

11. Inductancias

Circuitos RL

Circuitos RC

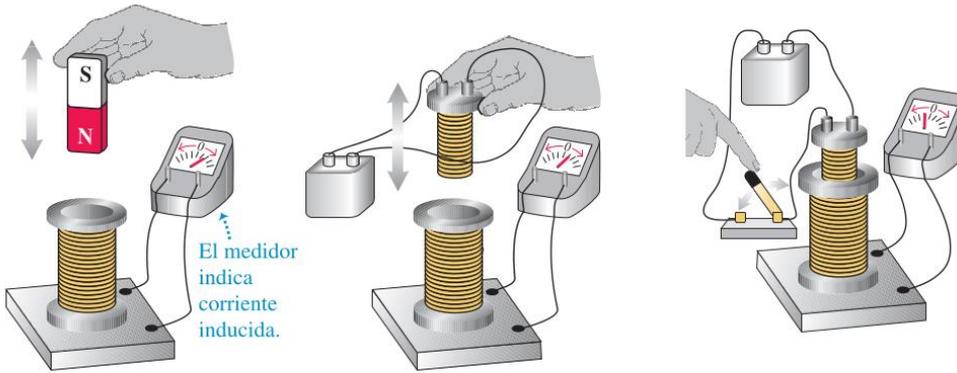
Ley de Faraday

Una ley, dos FEMs

$$\varepsilon = -\frac{d\phi_M}{dt}$$

$$\phi_M = \iint_S \vec{B} \cdot \hat{n} dS$$

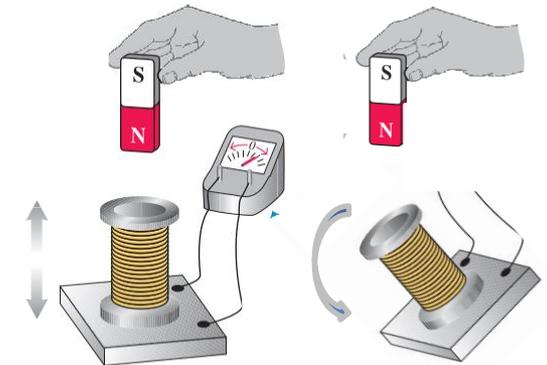
$\vec{B}(t) \rightarrow \vec{E}(t) \rightarrow$ FEM inducida



*Provocan que el campo magnético a través de la bobina cambie.

El flujo concatenado varía porque el campo \mathbf{B} que atraviesa la bobina **varía en el tiempo**

$\vec{B}(r)$ + desplazamiento \rightarrow FEM movimiento



Cambia orientación relativa

El campo \mathbf{B} se mantiene constante, pero flujo concatenado varía porque la bobina se desplaza hacia zonas de mayor intensidad de \mathbf{B}

Acoplamiento magnético

$$\varepsilon = -\frac{d\phi_M}{dt} \quad \phi_M = \iint_S \vec{B} \cdot \hat{n} dS$$



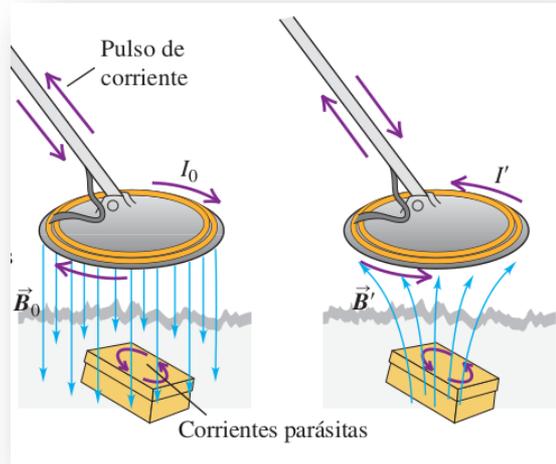
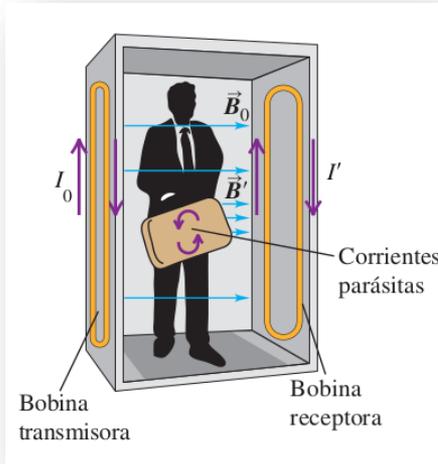
El campo \mathbf{B} generado por la **bobina primaria** atraviesa la **bobina secundaria**.

$$i_{prim}(t) \rightarrow \underline{B_{prim}(t)} \rightarrow \phi_{sec}(t) \rightarrow \varepsilon_{sec}(t) \rightarrow i_{sec}(t)$$

Ambos circuitos están **magnéticamente acoplados**

Detector de metales

$$\varepsilon = -\frac{d\phi_M}{dt} \quad \phi_M = \iint_S \vec{B} \cdot \hat{n} dS$$



El campo \mathbf{B} generado por **la bobina primaria** atraviesa la **bobina secundaria**.

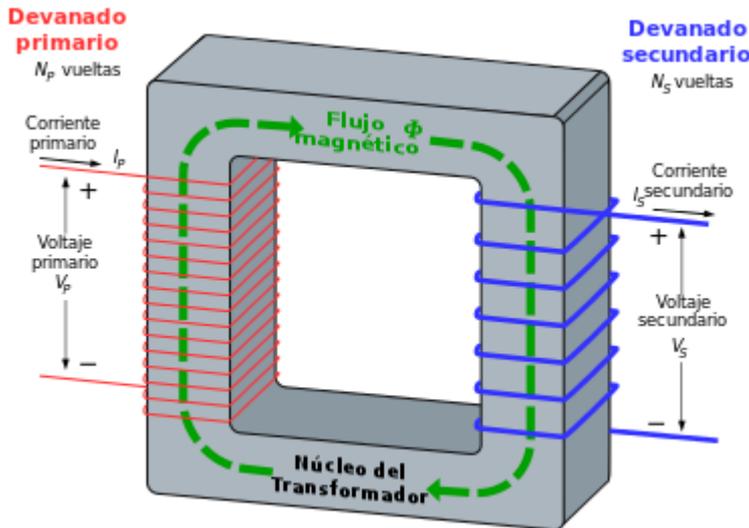
$$i_{prim}(t) \rightarrow B_{prim}(t) \rightarrow \phi_{sec}(t) \rightarrow \varepsilon_{sec}(t) \rightarrow i_{sec}(t) \rightarrow B_{sec}(t)$$



Ambos circuitos están **magnéticamente acoplados**

Transformador

$$\varepsilon = -\frac{d\phi_M}{dt} \quad \phi_M = \iint_S \vec{B} \cdot \hat{n} \, dS$$

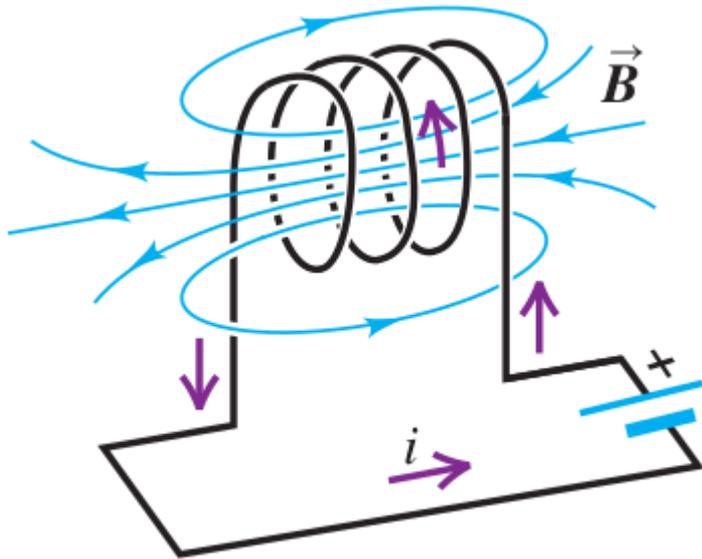


El campo \mathbf{B} generado por **la bobina primaria** atraviesa la **bobina secundaria**.

$$\varepsilon_{prim}(t) \rightarrow i_{prim}(t) \rightarrow \underline{B_{prim}(t)} \rightarrow \phi_{sec}(t) \rightarrow \varepsilon_{sec}(t)$$

Ambos circuitos están **magnéticamente acoplados**

Autoinductancia



En el caso en que $i=i(t)$

$$\varepsilon_{\text{auto-inducida}} = -\frac{d\phi_M}{dt} = -L \frac{di}{dt}$$

proporcional a i

$$\phi_M = \iint_S \vec{B} \cdot \hat{n} dS \sim L i$$

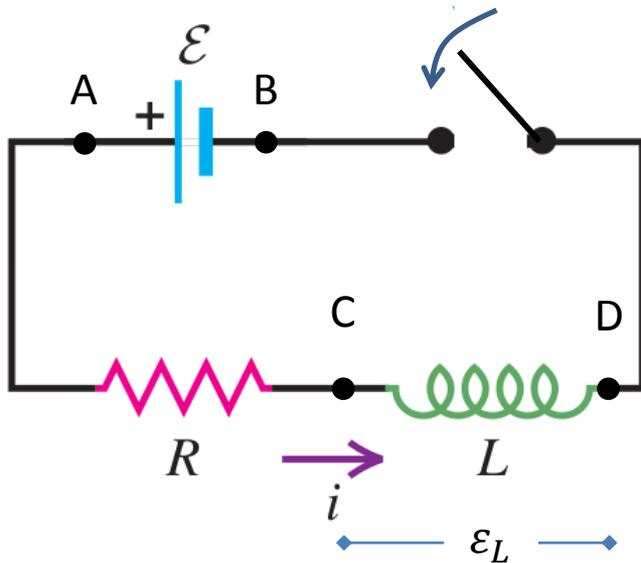
Coefficiente de auto-inducción

Es una constante que depende de la geometría de las espiras

$$[L] = \frac{\text{Weber}}{A} = H \text{ (Henry)}$$

$$\phi_M = L i$$

Circuito RL



$$i(t \leq 0) = 0$$

A $t=0$ se cierra la llave y comienza a circular $i(t)$

Sobre la bobina aparecera una FEM inducida (se opone al cambio, i.e. al crecimiento de i)

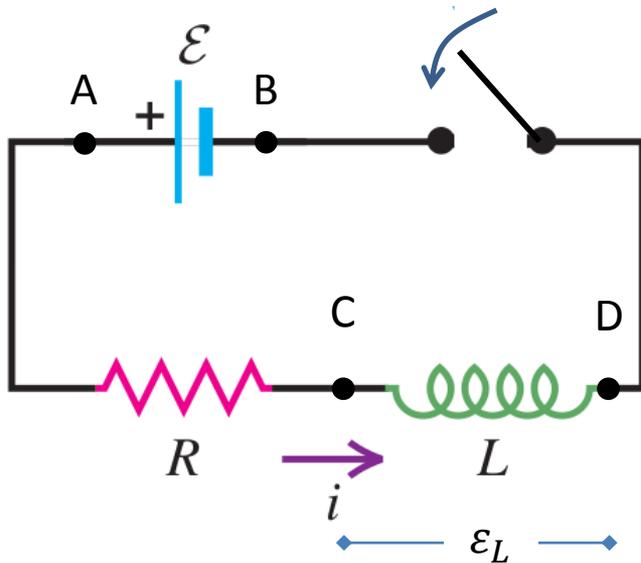
$$\varepsilon_L = -L \frac{di}{dt}$$

Ecuación para el circuito: $\varepsilon + \varepsilon_L = i R \longrightarrow \varepsilon = i R + L \frac{di}{dt}$

Ec diferencial para encontrar $i(t)$

$$\frac{di}{dt} = \frac{\varepsilon}{L} - \frac{R}{L} i$$

Circuito RL



A $t=0$ se cierra la llave y comienza a circular $i(t)$

$$\frac{di}{dt} = \frac{\varepsilon}{L} - \frac{R}{L}i \quad i(t \leq 0) = 0$$

Para resolver esta ecuación hacemos lo mismo que hicimos para analizar la carga de un capacitor

$$\frac{di}{dt} = -\frac{R}{L}\left(i - \frac{\varepsilon}{R}\right)$$

$$\frac{di}{\left(i - \frac{\varepsilon}{R}\right)} = -\frac{R}{L}dt$$

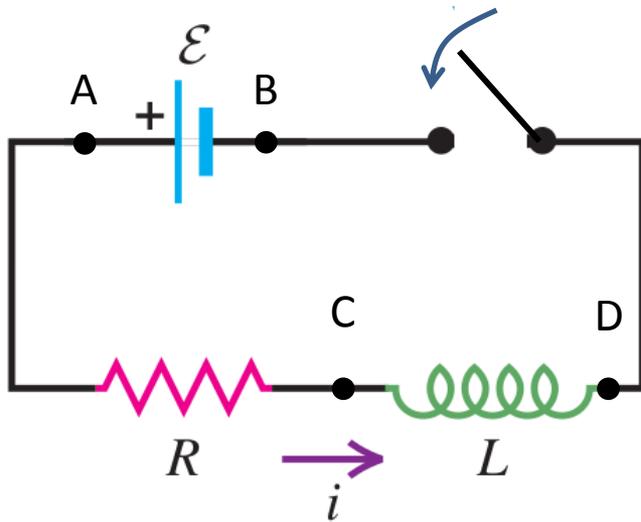
$$\int_0^i \frac{di'}{\left(i' - \frac{\varepsilon}{R}\right)} = -\int_0^t \frac{R}{L}dt'$$

$$\ln\left(\frac{i - \frac{\varepsilon}{R}}{-\frac{\varepsilon}{R}}\right) = -\frac{R}{L}t$$

$$\frac{i - \frac{\varepsilon}{R}}{-\frac{\varepsilon}{R}} = e^{-\frac{R}{L}t}$$

$$i(t) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

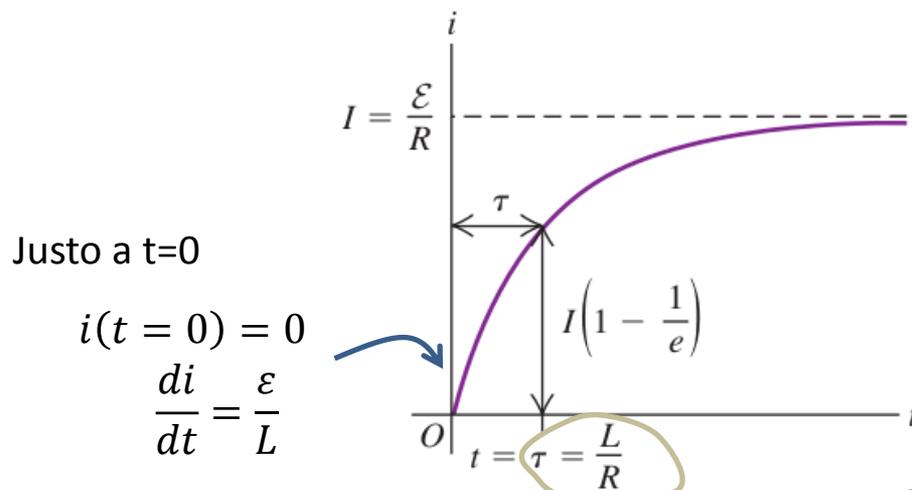
Circuito RL



A $t=0$ se cierra la llave y comienza a circular $i(t)$

$$\frac{di}{dt} = \frac{\varepsilon}{L} - \frac{R}{L}i \quad i(t \leq 0) = 0$$

Para resolver esta ecuación hacemos lo mismo que hicimos para analizar la carga de un capacitor

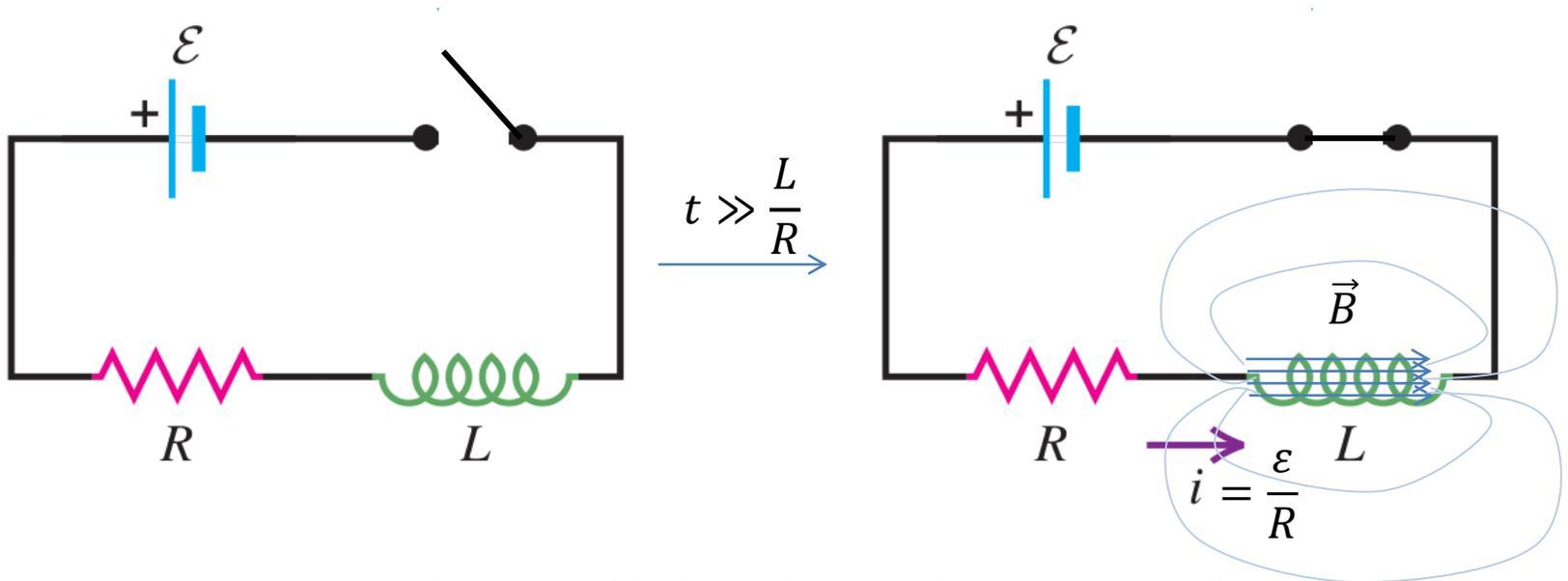


En el estacionario ($t \gg L/R$) la bobina se comporta como un cable

$$i(t) = \frac{\varepsilon}{R} \left(1 - e^{-\frac{R}{L}t}\right)$$

Si $L \rightarrow \infty, \tau \rightarrow \infty$

Energía almacenada en un circuito RL

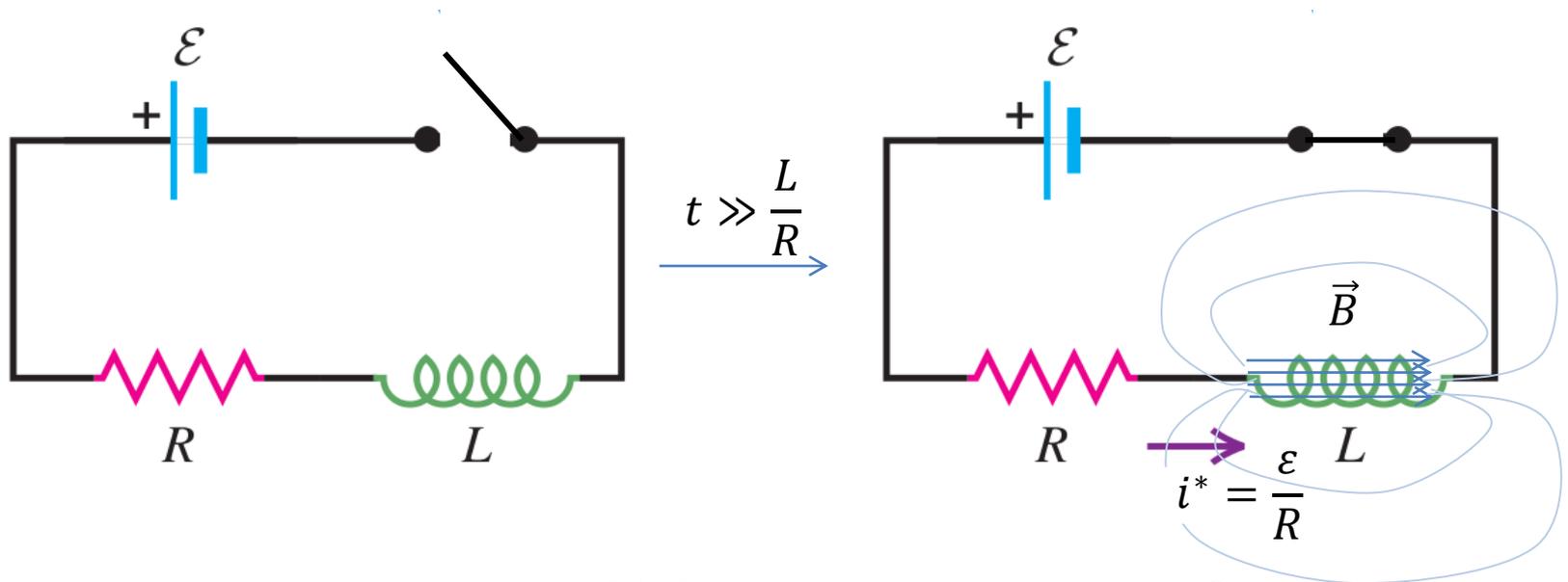


Cuanta energía entregó la batería en todo este tiempo?

$$P(t) = i(t)\varepsilon = i \left(iR + L \frac{di}{dt} \right) \longrightarrow dE(t) = P(t)dt = \underbrace{i^2 R dt}_{\substack{\text{energía disipada} \\ \text{en la resistencia}}} + \underbrace{Li di}_{\substack{\text{energía almacenada} \\ \text{en } L}}$$

Potencia instantánea entregada por la batería Energía entregada entre t y $t + dt$ dE_R dE_L

Energía almacenada en un circuito RL



Cuanta energía entregó la batería en todo este tiempo?

$$dE(t) = dE_R(t) + dE_L(i(t))$$

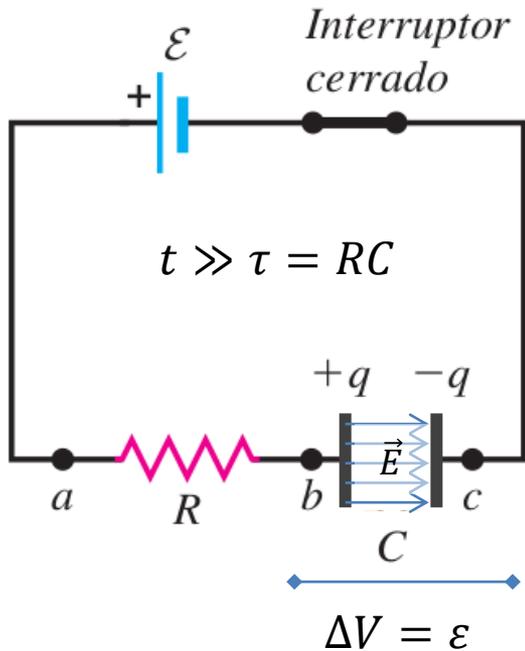
$$= i^2 R dt + Li \, di$$

$$E_L = \int_{i=0}^{i=\frac{\mathcal{E}}{R}} Li \, di = \frac{Li^2}{2} \Big|_0^{\frac{\mathcal{E}}{R}}$$

$$E_L = \frac{Li^{*2}}{2}$$

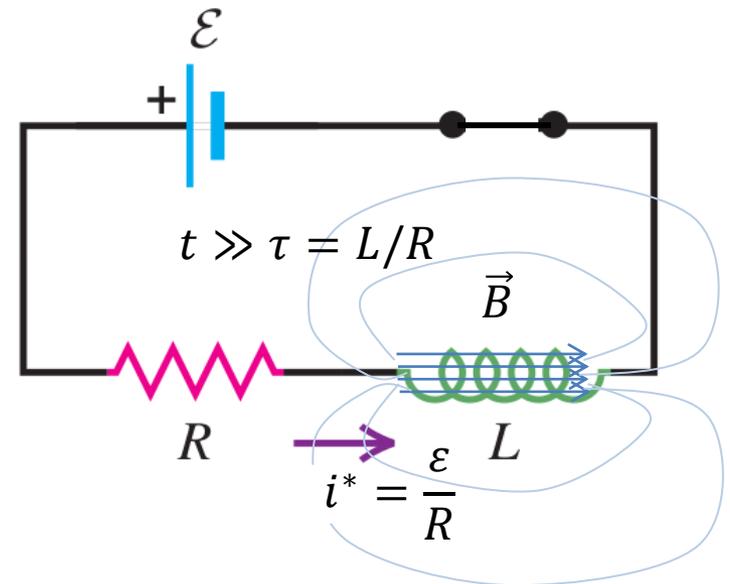
Energía almacenada en la bobina (campo \mathbf{B} creado en su interior)

Dispositivos para almacenar energía



$$E_C = \frac{C\varepsilon^2}{2}$$

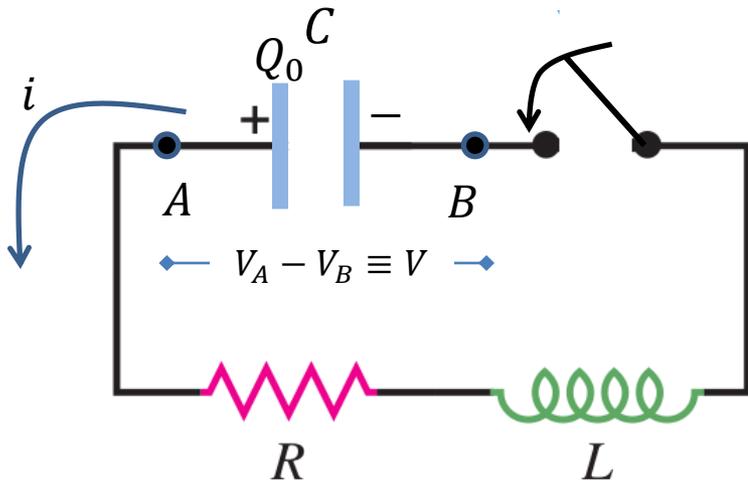
Energía almacenada en el capacitor
(campo \mathbf{E} creado en su interior)



$$E_L = \frac{Li^{*2}}{2}$$

Energía almacenada en la bobina
(campo \mathbf{B} creado en su interior)

Circuito RLC



$q(t)$: carga en el capacitor

$i(t)$: intensidad en el circuito

$$q(t = 0) = Q_0$$

$$i(t = 0) = 0$$

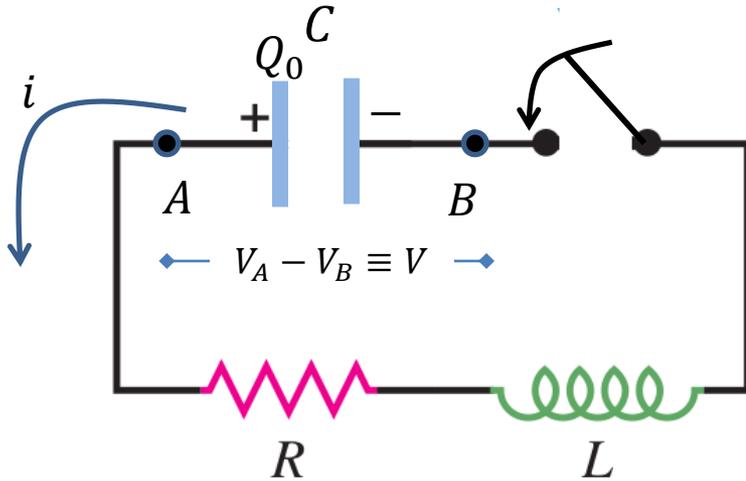
$$i(t) = -\frac{dq(t)}{dt}$$

$$v(t) - i(t)R - L \frac{di(t)}{dt} = 0$$

$$\frac{q(t)}{C} + \frac{dq(t)}{dt}R + L \frac{d^2q(t)}{dt^2} = 0$$

$$L \frac{d^2q(t)}{dt^2} + \frac{dq(t)}{dt}R + \frac{q(t)}{C} = 0$$

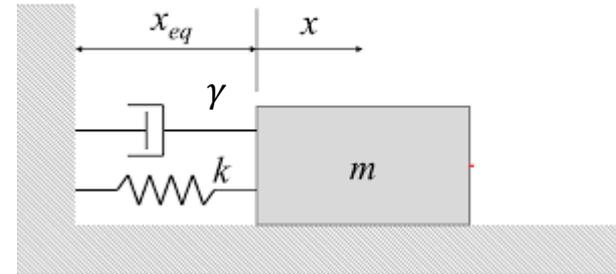
Sistemas equivalentes



$$L \frac{d^2 q(t)}{dt^2} + \frac{dq(t)}{dt} R + \frac{q(t)}{C} = 0$$

$$q(t = 0) = Q_0$$

$$i(t = 0) = \frac{dq}{dt}(t = 0) = 0$$

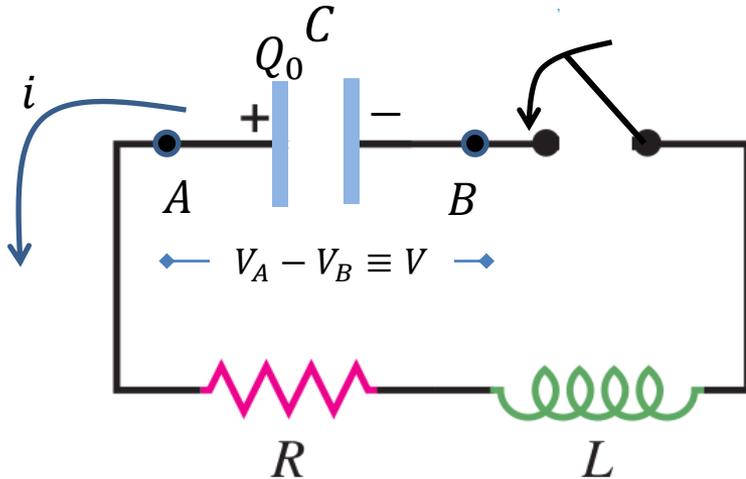


$$F = m \frac{d^2 x(t)}{dt^2}$$

$$-kx - \gamma \frac{dx}{dt} = m \frac{d^2 x(t)}{dt^2}$$

$$m \frac{d^2 x(t)}{dt^2} + \gamma \frac{dx}{dt} + kx = 0$$

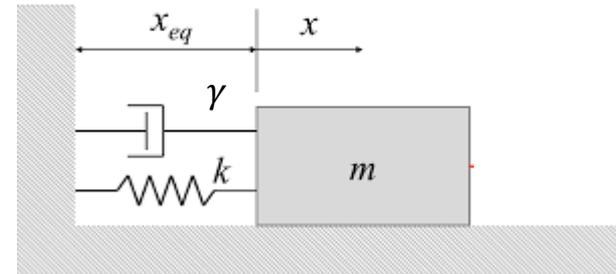
Sistemas equivalentes



$$L \frac{d^2 q(t)}{dt^2} + \frac{dq(t)}{dt} R + \frac{q(t)}{C} = 0$$

$$q(t = 0) = Q_0$$

$$i(t = 0) = \frac{dq}{dt}(t = 0) = 0$$



$$m \frac{d^2 x(t)}{dt^2} + \gamma \frac{dx}{dt} + kx = 0$$

$$x(t = 0) = x_0$$

$$v(t = 0) = \frac{dx}{dt}(t = 0) = v_0$$

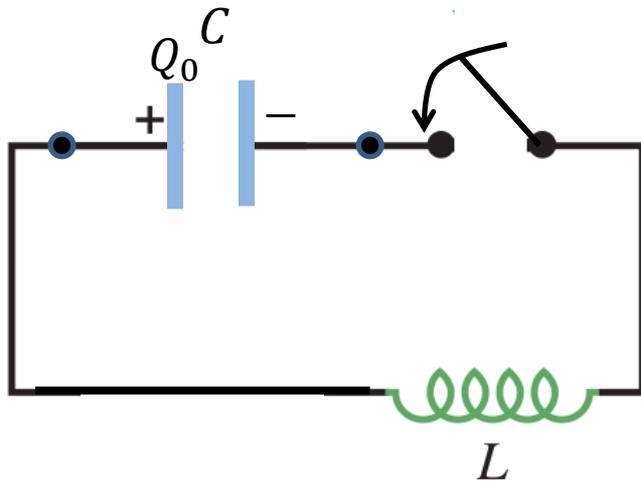
$$q \leftrightarrow x$$

$$L \leftrightarrow m$$

$$R \leftrightarrow \gamma$$

$$k \leftrightarrow \frac{1}{C}$$

Circuito LC (R=0)

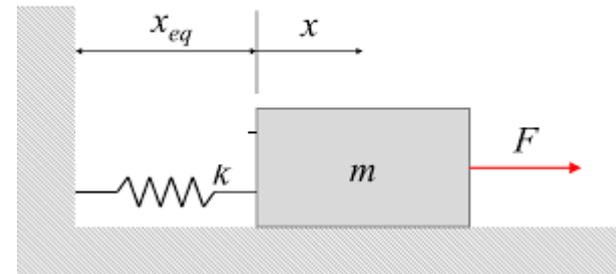


$$L \frac{d^2 q(t)}{dt^2} + \cancel{\frac{dq(t)}{dt} R} + \frac{q(t)}{C} = 0$$

$$\frac{d^2 q}{dt^2} = - \frac{1}{LC} q \quad \omega^2$$

Soluciones: $q(t) = Q_0 \cos(\omega t + \varphi)$

$$i(t) = - \frac{dq(t)}{dt} = Q_0 \omega \sin(\omega t + \varphi)$$



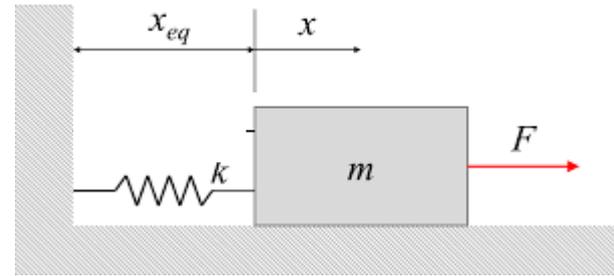
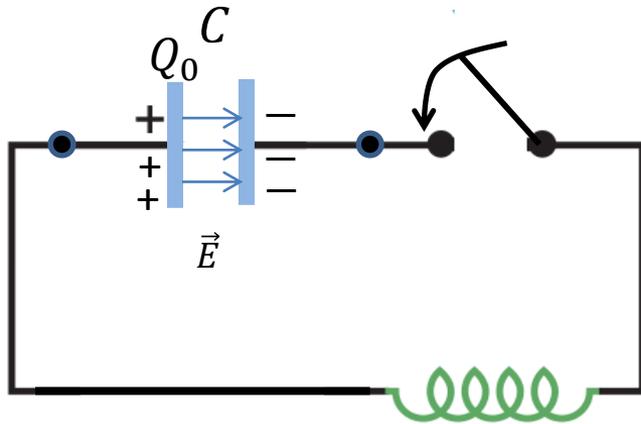
Ec. de oscilador armónico

$$m \frac{d^2 x(t)}{dt^2} + \cancel{\gamma \frac{dx}{dt}} + kx = 0$$

Circuito LC (R=0)

Yo ya se que la dinámica dada por las ecs del O.A. conserva la suma $E = K + U$

$$\frac{d^2q}{dt^2} = -\omega^2 q$$



$$q \leftrightarrow x \quad L \leftrightarrow m \quad k \leftrightarrow \frac{1}{C}$$

$$K = \frac{mv^2}{2} = \frac{Li^2}{2}$$

$$U = \frac{kx^2}{2} = \frac{q^2}{2C}$$

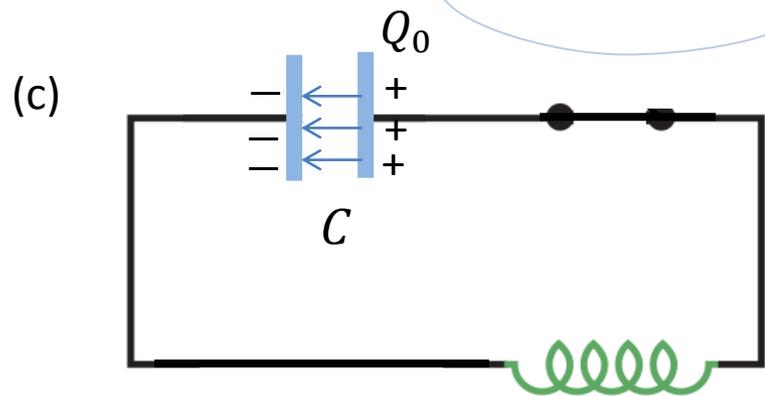
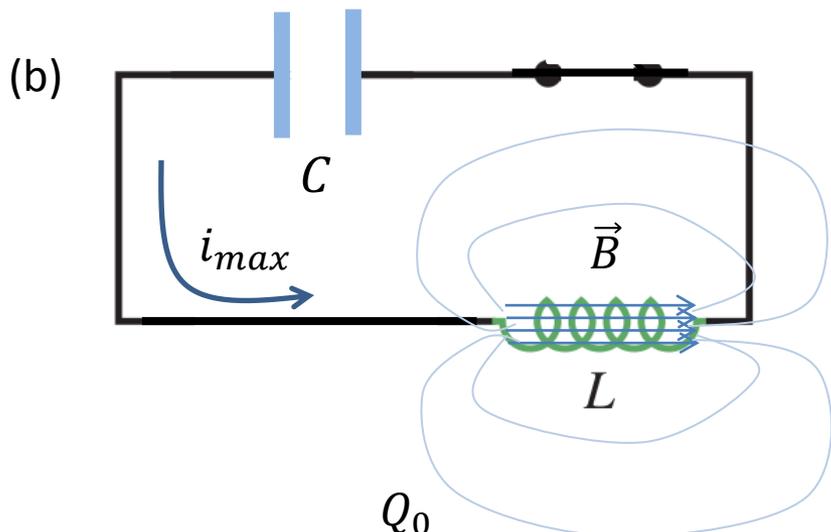
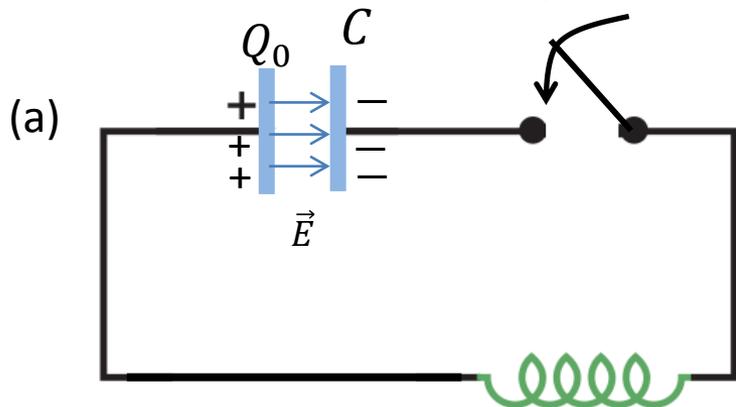
energía almacenada en L hace las veces de energía cinética

energía almacenada en C hace las veces de energía potencial

A lo largo de la dinámica

$$U_{tot} = U_L(t) + U_C(t) = cte$$

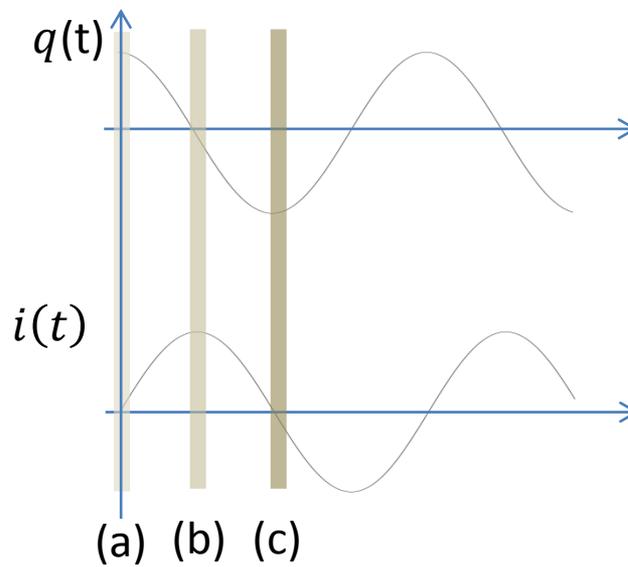
Circuito LC (R=0)

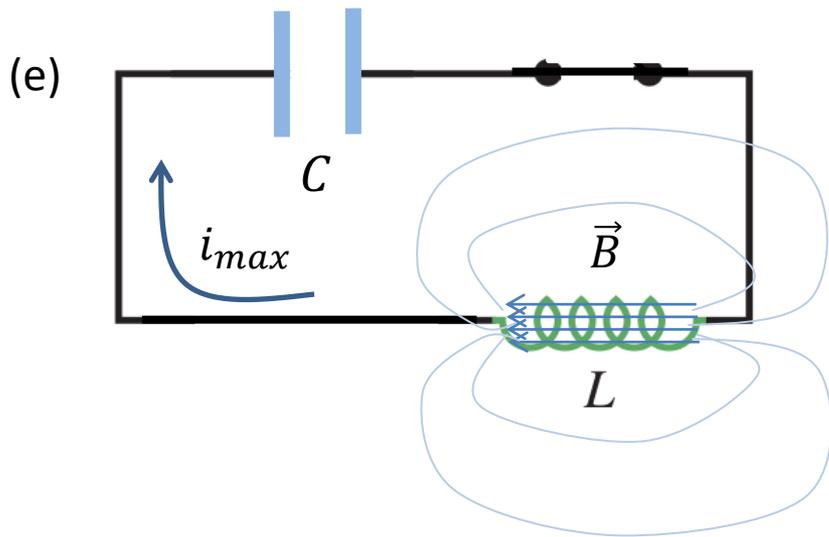
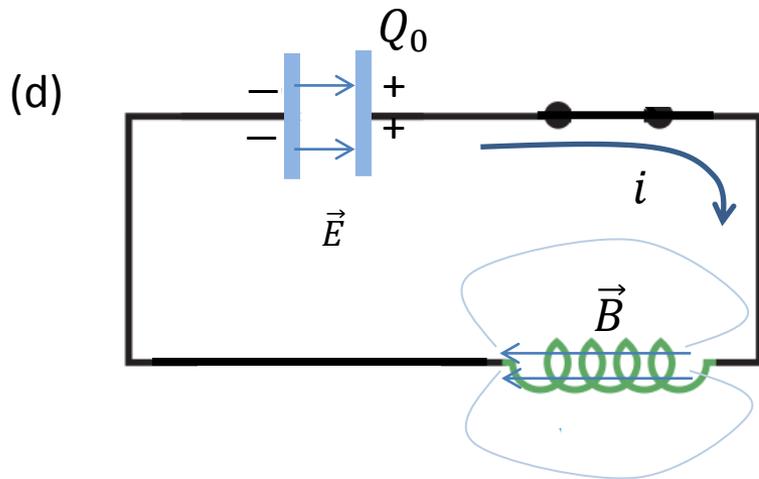


$$\frac{d^2q}{dt^2} = -\omega^2q$$

$$q(t) = Q_0 \cos(\omega t + \varphi)$$

$$i(t) = -\frac{dq(t)}{dt} = Q_0\omega \sin(\omega t + \varphi)$$



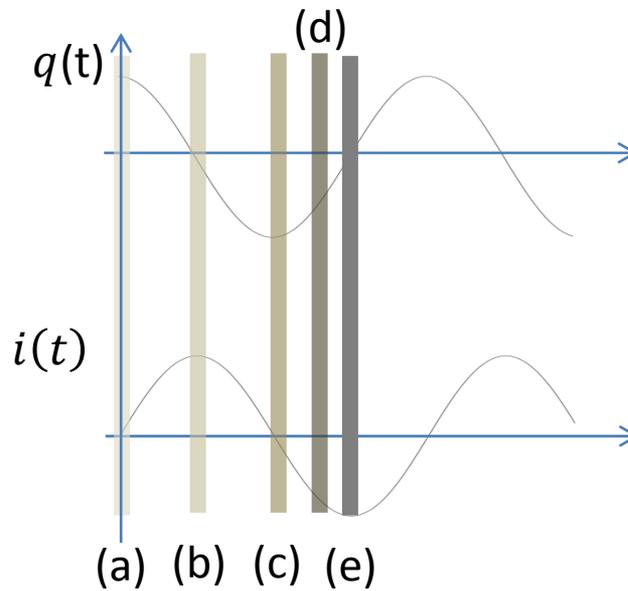


Circuito LC ($R=0$)

$$\frac{d^2q}{dt^2} = -\omega^2q$$

$$q(t) = Q_0 \cos(\omega t + \varphi)$$

$$i(t) = -\frac{dq(t)}{dt} = Q_0\omega \sin(\omega t + \varphi)$$



Desde el pto de vista energético

Circuito LC (R=0)

$$\frac{d^2 q}{dt^2} = -\omega^2 q$$

$$q(t) = Q_0 \cos(\omega t + \varphi)$$

$$i(t) = -\frac{dq(t)}{dt} = Q_0 \omega \sin(\omega t + \varphi)$$

A lo largo de la dinámica

$$U_{tot} = U_L(t) + U_C(t) = cte$$

\uparrow \uparrow
 $\frac{Li^2}{2}$ $\frac{q^2}{2C}$

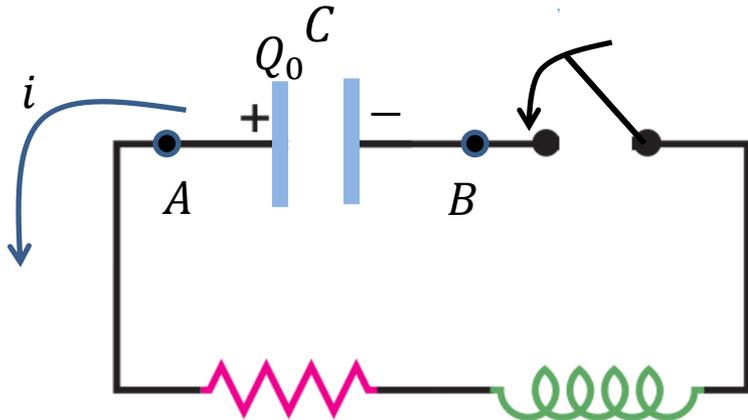
$$U_L(t) + U_C(t) = \frac{L(Q_0 \omega \sin(\omega t + \varphi))^2}{2} + \frac{(Q_0 \cos(\omega t + \varphi))^2}{2C}$$

$\frac{1}{LC}$ ←

$$= \frac{LQ_0^2 \omega^2}{2} (\sin(\omega t + \varphi))^2 + \frac{Q_0}{2C} (\cos(\omega t + \varphi))^2$$
$$= \frac{Q_0}{2C} [(\sin(\omega t + \varphi))^2 + (\cos(\omega t + \varphi))^2] = \frac{Q_0}{2C}$$

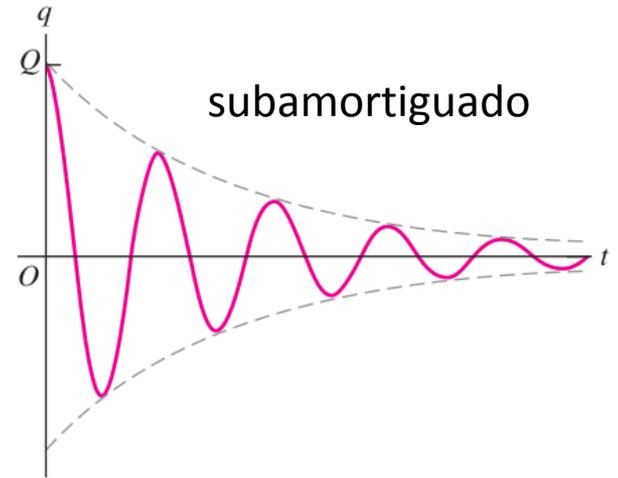
Energía inicial
del sistema

Circuito RLC

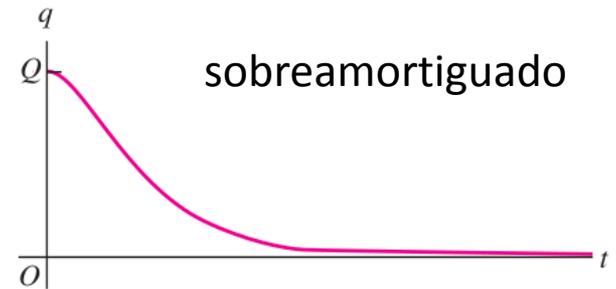


$$L \frac{d^2 q(t)}{dt^2} + \frac{dq(t)}{dt} R + \frac{q(t)}{C} = 0$$

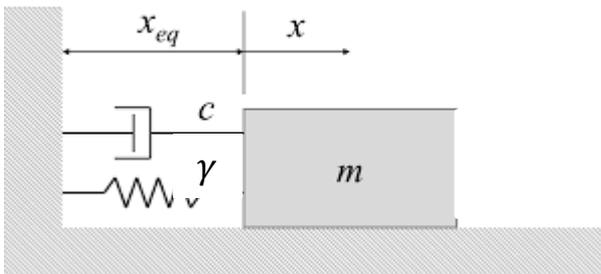
$$R < \sqrt{\frac{4L}{C}}$$



$$R > \sqrt{\frac{4L}{C}}$$



$$m \frac{d^2 x(t)}{dt^2} + \gamma \frac{dx}{dt} + kx = 0$$



$$R = \sqrt{\frac{4L}{C}}$$

