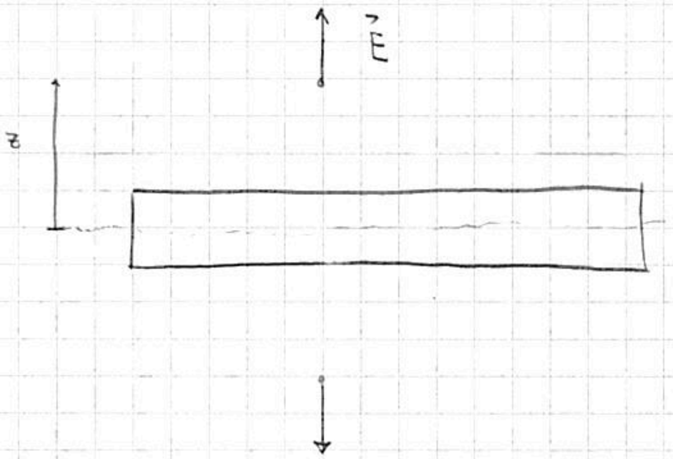
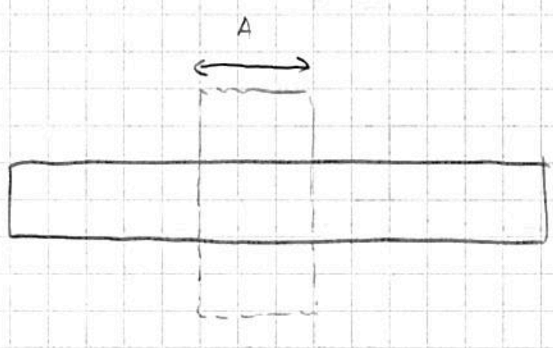
1. (a) Por simetría, $\bar{E} = E(z)\,\hat{z}$ con $E(-z) = -E(z)$



Superficie de Gauss: cilindro



$$\oint \bar{E} \cdot d\bar{S} = 2\,E(z)\,A \qquad z > 0$$

$$Q_{enc} = \begin{cases} \rho A \ell & \text{si} \quad z > \ell/2 \\ \rho A\, 2z & \text{si} \quad 0 < z < \ell/2 \end{cases}$$

Gauss: $\oint \vec{E} \cdot \vec{dS} = \frac{Q_{enc}}{\varepsilon_0}$

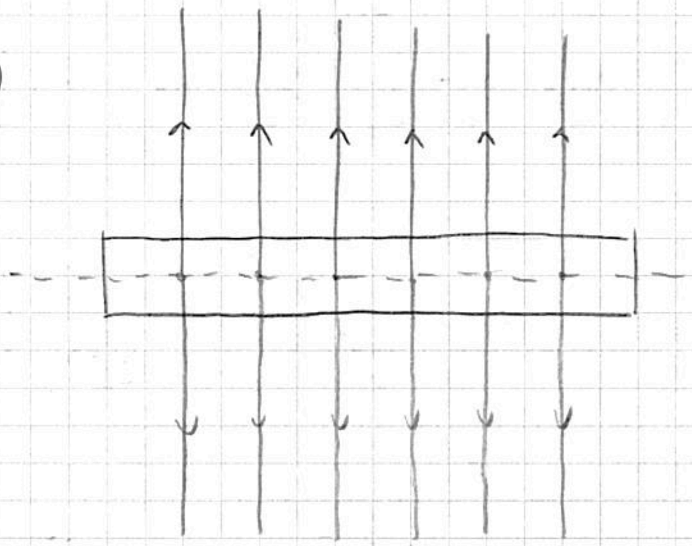
$\rightarrow \quad 2E(z)A = \frac{Q_{enc}}{\varepsilon_0} \quad \rightarrow \quad E(z) = \frac{Q_{enc}}{2\varepsilon_0 A}$

$$= \begin{cases} \frac{\rho l}{2\varepsilon_0} & \text{si } z > l/2 \\ \frac{\rho z}{\varepsilon_0} & \text{si } 0 < z < l/2 \end{cases}$$

$$\Rightarrow \boxed{\vec{E} = \begin{cases} \frac{\rho l}{2\varepsilon_0}\hat{z} & \text{si } z > l/2 \\ \frac{\rho z}{\varepsilon_0}\hat{z} & \text{si } -l/2 < z < l/2 \\ -\frac{\rho l}{2\varepsilon_0}\hat{z} & \text{si } z < -l/2 \end{cases}}$$

$$\vec{F} = q\vec{E} = \begin{cases} \frac{q\rho l}{2\varepsilon_0}\hat{z} & \text{si } z > l/2 \leftarrow \text{Apunta hacia abajo} \\ -\frac{q\rho l}{2\varepsilon_0}\hat{z} & \text{si } z < -l/2 \leftarrow \text{Apunta hacia arriba} \end{cases}$$

(b)



(c) $$V(z) = V(z) - V(0) = \int_z^0 \bar{E} \cdot d\bar{r}$$

$$= \int_z^0 E(z')\, dz' = - \int_0^z E(z')\, dz'$$

$$-l/2 < z < l/2 \implies V(z) = - \int_0^z \frac{\rho z}{\varepsilon_0}\, dz$$

$$= - \frac{\rho}{\varepsilon_0} z^2$$

$$z > l/2 \implies V(z) = - \int_0^{l/2} \frac{\rho z'}{\varepsilon_0}\, dz' - \int_{l/2}^{z} \frac{\rho l}{2\varepsilon_0}\, dz'$$

$$= \cancel{- \frac{\rho}{\varepsilon_0} \left(\frac{l}{2}\right)^2} - \frac{\rho l}{2\varepsilon_0} \left(z - \cancel{l/2} \right)$$

$$= - \frac{\rho l z}{2\varepsilon_0}$$

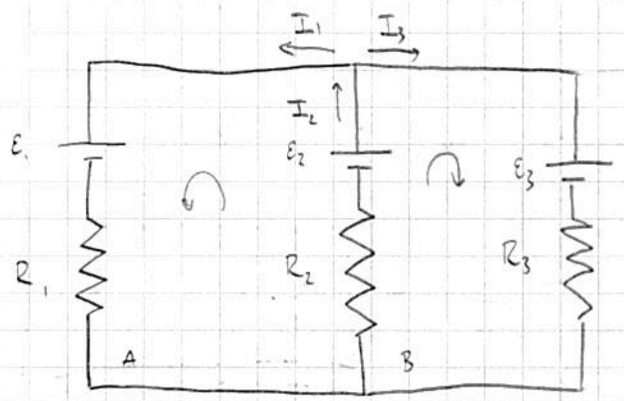
$$z < -\ell/2 \Rightarrow \quad V(z) = -\int_0^{-\ell/2} \frac{\rho z'}{\varepsilon_0}\, dz' + \int_{-\ell/2}^{z} \frac{\rho \ell}{2\varepsilon_0}\, dz'$$

$$= -\cancel{\frac{\rho}{\varepsilon_0}\left(\frac{\ell}{2}\right)^2} + \frac{\rho\ell}{2\varepsilon_0}\left(z + \cancel{\frac{\ell}{2}}\right)$$

$$= \frac{\rho\ell}{2\varepsilon_0}\, z$$

$$\Rightarrow \boxed{V(z) = \begin{cases} -\dfrac{\rho\ell}{2\varepsilon_0}\,|z| & \text{si} \quad |z| > \ell/2 \\[2ex] -\dfrac{\rho}{\varepsilon_0}\, z^2 & \text{si} \quad |z| < \ell/2 \end{cases}}$$

2.



$R_1 = 2\Omega$

$R_2 = 1\Omega$

$R_3 = 2\Omega$

$\mathcal{E}_1 = 4V$ $\quad \mathcal{E}_3 = 2V$

$\mathcal{E}_2 = 6V$

Capacitor cargado $\Rightarrow$ No circula corriente $\Rightarrow$ Lo ignoro por el momento

(a) Nodos: $I_2 = I_1 + I_3$

Mallas:

A: $-\mathcal{E}_2 + \mathcal{E}_1 + I_1 R_1 + I_2 R_2 = 0$

B: $-\mathcal{E}_2 + \mathcal{E}_3 + I_3 R_3 + I_2 R_2 = 0$

Reemplazo números

A: $2 I_1 + I_2 = 2 \Rightarrow I_1 = 1 - I_2/2$

B: $2 I_3 + I_2 = 4 \Rightarrow I_3 = 2 - I_2/2$

Reemplazo en nodos

$I_2 = 3 - I_2 \; - \; 2 I_2 = 3 \Rightarrow \boxed{I_2 = \frac{3}{2} A}$

Reemplazo el resultado en A y B

A: $I_1 = 1 - \frac{3}{4} = \boxed{\frac{1}{4}\ A = I_1}$

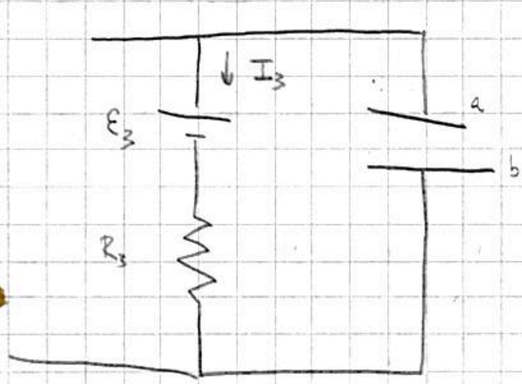
B: $I_3 = 2 - \frac{3}{4} = \boxed{\frac{5}{4}\ A = I_3}$

(b) $Q = C\Delta V$

$$\Delta V = \varepsilon_3 + R_3 I_3 = 2 + 2 \times \frac{5}{4}$$

$$= \frac{9}{2}\ V$$

$$\Longrightarrow \boxed{Q = \frac{9}{2}\ C}$$

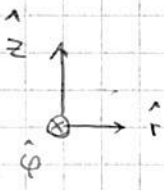


$\Delta V = V_a - V_b > 0$

$\Longrightarrow$ Carga positiva en la placa de arriba

(c) $P = I^2 R \Longrightarrow P_2 = I_2^2 R_2 = \boxed{\frac{9}{4}\ W = P_2}$

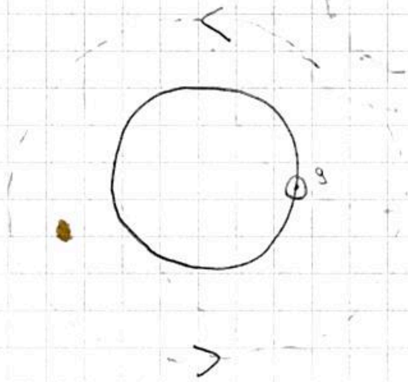
3. (a)



Por simetría, $\vec{B} = B(r)\,\hat{\varphi}$

Ampèr : $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_c$

Circuito d Ampèr : círculo



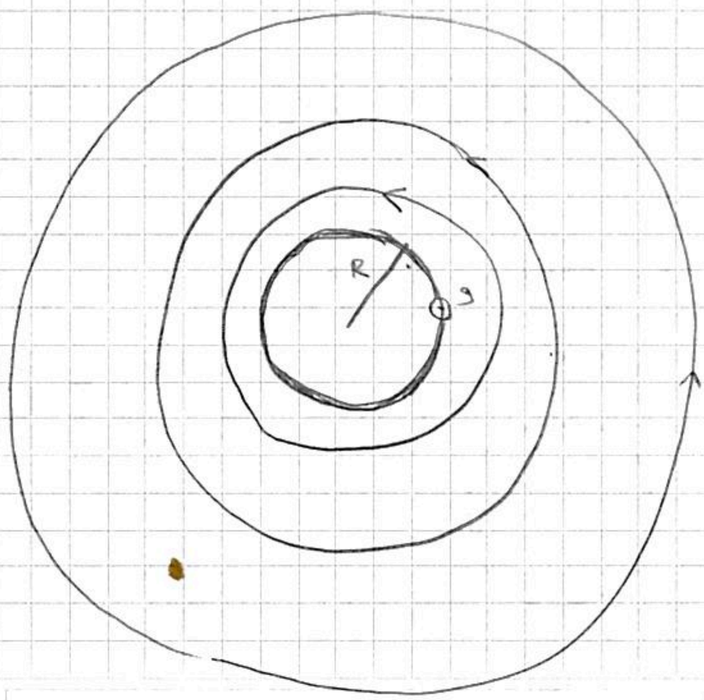
$\Longrightarrow \oint \vec{B} \cdot d\vec{l} = B(r)\,2\pi r$

$$I_c = \begin{cases} g\,2\pi R & \text{si} \quad r > R \\ 0 & \text{si} \quad r < R \end{cases}$$

$$\Rightarrow \quad B(r) = \frac{\mu_0 I_c}{2\pi r} = \begin{cases} \dfrac{\mu_0 g R}{r} & \text{si} \quad r > R \\ 0 & \text{si} \quad r < R \end{cases}$$

$$\Rightarrow \quad \boxed{\vec{B} = \begin{cases} \dfrac{\mu_0 g R}{r}\,\hat{\varphi} & \text{si} \quad r > R \\ 0 & \text{si} \quad r < R \end{cases}}$$

(b)



(c) $\vec{F} = q\vec{v}\times\vec{B} = q v B(r) \underbrace{\hat{z}\times\hat{\varphi}}_{= -\hat{r}}$

$= - q v B(r)\,\hat{r}$

$$\Rightarrow \boxed{\vec{F} = \begin{cases} 0 & \text{si} \quad r < R \\ -\dfrac{q v \mu_0 g R}{r}\,\hat{r} & \text{si} \quad r > R \end{cases}}$$