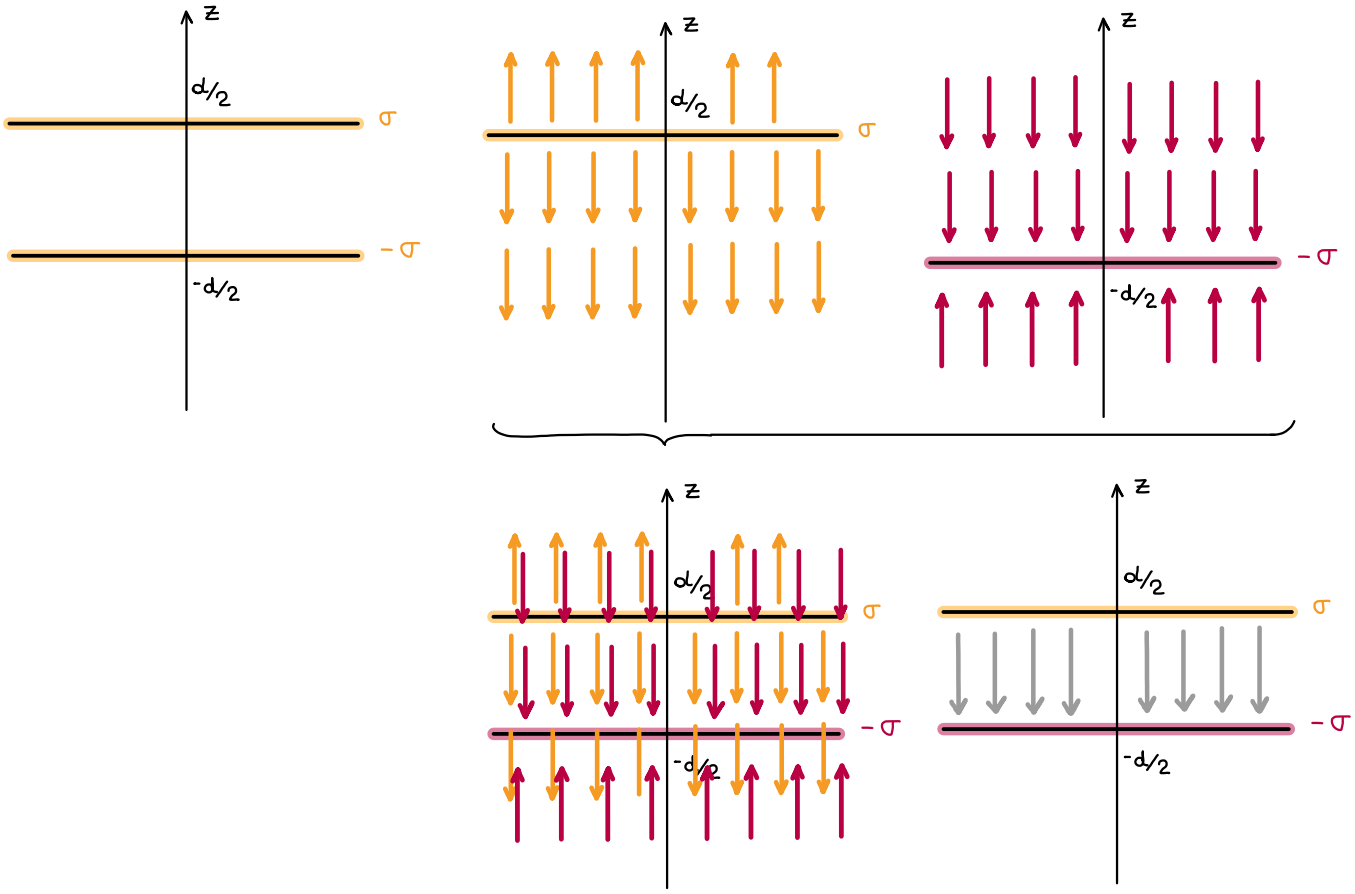
i)




ii) Por el principio de superposición sabemos que

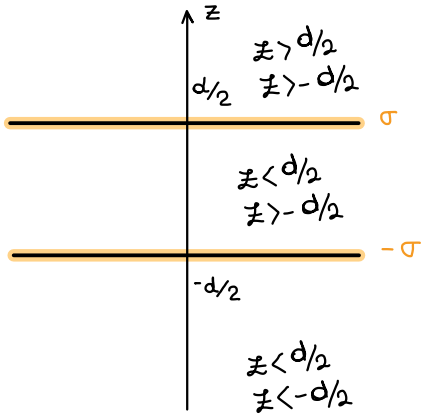
$$\bar{E}(\bar{r}) = \bar{E}_{\sigma}(\bar{r}) + \bar{E}_{(-\sigma)}(\bar{r})$$

Pero ya conocemos el campo generado por un plano infinito cargado

$$\bar{E}_{plano}(\bar{r}) = \begin{cases} 2k\pi\sigma_{plano}\,\hat{z} & si \quad z > z_{plano} \\ -2k\pi\sigma_{plano}\,\hat{z} & si \quad z < z_{plano} \end{cases} \qquad \bar{E}(\bar{r}) = \begin{cases} 2\pi k\sigma_0\,\hat{z} & z > z_{plano} \\ -2\pi k\sigma_0\,\hat{z} & z < z_{plano} \end{cases}$$

$$\bar{E}_{\sigma}(\bar{r}) = \begin{cases} 2k\pi\sigma & \hat{z} \text{ si } z > d/2 \\ -2k\pi\sigma & \hat{z} \text{ si } z < d/2 \end{cases}$$

$$\bar{E}_{(-\sigma)}(\bar{r}) = \begin{cases} -2k\pi\sigma & \hat{z} \text{ si } z > -d/2 \\ 2k\pi\sigma & \hat{z} \text{ si } z < -d/2 \end{cases}$$




$$\bar{E}(\bar{r}) = \bar{E}_{\sigma}(\bar{r}) + \bar{E}_{(-\sigma)}(\bar{r})$$

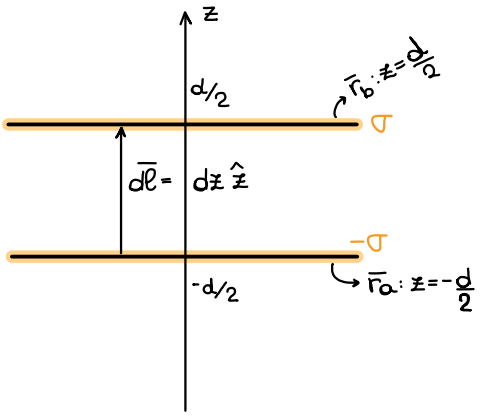
$$\begin{cases} 2k\pi\sigma & \hat{z} \text{ si } z > d/2 \\ -2k\pi\sigma & \hat{z} \text{ si } z < d/2 \end{cases} \qquad \begin{cases} -2k\pi\sigma & \hat{z} \text{ si } z > -d/2 \\ 2k\pi\sigma & \hat{z} \text{ si } z < -d/2 \end{cases}$$

$$\begin{cases} 2k\pi\sigma\,\hat{z} - 2k\pi\sigma\,\hat{z} & d/2 < z \\ -2k\pi\sigma\,\hat{z} - 2k\pi\sigma\,\hat{z} & -d/2 < z < d/2 \\ -2k\pi\sigma\,\hat{z} + 2k\pi\sigma\,\hat{z} & z < -d/2 \end{cases} \qquad (0,0,2k\pi\sigma) + (0,0,-2k\pi\sigma)$$

$$\bar{E}(\bar{r}) = \begin{cases} 0 & d/2 < z \\ -4\pi k\sigma\,\hat{z} & -d/2 < z < d/2 \\ 0 & z < -d/2 \end{cases}$$

iv) $\bar{E}(\bar{r}) = -\bar{\nabla}\Phi(\bar{r})$

$$\underbrace{\Delta\Phi_{ab}}_{\Delta\varphi_{ab}} = -\int_{\mathcal{C}_{ab}} d\vec{\ell}\cdot\bar{E}(\bar{r})$$




$$\varphi(\bar{r}_b) - \varphi(\bar{r}_a) = -\int_{\bar{r}_a}^{\bar{r}_b} \bar{E}(\bar{r})\cdot d\bar{\ell}$$

$$\varphi(d/2) - \varphi(-d/2) = -\int_{-\frac{d}{2}}^{d/2} \left(-4\pi k\sigma\,\hat{z}\right) dz\,\hat{z}$$

$$= 4\pi k\sigma \int_{-d/2}^{d/2} \underbrace{(\hat{z}\cdot\hat{z})}_{=1}\, dz$$

$$= 4\pi k\sigma\left(\frac{d}{2} - \left(\frac{-d}{2}\right)\right)$$

$$\Delta\varphi = 4\pi k\sigma d$$