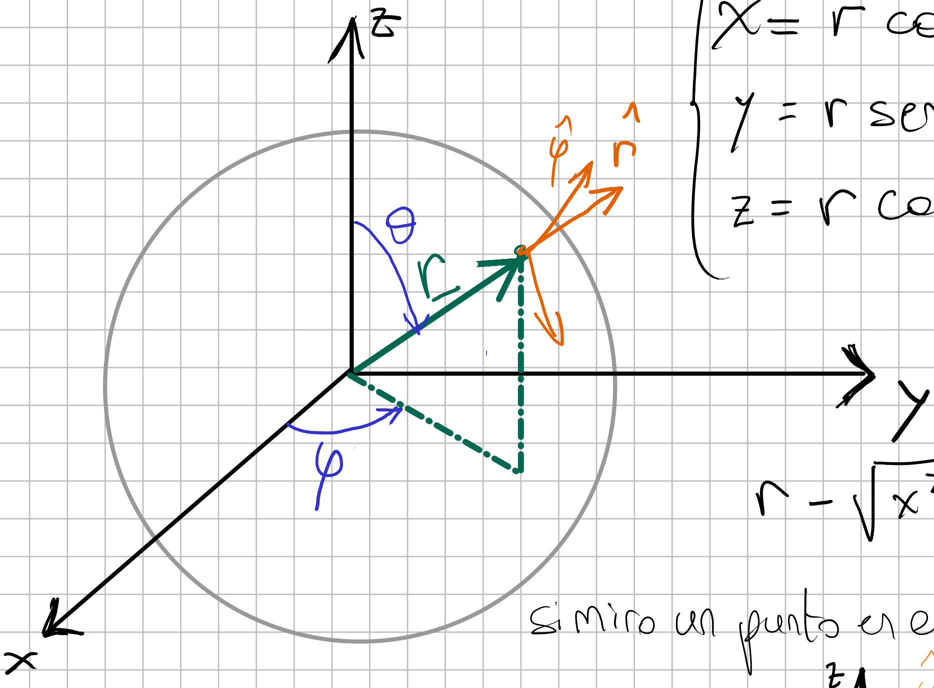
Coordenadas Esféricas:



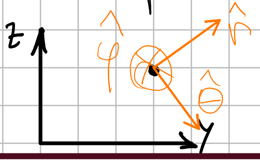
$$\begin{cases} x = r\cos(\varphi)\ \text{sen}(\theta) \\ y = r\ \text{sen}(\varphi)\ \text{sen}(\theta) \\ z = r\cos(\theta) \end{cases}$$

$0 \leq \varphi < 2\pi$

$0 \leq \theta < \pi$

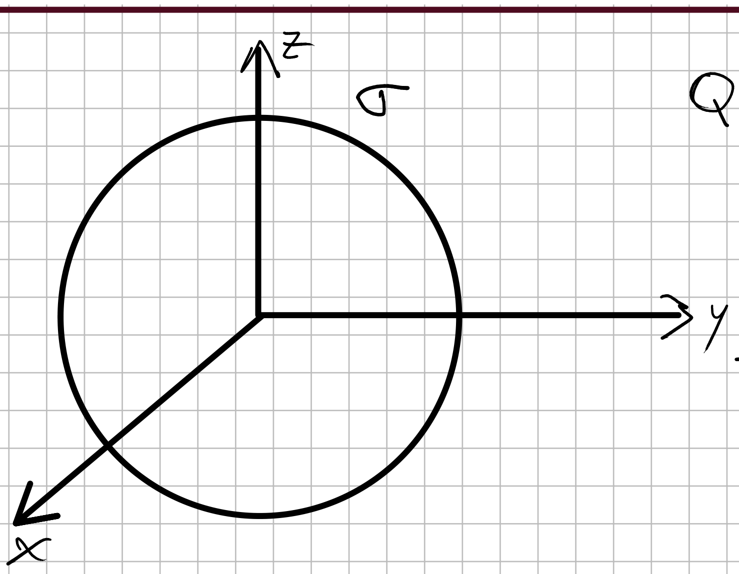
$$r - \sqrt{x^2+y^2+z^2}$$

si miro un punto en el plano y-z:



7. Para las siguientes configuraciones de carga dibuje las líneas de campo eléctrico y las superficies equipotenciales. Calcule el campo eléctrico y el potencial en todo el espacio.

   a) Un hilo recto infinito con densidad lineal uniforme λ.
   b) Una superficie esférica de radio R con densidad superficial uniforme σ.
   c) Una esfera maciza de radio R con densidad volumétrica uniforme ρ.
   d) Un plano infinito con densidad superficial uniforme σ.
   e) Un cilindro hueco infinito con densidad superficial uniforme σ.
   f) Un cilindro macizo infinito con densidad volumétrica uniforme ρ.

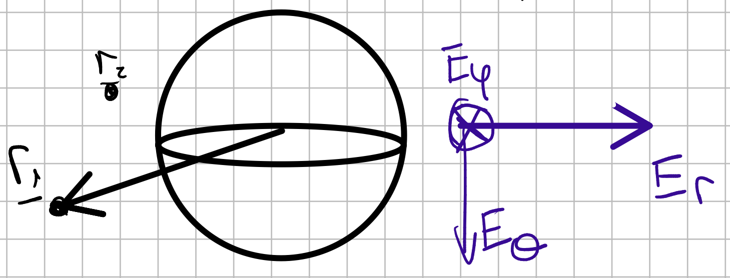


$$Q = \iint_S \sigma\, ds = \sigma\, \text{Área}$$

$$\sigma = \frac{Q}{A}$$

$$[\sigma] = \frac{[Q]}{[A]} = \frac{C}{m^2}$$

Direcciones y dependencias:



$$\underline{E}(\underset{\substack{\downarrow \\ (r,\varphi,\theta)}}{\underline{r}}) = E_r(\underline{r})\,\hat{r} + E_\varphi(\underline{r})\,\hat{\varphi} + E_\theta(\underline{r})\,\hat{\theta}$$

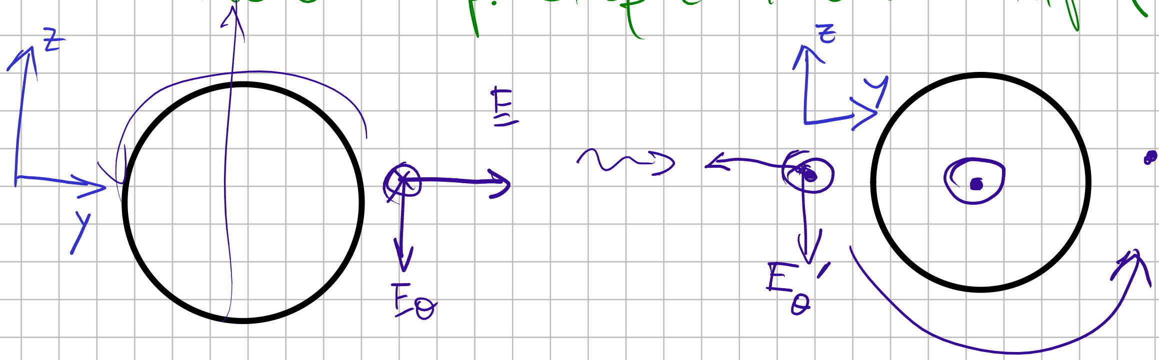
$$\underline{r}_1 = (r_1, \varphi_1, \theta_1)$$

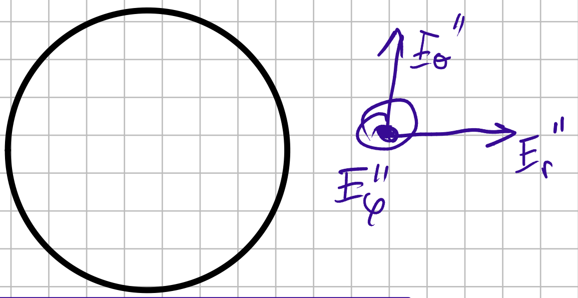
$$\underline{r}_2 = (r_2, \varphi_2, \theta_2)$$

$$\underline{E}(\underline{r}) = \underbrace{E_r}(\underbrace{r,\varphi,\theta})\underbrace{\hat{r}} + \underbrace{E_\varphi}(\underbrace{r,\varphi,\theta})\underbrace{\hat{\varphi}} + \underbrace{E_\theta}(\underbrace{r,\varphi,\theta})\underbrace{\hat{\theta}}$$

Componentes (Dirección del campo)

Punto del esp. en que miro el campo (Dependencias)





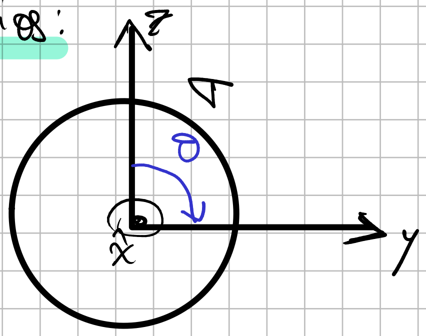
$$E_\theta'' = -E_\theta \equiv 0$$

$$E_\varphi'' = -E_\varphi \equiv 0$$

$$\boxed{\underline{E}(\underline{r}) = E(\underline{r})\,\hat{r}}$$

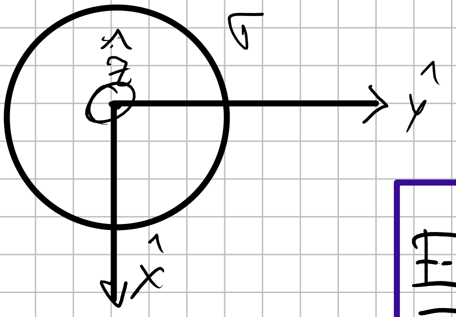
- Si roto sobre el eje $\hat{z}$ y luego sobre el eje $\hat{x}$ los componentes "angulares" $(\hat{\theta},\hat{\varphi})$ quedan distintos, pero ∇ es igual $\Rightarrow$ $E_\theta = E_\varphi = 0$

Dependencias:



Rotando continuamente sobre $\hat{x}$ no cambia ∇ $\Rightarrow$

No puede depender de θ



No puede depender de φ

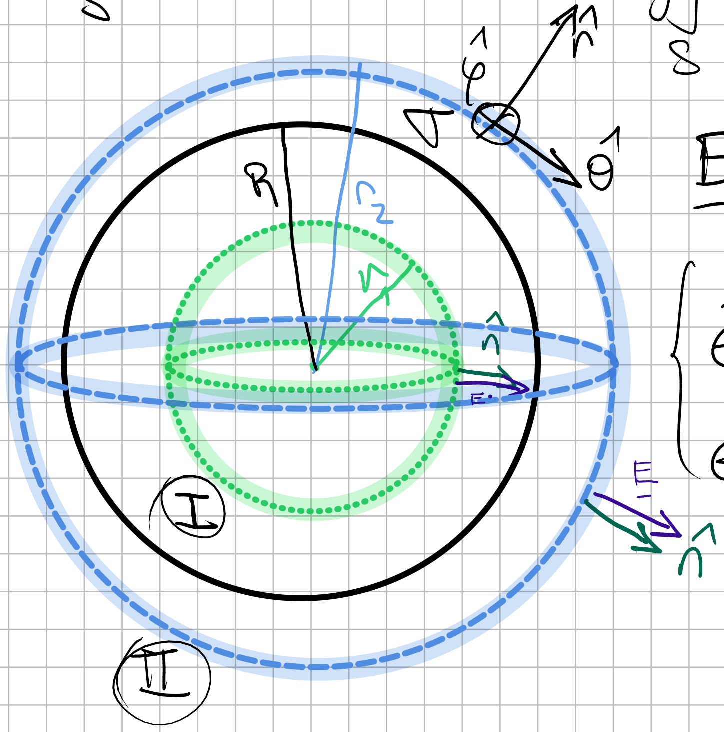
$$\boxed{\underline{E}(\underline{r}) = E(r)\hat{r}}$$

Quiero el módulo de E, ¡la función E(r)!

Ley de Gauss: Elegimos superficies imaginarios para usar Gauss. Dos esferas con $r_1 < R$
$r_2 > R$

$$\oiint_S \underline{E}\cdot\underline{dS} = \frac{Q_{enc}}{\epsilon_0}$$

$$\oiint_S \underline{E}\cdot\underline{dS} = E\oiint_S \hat{e}\cdot\underline{dS}$$



$$\underline{E} = E\hat{e}$$

$$\begin{cases} \hat{e} \perp dS\,\hat{n} \Rightarrow \hat{e}\cdot\hat{n} = 0 \\ \hat{e} \,//\, dS\,\hat{n} \Rightarrow \hat{e}\cdot\hat{n} = 1 \end{cases}$$

Ⓘ $\oiint_S \underline{E}(r_1)\cdot\underline{dS} = \frac{Q_{enc}}{\epsilon_0} = 0$ $r_1 < R$

(II) $\oiint_S \underline{E}(r_2)\cdot\underline{dS} = \frac{Q_{enc}}{\epsilon_0} = \frac{\sigma S}{\epsilon_0}$ $r_2 > R$

$S = 4\pi R^2 \Rightarrow Q_{enc} = 4\pi R^2\sigma$

$$\oiint_S E(r_{1,2})\underbrace{\hat{r}\cdot ds_{1,2}\overset{\hat{n}=\hat{r}}{\hat{r}}}_{1} = E(r_{1,2})\oiint_S ds_{1,2} = E(r_{1,2})S_{1,2}$$

$$\left.\begin{array}{l} (I)\ \ \underline{E}(r_1)\cdot S_1 = 4\pi r_1^2\,\underline{E}(r_1) \\ (II)\ \ \underline{E}(r_2)\cdot S_2 = 4\pi r_2^2\,E(r_2) \end{array}\right\} \oiint_S \underline{E}(r)\cdot\underline{ds} = 4\pi r^2 E(r)$$

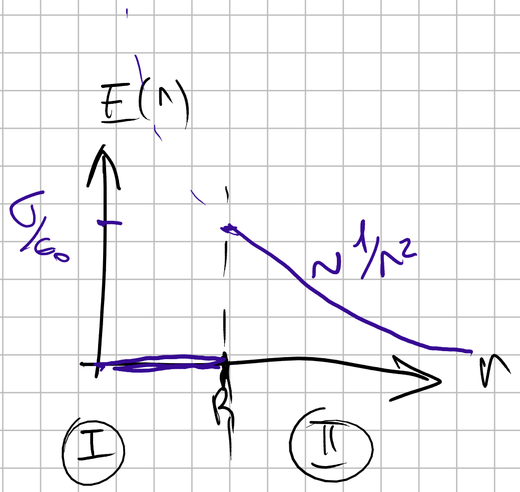
(I) $4\pi r^2 \underline{E}(r) = 0$ $\quad r<R$

$$E(r) = 0$$

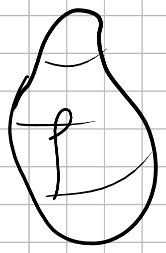
(II) $\cancel{4\pi} r^2 \underline{E}(r) = \cancel{4\pi} R^2 \frac{\sigma}{\epsilon_0}$ $\quad r>R$

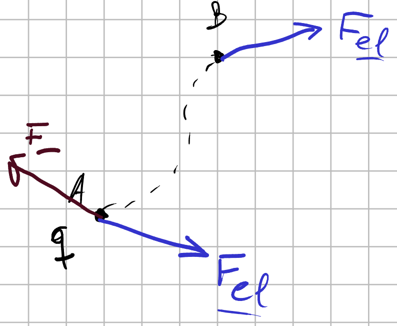
$$\underline{E}(r) = \frac{\sigma}{\epsilon_0}\left(\frac{R}{r}\right)^2$$

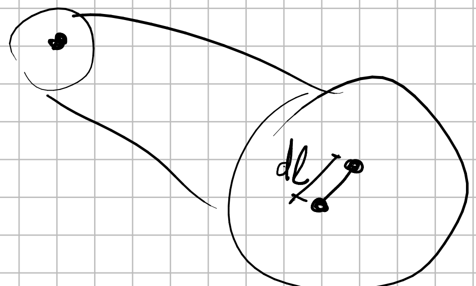
$$\boxed{\underline{E}(r) = \begin{cases} 0 & r<R \\ \frac{\sigma}{\epsilon_0}\left(\frac{R}{r}\right)^2 & r>R \end{cases}}$$



Potencial electrostatico:





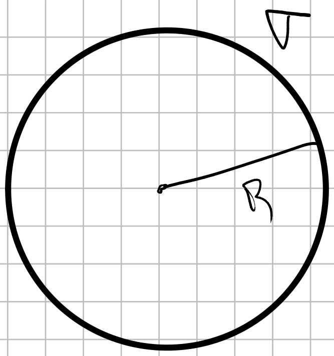


$$\Sigma \underline{F}\cdot\underline{dl} = 0$$

$$\underline{F} = -\underline{F_{el}}$$

$$W_{AB} = \int_C \underline{F}\cdot\underline{dl} = -\int_C \underline{F_{el}}\cdot\underline{dl} = -q\int_C \underline{E}\cdot\underline{dl} \Rightarrow \frac{W_{AB}}{q} = \boxed{-\int_A^B \underline{E}\cdot\underline{dl} = V_{AB}}$$

$$[V] = \underset{V}{\text{Volts}} = \frac{J}{C} = \frac{N}{C} \cdot m$$



$$\underline{E}(r) = \begin{cases} 0\ \hat{r} & r < R \\ \frac{\sigma}{\epsilon_0}\left(\frac{R}{r}\right)^2 & r > R \end{cases}$$

$$V_{ab} = -\int_a^b \underline{E} \cdot d\underline{l}$$

a ≡ Referencia
b ≡ r variable genérico

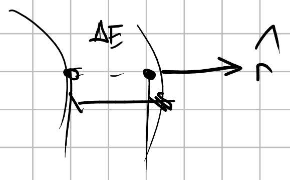
Elegir a

- ∞
- R
- 0
- 8 m

uso $a \to \infty$ porque tengo dist. de Q localizada

$\textcircled{II}$: $r > R$

$$V(r) = -\int_\infty^r \underline{E}(r) \cdot d\underline{l} = -\int_\infty^r E(r)\ \underbrace{\hat{r} \cdot dr\,\hat{r}}_{1} = -\int_\infty^r E(r)\, dr$$



$$V(r) = -\int_\infty^r \frac{\sigma}{\epsilon_0} R^2\, r^{-2}\, dr = -\frac{\sigma}{\epsilon_0} R^2 \left(-r^{-1}\right)\Big|_\infty^r$$

$$V(r) = \frac{\sigma R^2}{\epsilon_0}\left(\frac{1}{r} - \underbrace{\cancel{\lim_{r \to \infty} \frac{1}{r}}}_{V(\infty)}\right) \quad = 0$$

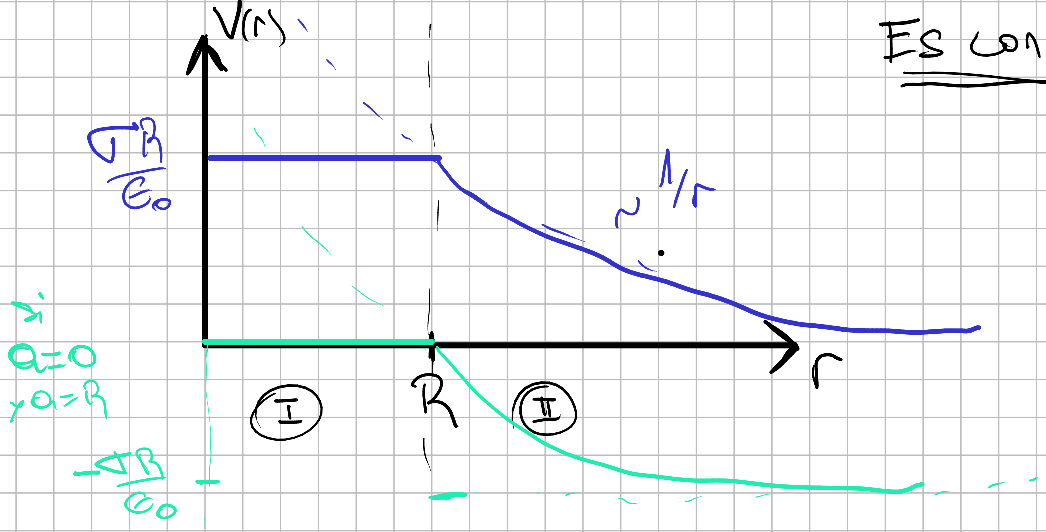
$$V(r) = \frac{\sigma R^2}{\epsilon_0\, r} \quad \text{si } r > R$$

$\textcircled{I}$ $r < R$

$$V(r) = \underbrace{-\int_\infty^R \underline{E}^{\textcircled{II}}(r) \cdot dr\,\hat{r}}\ \underbrace{-\cancel{\int_R^r \underline{E}^{\textcircled{I}}(r) \cdot dr\,\hat{r}}}_{} \overset{0}{=}$$

$$-\int_\infty^R \frac{\sigma}{\epsilon_0} \frac{R^2}{r^2}\, dr = \frac{\sigma}{\epsilon_0} R^2\ \frac{1}{r}\Big|_\infty^R = \frac{\sigma R^{\cancel{2}}}{\epsilon_0}\ \frac{1}{\cancel{R}} = \frac{\sigma R}{\epsilon_0}$$

$$\boxed{V(r) = \begin{cases} \dfrac{\sigma R}{\epsilon_0} & \text{si } r<R \\[2ex] \dfrac{\sigma R^2}{\epsilon_0 r} & \text{si } r>R \end{cases}}$$



Es continuo