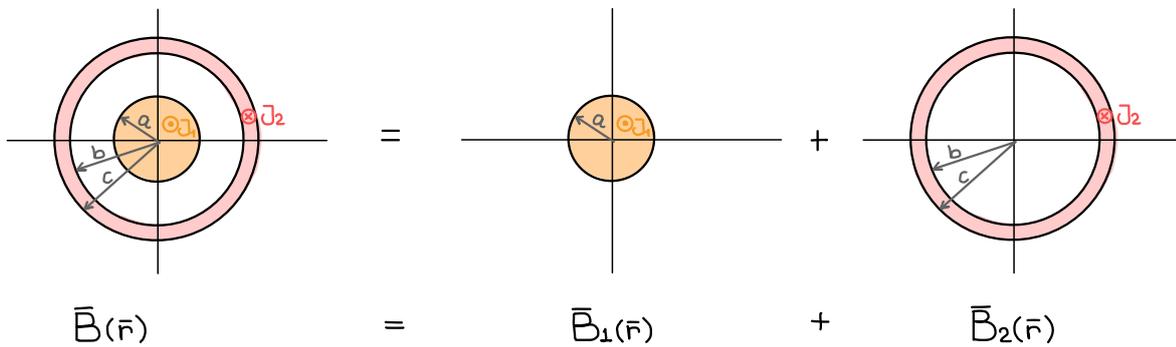
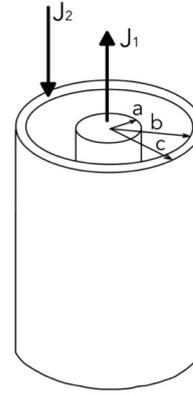
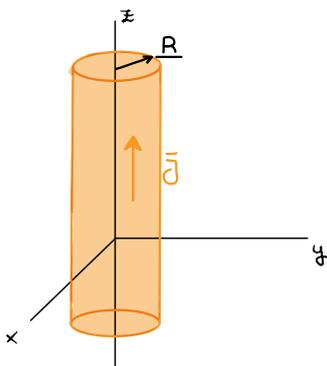


Ejercicio 5: Considere un par de cilindros infinitos concéntricos. El cilindro interior es macizo, de radio a , mientras que el cilindro exterior es hueco, con radio interno b y radio externo c . Por estos cilindros circulan corrientes \mathbf{J}_1 y \mathbf{J}_2 respectivamente, \mathbf{J}_1 y \mathbf{J}_2 son paralelas a los ejes de ambos cilindros y de sentido inverso.

- Calcule el campo magnético en todo el espacio
- Halle la relación que debe haber entre $|\mathbf{J}_1|$ y $|\mathbf{J}_2|$ para que el campo magnético sea nulo en el exterior del cilindro mayor.

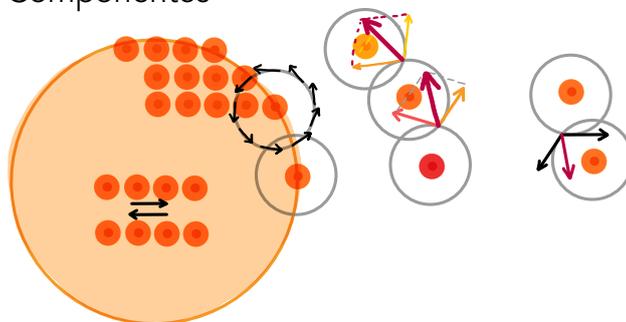


Campo magnético generado por un único cilindro



~~$\bar{\mathbf{B}}_1(\vec{r}) = B_{1r}(r, \varphi, z) \hat{r} + B_{1\varphi}(r, \varphi, z) \hat{\varphi} + B_{1z}(r, \varphi, z) \hat{z}$~~

Componentes



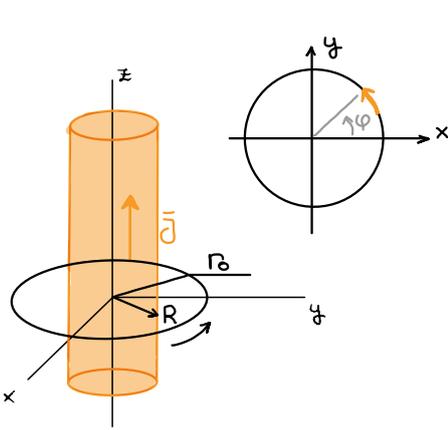
~~$\bar{\mathbf{B}}_1(\vec{r}) = B_{1\varphi}(r, \varphi, z) \hat{\varphi}$~~

$\bar{\mathbf{B}}_1(\vec{r}) = B_{1\varphi}(r) \hat{\varphi}$

Ley de Ampere

$$\oint_{\mathcal{C}} \vec{B}(\vec{r}) \cdot d\vec{\ell} = \mu_0 I_{\text{conc}}$$

Tomando como curva una circunferencia de radio r_0

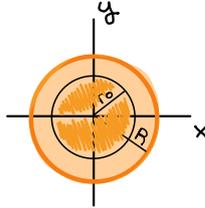


$$\oint_{\mathcal{C}} \vec{B}(\vec{r}) \cdot d\vec{\ell} = \int_{\varphi_0=0}^{\varphi_1=2\pi} B_{1\varphi}(r_0) \hat{\varphi} \cdot (\underbrace{r_0 d\varphi}_{d\vec{\ell}} \hat{\varphi}) = B_{1\varphi}(r_0) \cdot r_0 \int_0^{2\pi} d\varphi (\hat{\varphi} \cdot \hat{\varphi}) = 2\pi r_0 B_{1\varphi}(r_0)$$

La corriente encerrada depende del radio r_0 de la curva

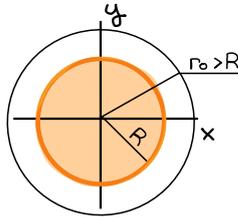
$$r_0 < R \Rightarrow I_{\text{conc}} = \pi r_0^2 J$$

$$\begin{aligned} \oint_{\mathcal{C}} \vec{B}(\vec{r}) \cdot d\vec{\ell} &= \mu_0 I_{\text{conc}} \\ 2\pi r_0 B_{1\varphi}(r_0) &= \mu_0 \pi r_0^2 J \\ B_{1\varphi}(r_0) &= \frac{\mu_0 J r_0}{2} \end{aligned}$$



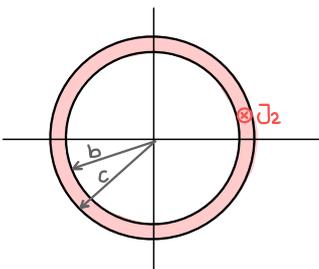
$$r_0 > R \Rightarrow I_{\text{conc}} = \pi R^2 J$$

$$\begin{aligned} \oint_{\mathcal{C}} \vec{B}(\vec{r}) \cdot d\vec{\ell} &= \mu_0 I_{\text{conc}} \\ 2\pi r_0 B_{1\varphi}(r_0) &= \mu_0 \pi R^2 J \\ B_{1\varphi}(r_0) &= \frac{\mu_0 J R^2}{2 r_0} \end{aligned}$$



$$\vec{B}(\vec{r}) = \begin{cases} \frac{\mu_0 J r}{2} \hat{\varphi} & r < R \\ \frac{\mu_0 J R^2}{2 r} \hat{\varphi} & r > R \end{cases}$$

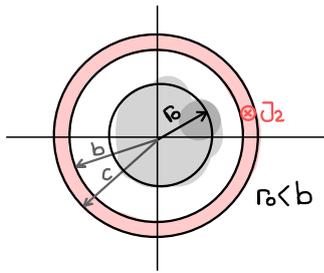
Campo magnético generado por un cilindro hueco



$$\begin{aligned} \vec{B}_2(\vec{r}) &= \cancel{B_{2r}(r, \varphi, z)} \hat{r} + B_{2\varphi}(r, \varphi, z) \hat{\varphi} + \cancel{B_{2z}(r, \varphi, z)} \hat{z} \\ &= B_{2\varphi}(r, \varphi, z) \hat{\varphi} \\ &= B_{2\varphi}(r) \hat{\varphi} \end{aligned}$$

Tomando como curva una circunferencia de radio r_0

$$\begin{aligned} \oint_{\mathcal{C}} \vec{B}(\vec{r}) \cdot d\vec{\ell} &= \int_0^{2\pi} B_{2\varphi}(r_0) \hat{\varphi} \cdot (r_0 d\varphi \hat{\varphi}) = B_{2\varphi}(r_0) r_0 \int_0^{2\pi} \hat{\varphi} \cdot (\hat{\varphi} d\varphi) = 2\pi r_0 B_{2\varphi}(r_0) \\ &= \int_0^{2\pi} r_0 d\varphi B_{2\varphi}(r_0) (\hat{\varphi} \cdot \hat{\varphi}) \end{aligned}$$

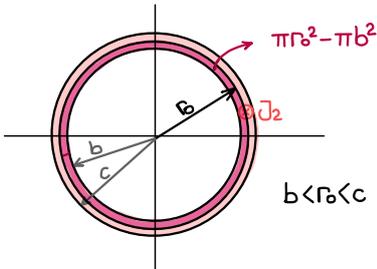


La corriente encerrada depende del radio r_0 de la curva

$$r_0 < b \quad I_{\text{conc}} = 0$$

$$b < r_0 < c \quad I_{\text{conc}} = -J_2 \pi (r_0^2 - b^2)$$

$$c < r_0 \quad I_{\text{conc}} = -J_2 \pi (c^2 - b^2)$$



$$r_0 < b \quad \oint \vec{B}(\vec{r}) \cdot d\vec{\ell} = \mu_0 I_{\text{conc}}$$

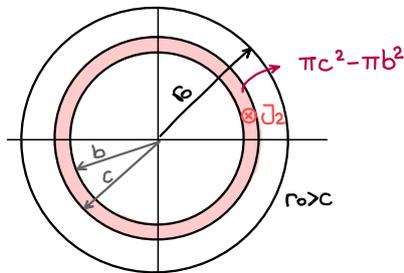
$$\oint 2\pi r_0 B_{2\varphi}(r_0) = 0$$

$$B_{2\varphi}(r_0) = 0$$

$$b < r_0 < c \quad \oint \vec{B}(\vec{r}) \cdot d\vec{\ell} = \mu_0 I_{\text{conc}}$$

$$2\pi r_0 B_{2\varphi}(r_0) = -\mu_0 J_2 \pi (r_0^2 - b^2)$$

$$B_{2\varphi}(r_0) = -\frac{\mu_0 J_2}{2} \frac{r_0^2 - b^2}{r_0}$$



$$c < r_0 \quad \oint \vec{B}(\vec{r}) \cdot d\vec{\ell} = \mu_0 I_{\text{conc}}$$

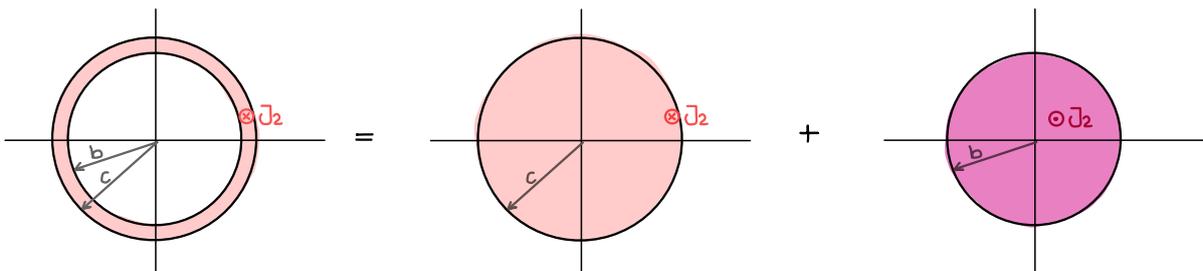
$$2\pi r_0 B_{2\varphi}(r_0) = -\mu_0 J_2 \pi (c^2 - b^2)$$

$$B_{2\varphi}(r_0) = -\frac{\mu_0 J_2}{2} \frac{c^2 - b^2}{r_0}$$

$$\vec{B}_2(\vec{r}) = \begin{cases} 0 & r < b \\ -\frac{\mu_0 J_2}{2} \frac{r^2 - b^2}{r} \hat{\varphi} & b < r < c \\ -\frac{\mu_0 J_2}{2} \frac{c^2 - b^2}{r} \hat{\varphi} & r > c \end{cases}$$

EXTRAS

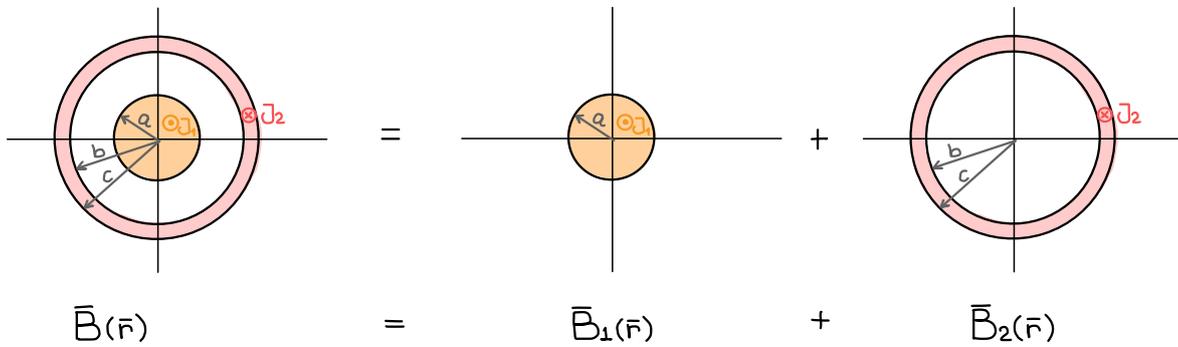
1) Otra forma de calcular este campo



$$\vec{B}(\vec{r}) = \begin{cases} -\frac{\mu_0 J_2}{2} r \hat{\varphi} & r < c \\ -\frac{\mu_0 J_2}{2} \frac{c^2}{r} \hat{\varphi} & r > c \end{cases}$$

$$\vec{B}(\vec{r}) = \begin{cases} \frac{\mu_0 J_2}{2} r \hat{\varphi} & r < b \\ \frac{\mu_0 J_2}{2} \frac{b^2}{r} \hat{\varphi} & r > b \end{cases}$$

$$\bar{B}(\bar{r}) = \begin{cases} (r < b) & \frac{\mu_0 \bar{J}_2 r}{2} \hat{\phi} - \frac{\mu_0 \bar{J}_2 r}{2} \hat{\phi} = 0 \\ b < r < c & \frac{\mu_0 \bar{J}_2 b^2}{2r} \hat{\phi} - \frac{\mu_0 \bar{J}_2 r}{2} \hat{\phi} = \frac{\mu_0 \bar{J}_2}{2} \left(\frac{b^2}{r} - r \right) \hat{\phi} \\ r > c & \frac{\mu_0 \bar{J}_2 b^2}{2r} \hat{\phi} - \frac{\mu_0 \bar{J}_2 c^2}{2r} \hat{\phi} = \frac{\mu_0 \bar{J}_2}{2} \frac{b^2 - c^2}{r} \hat{\phi} \end{cases}$$



$$\bar{B}_1(\bar{r}) = \begin{cases} \frac{\mu_0 \bar{J}_1 r}{2} \hat{\phi} & r < a \\ 0 & r > a \end{cases} \quad \bar{B}_2(\bar{r}) = \begin{cases} 0 & r < b \\ -\frac{\mu_0 \bar{J}_2}{2} \frac{r^2 - b^2}{r} \hat{\phi} & b < r < c \\ -\frac{\mu_0 \bar{J}_2}{2} \frac{c^2 - b^2}{r} \hat{\phi} & r > c \end{cases}$$

$$\bar{B}(\bar{r}) = \begin{cases} \frac{\mu_0 \bar{J}_1 r}{2} \hat{\phi} & r < a \\ \frac{\mu_0 \bar{J}_1 a^2}{2r} \hat{\phi} & a < r < b \\ \frac{\mu_0}{2} \left(\bar{J}_1 \frac{a^2}{r} - \bar{J}_2 \frac{r^2 - b^2}{r} \right) \hat{\phi} & b < r < c \\ \frac{\mu_0}{2} \left(\bar{J}_1 \frac{a^2}{r} - \bar{J}_2 \frac{c^2 - b^2}{r} \right) \hat{\phi} & r > c \end{cases}$$

$\bar{J}_1 = |\bar{J}_1|, \bar{J}_2 = |\bar{J}_2|$

b) $\bar{B}(r > c) = 0$

$$\frac{\mu_0}{2} \left(\bar{J}_1 \frac{a^2}{r} - \bar{J}_2 \frac{c^2 - b^2}{r} \right) = 0$$

$$\bar{J}_1 \frac{a^2}{r} - \bar{J}_2 \frac{c^2 - b^2}{r} = 0$$

$$\bar{J}_1 a^2 - \bar{J}_2 (c^2 - b^2) = 0$$

$$\bar{J}_1 a^2 = \bar{J}_2 (c^2 - b^2)$$

$$\bar{J}_1 = \frac{c^2 - b^2}{a^2} \bar{J}_2$$