11. Inductancias

Circuitos RL

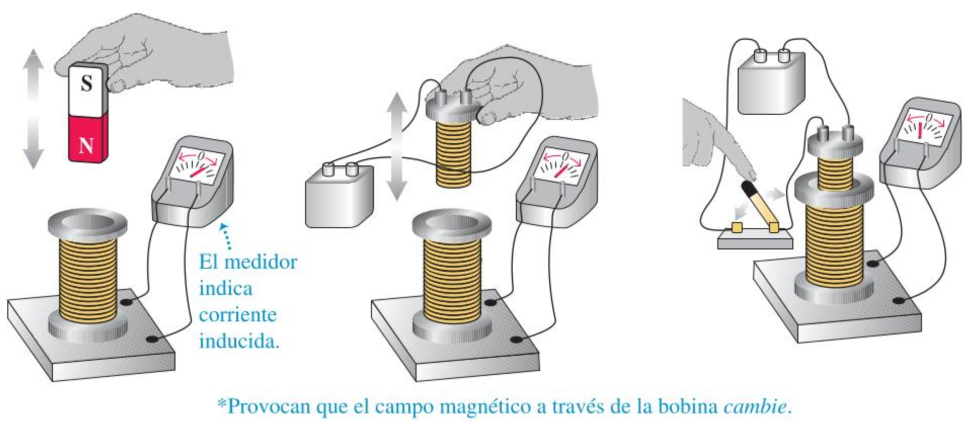
Circuitos RC

Ley de Faraday

Una ley, dos FEMs

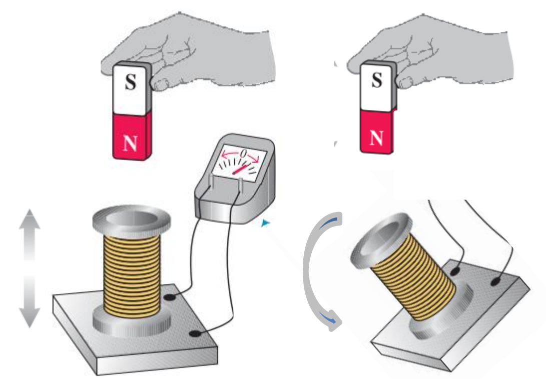
$$\varepsilon = -\frac{d\phi_M}{dt} \qquad \phi_M = \iint_S \vec{B}.\hat{n}\, dS$$

$\vec{B}(t) \rightarrow \vec{E}(t)$ →**FEM inducida**



El flujo concatenado varía porque el campo ***B*** que atraviesa la bobina **varía en el tiempo**

$\vec{B}(r)$ +desplazamiento→ **FEM movimiento**

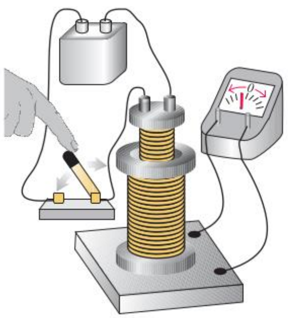


El campo ***B*** se mantiene constante, pero flujo concatenado varía porque la bobina se desplaza hacia zonas de mayor intensidad de ***B***

Cambia orientación relativa

Acoplamiento magnético

$$\varepsilon = -\frac{d\phi_M}{dt} \qquad \phi_M = \iint_S \vec{B}.\hat{n}\, dS$$



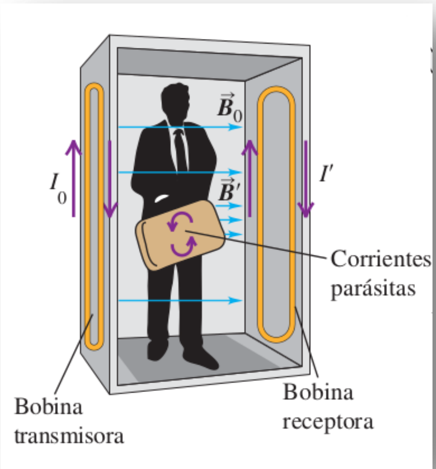
El campo ***B*** generado por **la bobina primaria** atraviesa la **bobina secundaria**.

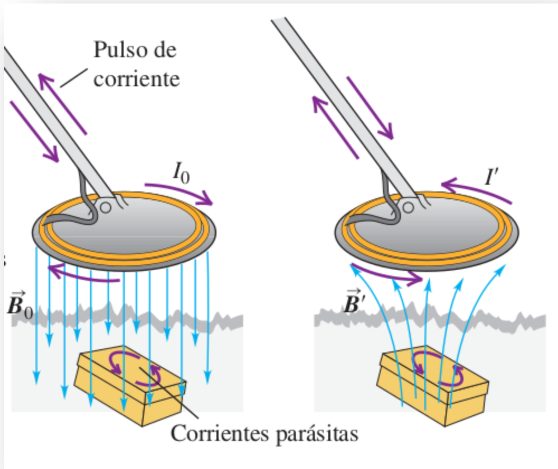
$$i_{prim}(t) \to \underline{B_{prim}(t) \to \phi_{sec}}(t) \to \varepsilon_{sec}(t) \to i_{sec}\ (t)$$

Ambos circuitos están **magnéticamente acoplados**

Detector de metales

$$\varepsilon = -\frac{d\phi_M}{dt} \qquad \phi_M = \iint_S \vec{B}.\hat{n}\,dS$$





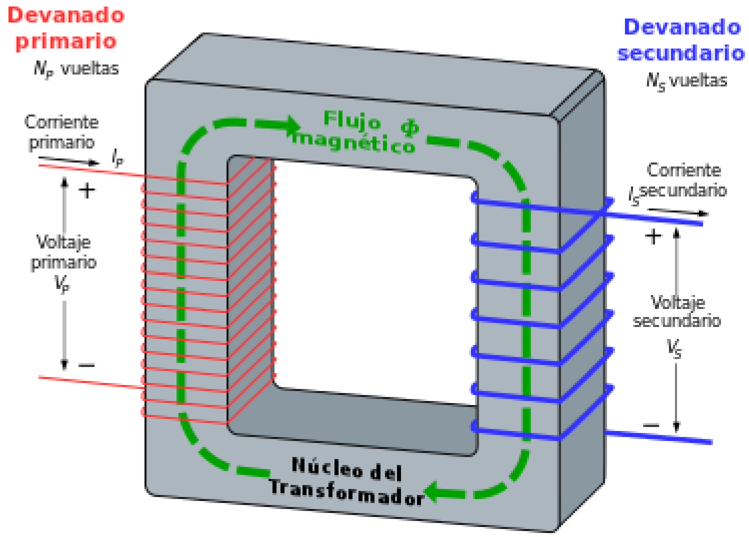
El campo ***B*** generado por **la bobina primaria** atraviesa la **bobina secundaria**.

$$i_{prim}(t) \to \underline{B_{prim}(t) \to \phi_{sec}(t)} \to \varepsilon_{sec}(t) \to i_{sec}\,(t) \to B_{sec}(t)$$

Ambos circuitos están **magnéticamente acoplados**

Transformador

$$\varepsilon = -\frac{d\phi_M}{dt} \qquad \phi_M = \iint_S \vec{B}.\hat{n}\, dS$$

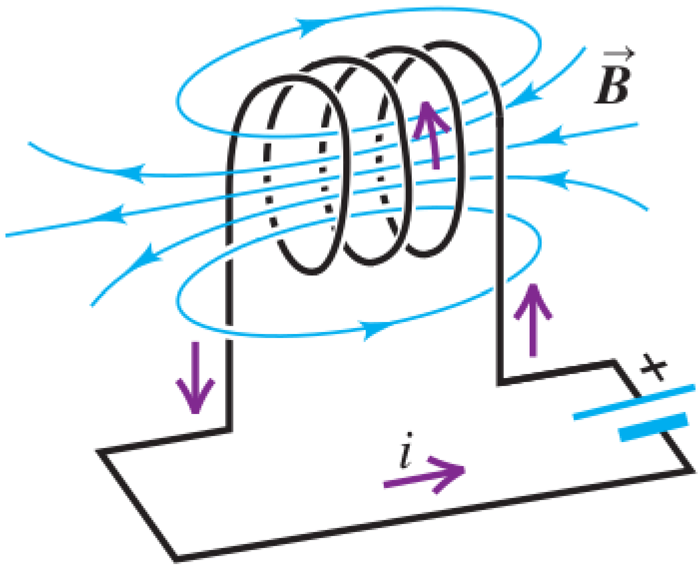


El campo ***B*** generado por **la bobina primaria** atraviesa la **bobina secundaria**.

$$\varepsilon_{prim}(t) \rightarrow i_{prim}(t) \rightarrow \underline{B_{prim}(t) \rightarrow \phi_{sec}(t)} \rightarrow \varepsilon_{sec}(t)$$

Ambos circuitos están **magnéticamente acoplados**

Autoinductancia



proporcional a *i*

$$\phi_M = \iint_S \vec{B}.\hat{n}\, dS \quad \sim L\, i$$

Coeficiente de auto-inducción

Es una constante que depende
de la geometría de las espiras

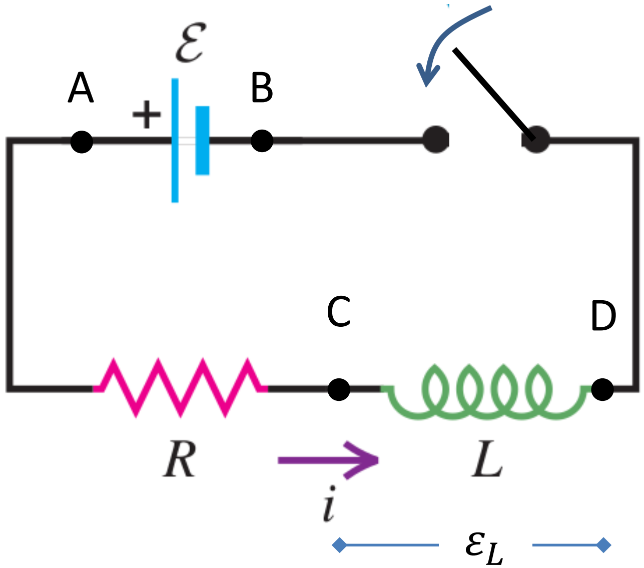
$$[L] = \frac{Webber}{A} = H \text{ (Henry)}$$

$$\boxed{\phi_M = L\, i}$$

En el caso en que *i=i(t)*

$$\varepsilon_{auto-inducida} = -\frac{d\phi_M}{dt} = -L\frac{di}{dt}$$

Circuito RL



$i(t \leq 0) = 0$

A t=0 se cierra la llave y comienza a circular $i(t)$

Sobre la bobina aparecera una FEM inducida (se opone al cambio, i.e. al crecimiento de *i*)

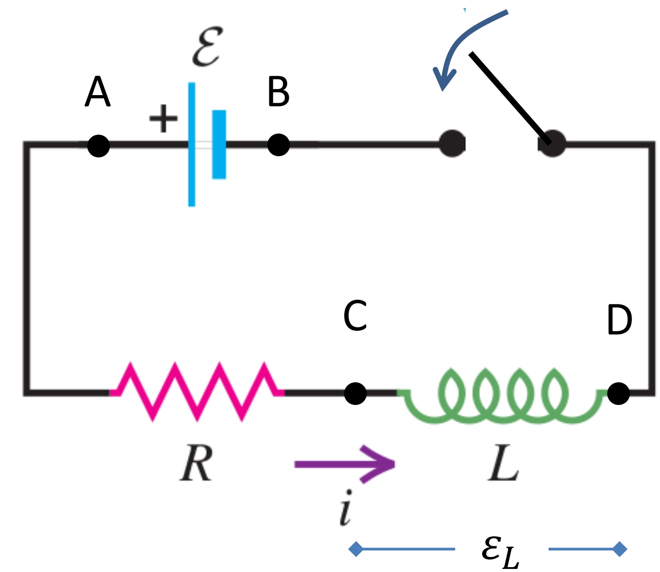
$$\varepsilon_L = -L\frac{di}{dt}$$

Ecuación para el circuito: $\varepsilon_L + \varepsilon = i\ R \longrightarrow \varepsilon = i\ R + L\frac{di}{dt}$

Ec diferencial para encontrar *i(t)*

$$\frac{di}{dt} = \frac{\varepsilon}{L} - \frac{R}{L}i$$

Circuito RL



A t=0 se cierra la llave y comienza a circular $i(t)$

$$\frac{di}{dt} = \frac{\varepsilon}{L} - \frac{R}{L} i \qquad\qquad i(t \leq 0) = 0$$

Para resolver esta ecuación hacemos lo mismo que hicimos para analizar la carga de un capacitor

$$\frac{di}{dt} = -\frac{R}{L}\left(i - \frac{\varepsilon}{R}\right)$$

$$\frac{di}{\left(i - \frac{\varepsilon}{R}\right)} = -\frac{R}{L} dt$$

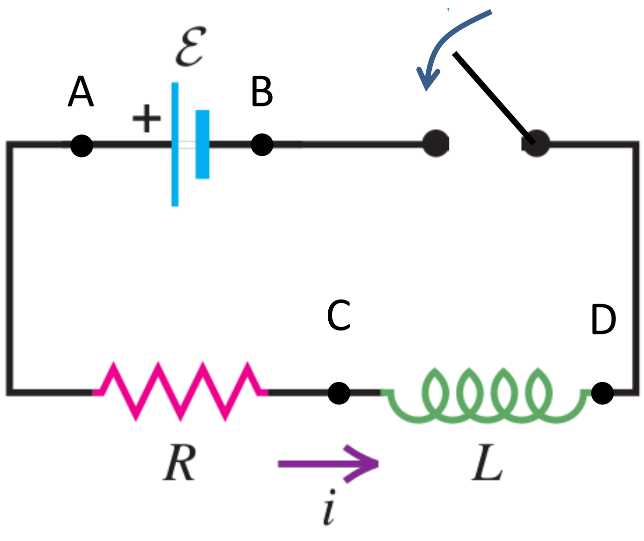
$$\int_0^i \frac{di'}{\left(i' - \frac{\varepsilon}{R}\right)} = -\int_0^t \frac{R}{L} dt'$$

$$\ln\left(\frac{i - \frac{\varepsilon}{R}}{-\frac{\varepsilon}{R}}\right) = -\frac{R}{L} t$$

$$\frac{i - \frac{\varepsilon}{R}}{-\frac{\varepsilon}{R}} = e^{-\frac{R}{L}t}$$

$$\boxed{i(t) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$

Circuito RL

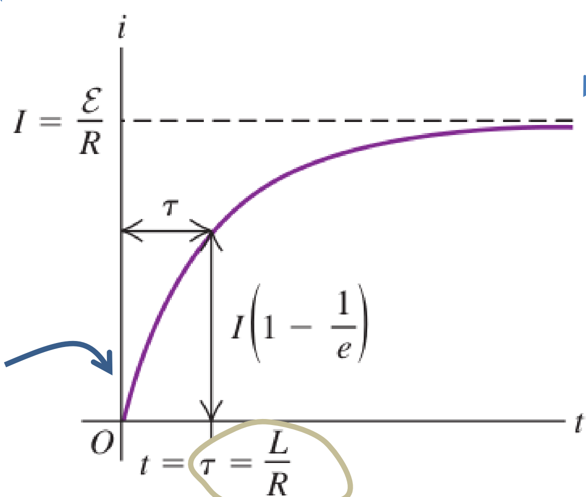


A t=0 se cierra la llave y comienza a circular $i(t)$

$$\frac{di}{dt} = \frac{\varepsilon}{L} - \frac{R}{L}i \qquad\qquad i(t \leq 0) = 0$$

Para resolver esta ecuación hacemos lo mismo que hicimos para analizar la carga de un capacitor

En el estacionario (t>>L/R) la bobina se comporta como un cable



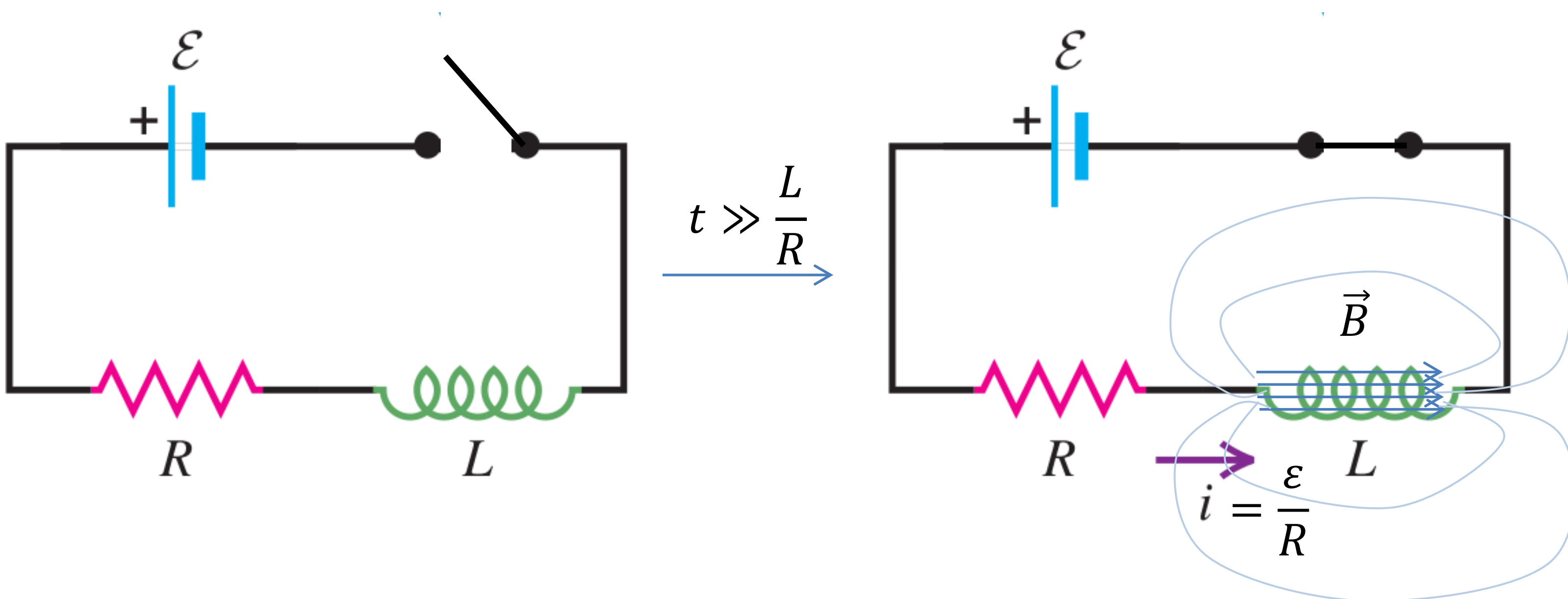
$$\boxed{i(t) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$

Justo a t=0

$$i(t = 0) = 0$$
$$\frac{di}{dt} = \frac{\varepsilon}{L}$$

Si $L \to \infty, \tau \to \infty$

Energía almacenada en un circuito RL



Cuanta energía entregó la batería en todo este tiempo?

$$P(t) = i(t)\varepsilon \;\; = i\left(i\,R + L\,\frac{di}{dt}\right) \longrightarrow \quad dE(t) = P(t)dt = i^2Rdt + Li\,di$$

Potencia instantanea entregada por la bateria

Energia entregada entre t y $t + dt$

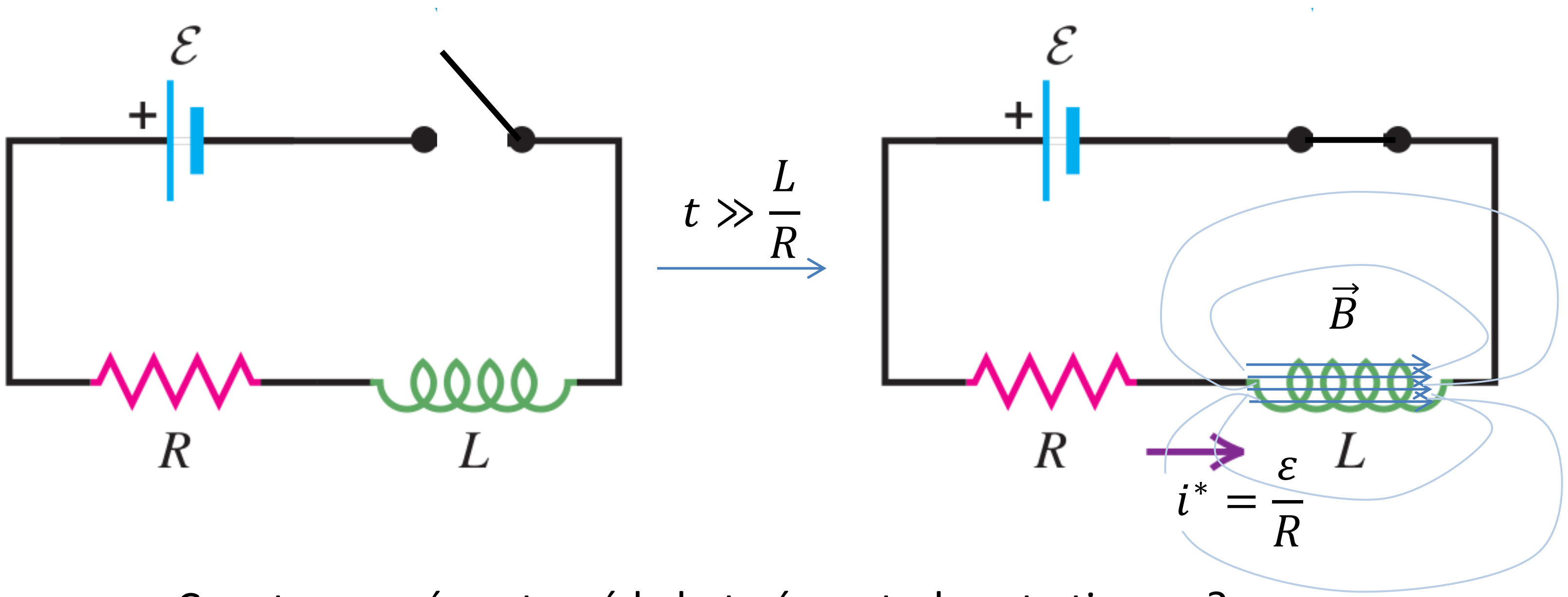
energia disipada en la resistencia

dE_R

energia alamacenada en L

dE_L

Energía almacenada en un circuito RL



Cuanta energía entregó la batería en todo este tiempo?

$$dE(t) = dE_R(t) + dE_L(i(t))$$

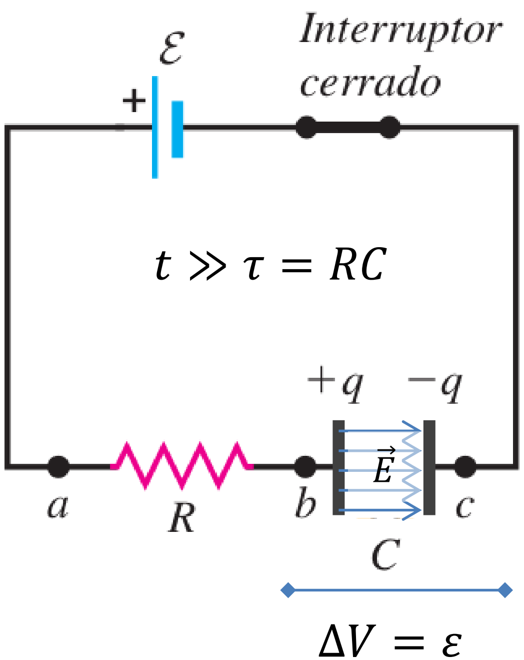
$$= i^2 R dt + Li\, di$$

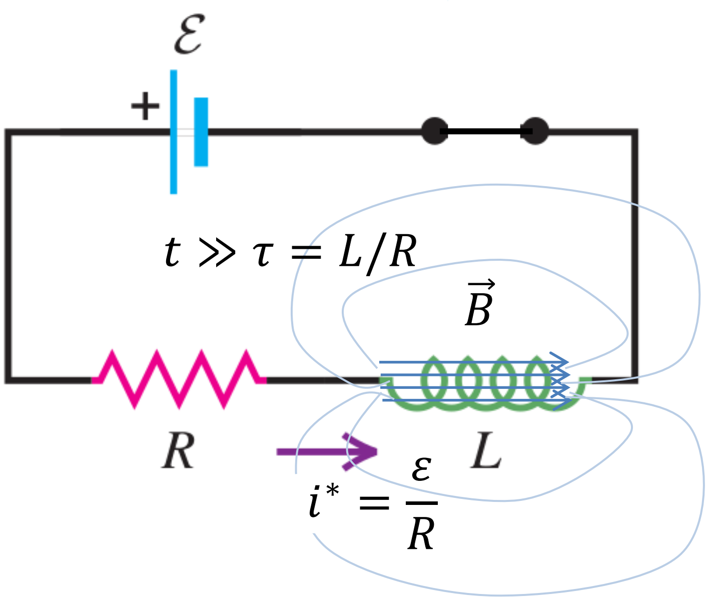
$$E_L = \int_{i=0}^{i=\frac{\varepsilon}{R}} Li\, di = \frac{Li^2}{2}\Bigg|_0^{\frac{\varepsilon}{R}}$$

$$\boxed{E_L = \frac{Li^{*2}}{2}}$$

Energia almacenada en la bobina (campo ***B*** creado en su interior)

Dispositivos para almacenar energía





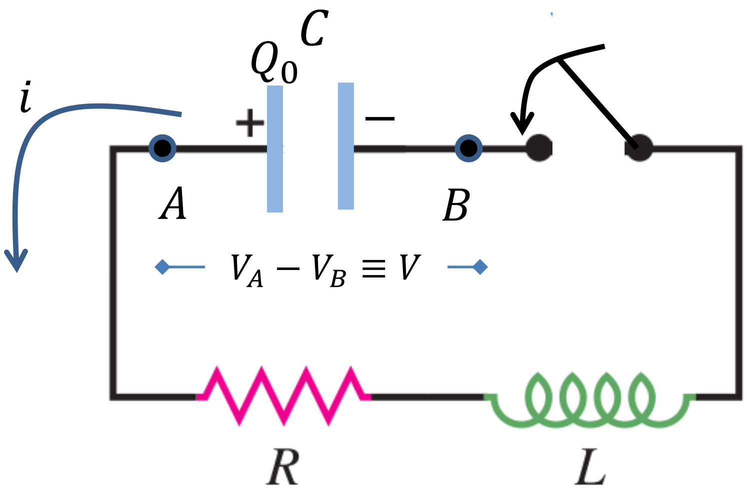
$$\mathrm{E}_C = \frac{C\varepsilon^2}{2}$$

Energia almacenada en el capacitor
(campo ***E*** creado en su interior)

$$\mathrm{E}_L = \frac{Li^{*2}}{2}$$

Energia almacenada en la bobina
(campo ***B*** creado en su interior)

Circuito RLC



$q(t)$: carga en el capacitor

$i(t)$: intensidad en el circuito

$$q(t=0) = Q_0$$

$$i(t=0) = 0$$

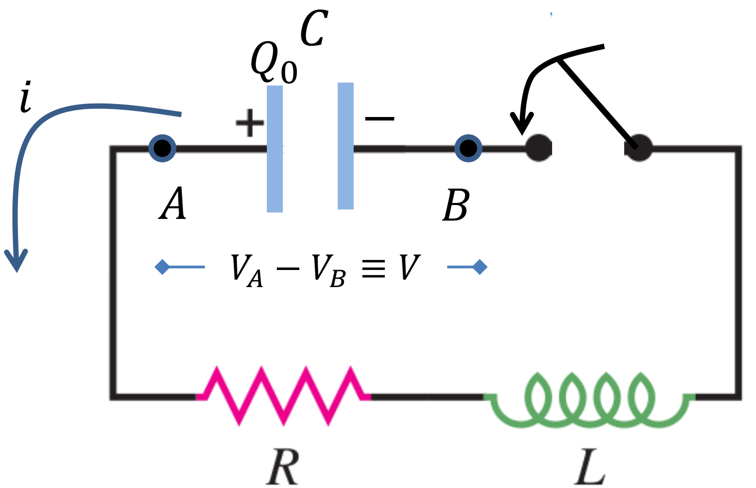
$$i(t) = -\frac{dq(t)}{dt}$$

$$v(t) - i(t)R \quad - L\frac{di(t)}{dt} = 0$$

$$\frac{q(t)}{C} + \frac{dq(t)}{dt}R + L\frac{d^2q(t)}{dt^2} = 0$$

$$\boxed{L\frac{d^2q(t)}{dt^2} + \frac{dq(t)}{dt}R + \frac{q(t)}{C} = 0}$$

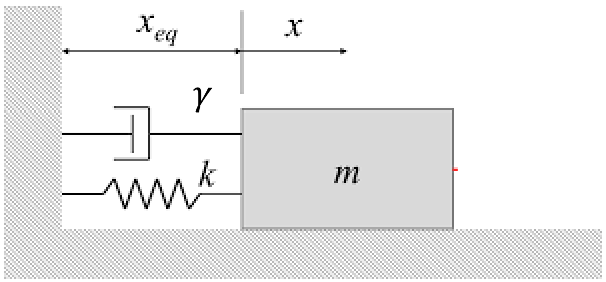
Sistemas equivalentes



$$L\frac{d^2q(t)}{dt^2} + \frac{dq(t)}{dt}R + \frac{q(t)}{C} = 0$$

$$q(t=0) = Q_0$$

$$i(t=0) = \frac{dq}{dt}(t=0) = 0$$

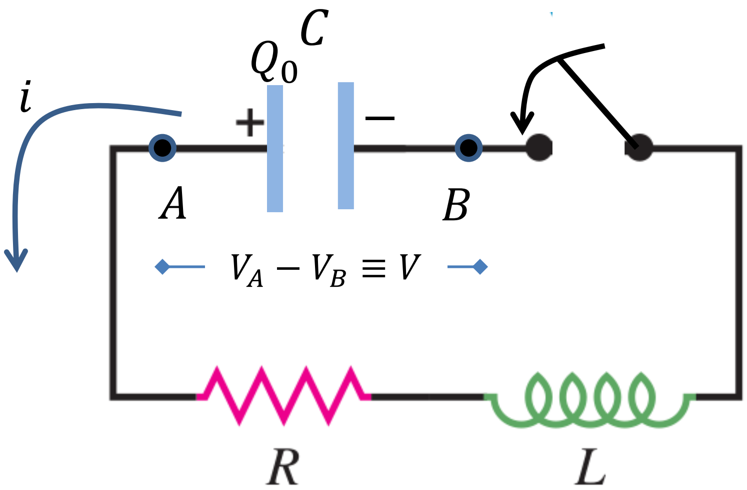


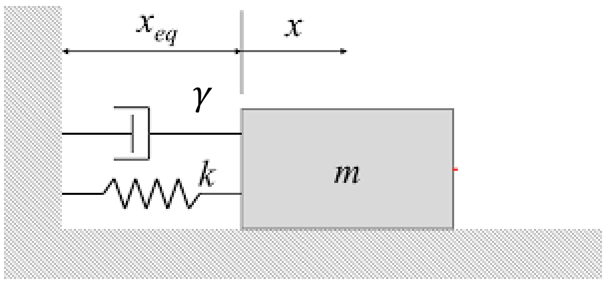
$$F = m\frac{d^2x(t)}{dt^2}$$

$$-kx - \gamma\frac{dx}{dt} = m\frac{d^2x(t)}{dt^2}$$

$$m\frac{d^2x(t)}{dt^2} + \gamma\frac{dx}{dt} + kx = 0$$

Sistemas equivalentes





$$L\frac{d^2q(t)}{dt^2} + \frac{dq(t)}{dt}R + \frac{q(t)}{C} = 0$$

$$q(t=0) = Q_0$$

$$i(t=0) = \frac{dq}{dt}(t=0) = 0$$

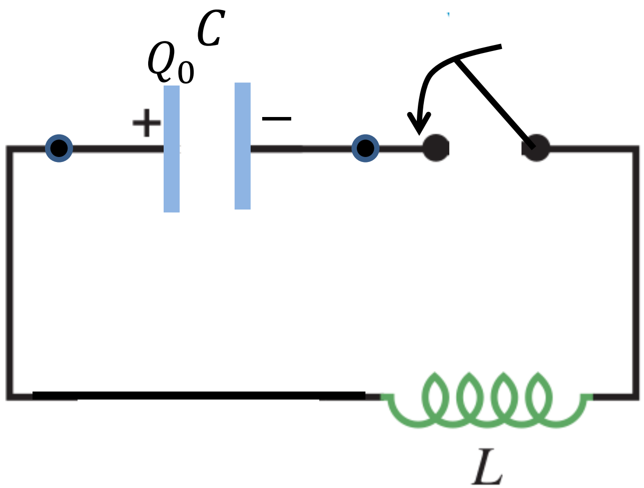
$$m\frac{d^2x(t)}{dt^2} + \gamma\frac{dx}{dt} + kx = 0$$

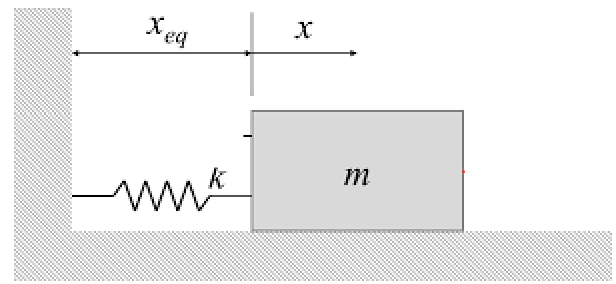
$$x(t=0) = x_0$$

$$v(t=0) = \frac{dx}{dt}(t=0) = v_0$$

$q \leftrightarrow x$ $\qquad$ $L \leftrightarrow m$ $\qquad$ $R \leftrightarrow \gamma$ $\qquad$ $k \leftrightarrow \frac{1}{C}$

Circuito LC (R=0)





$$L\frac{d^2q(t)}{dt^2} + \cancel{\frac{dq(t)}{dt}R} + \frac{q(t)}{C} = 0$$

$$\frac{d^2q}{dt^2} = -\overbrace{\frac{1}{LC}}^{\omega^2}q$$

Ec. de oscilador armónico

$$m\frac{d^2x(t)}{dt^2} + \gamma\cancel{\frac{dx}{dt}} + kx = 0$$

$$\frac{d^2x}{dt^2} = -\overbrace{\frac{k}{m}}^{\omega^2}x$$

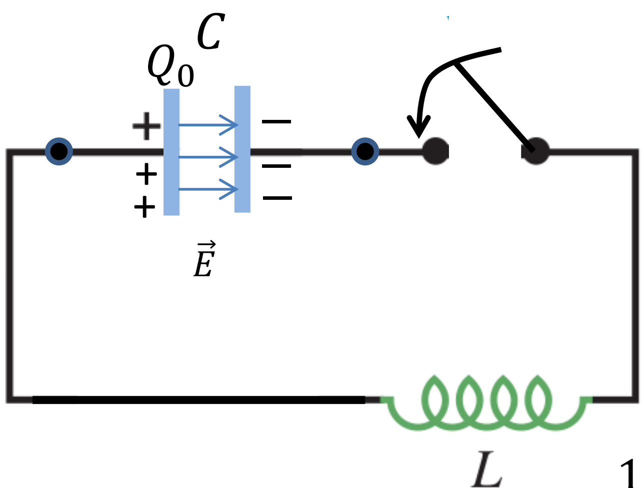
Soluciones: $q(t) = Q_0\cos(\omega t + \varphi)$

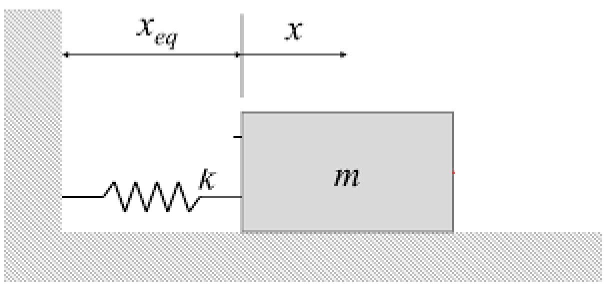
$$i(t) = -\frac{dq(t)}{dt} = Q_0\omega\sin(\omega t + \varphi)$$

Circuito LC (R=0)

Yo ya se que la dinámica dada por las ecs del O.A. conserva la suma $\mathrm{E} = K + \mathrm{U}$

$$\frac{d^2q}{dt^2} = -\omega^2 q$$





$q \leftrightarrow x \qquad L \leftrightarrow m \qquad k \leftrightarrow \dfrac{1}{C}$

$$K = \frac{mv^2}{2} = \frac{Li^2}{2}$$

$$U = \frac{kx^2}{2} = \frac{q^2}{2C}$$

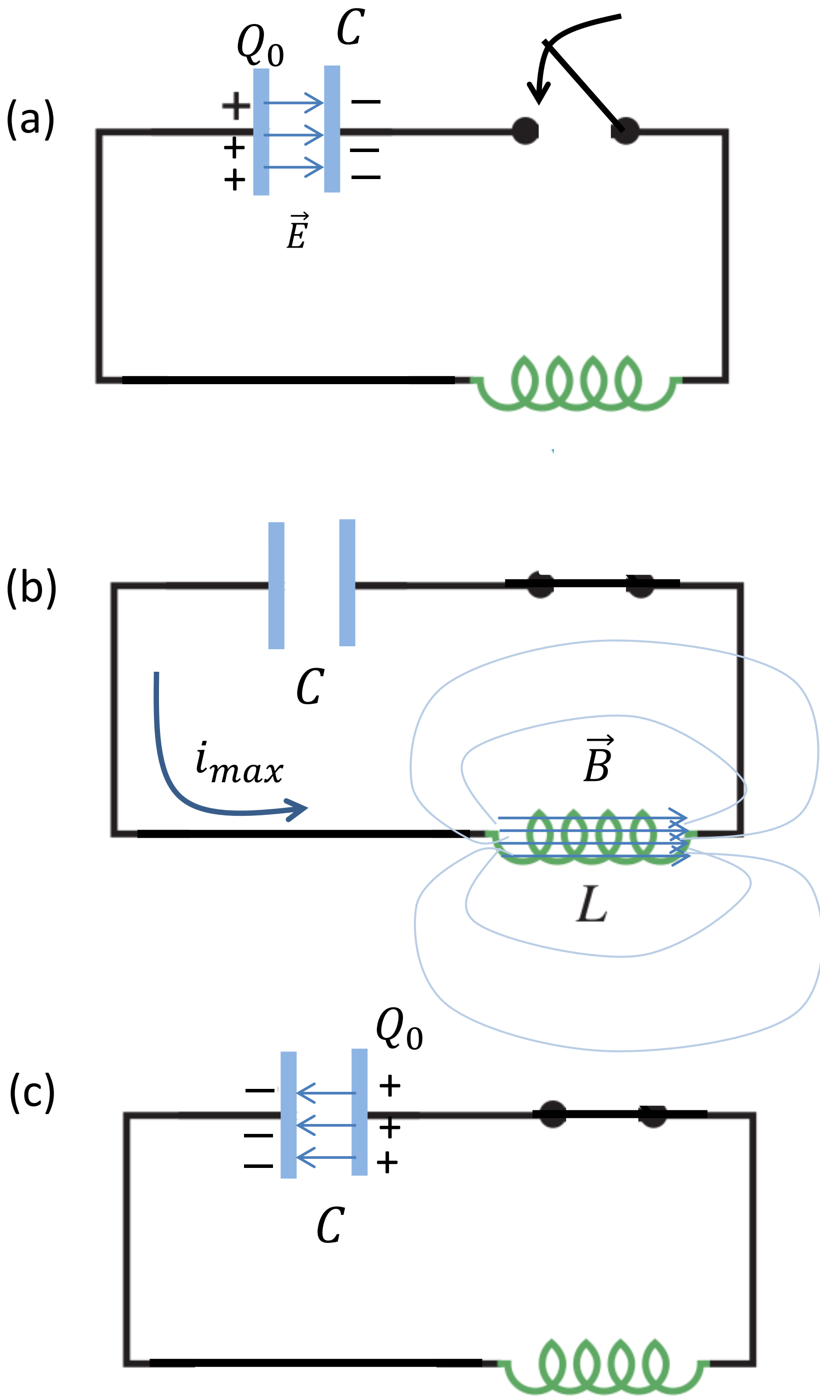
energía almacenada en L hace las veces de energía cinetica

energía almacenada en C hace las veces de energía potencial

A lo largo de la dinámica

$$U_{tot} = U_L(t) + U_C(t) = cte$$

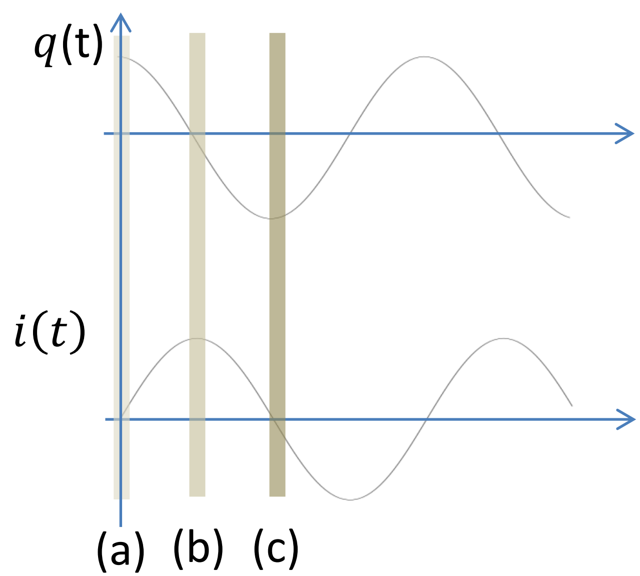
Circuito LC (R=0)



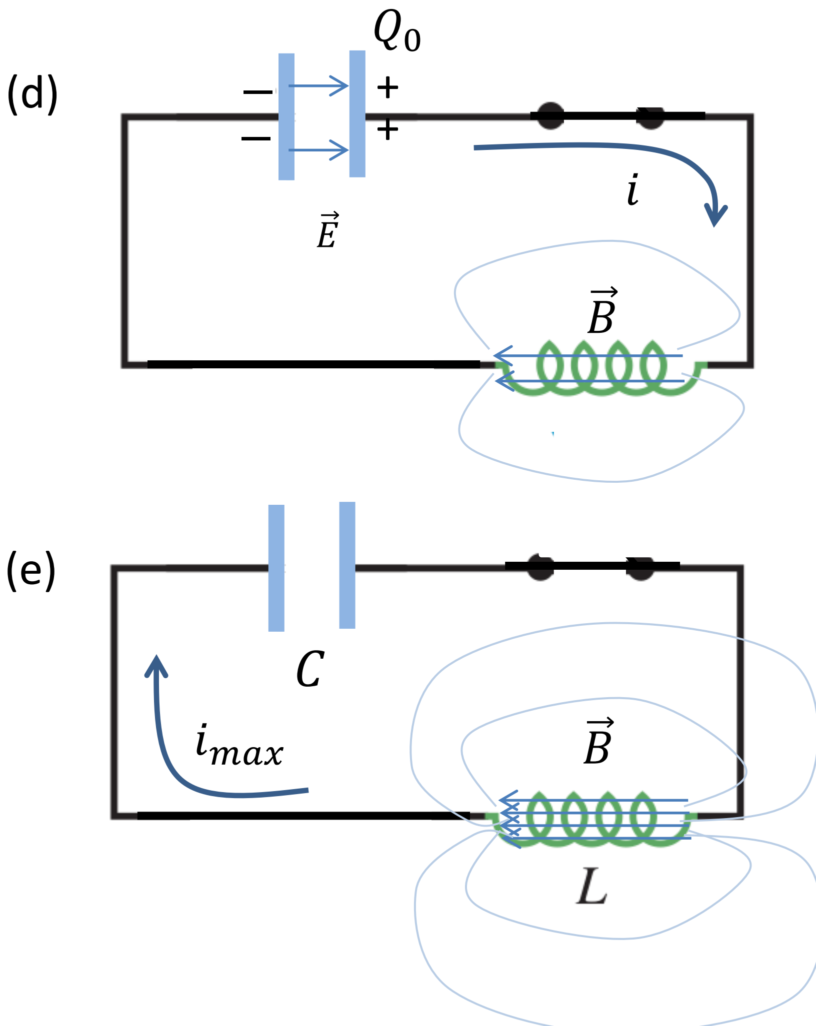
$$\frac{d^2q}{dt^2} = -\omega^2 q$$

$$q(t) = Q_0 \cos(\omega t + \varphi)$$

$$i(t) = -\frac{dq(t)}{dt} = Q_0\omega \sin(\omega t + \varphi)$$



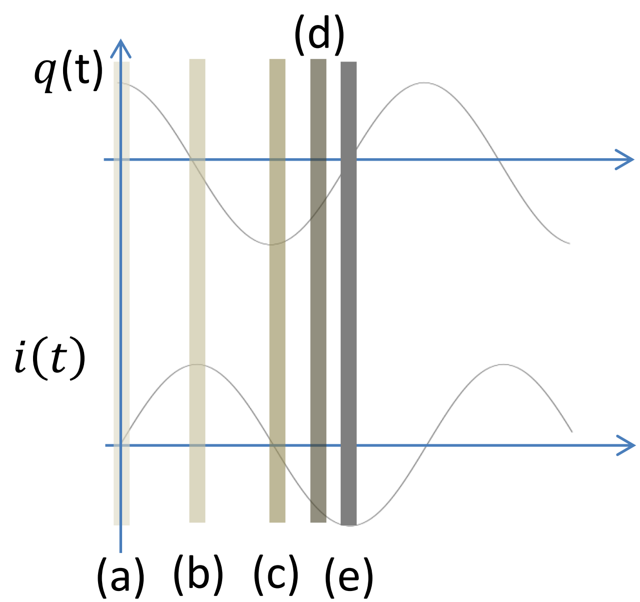
Circuito LC (R=0)



$$\frac{d^2q}{dt^2} = -\omega^2 q$$

$$q(t) = Q_0 \cos(\omega t + \varphi)$$

$$i(t) = -\frac{dq(t)}{dt} = Q_0 \omega \sin(\omega t + \varphi)$$



Desde el pto de vista energético
Circuito LC (R=0)

$$\frac{d^2q}{dt^2} = -\omega^2 q$$

$$q(t) = Q_0 \cos(\omega t + \varphi)$$

$$i(t) = -\frac{dq(t)}{dt} = Q_0 \omega \sin(\omega t + \varphi)$$

A lo largo de la dinámica

$$U_{tot} = \underset{\substack{\uparrow \\ \frac{Li^2}{2}}}{U_L(t)} + \underset{\substack{\uparrow \\ \frac{q^2}{2C}}}{U_C(t)} = cte$$

$$U_L(t) + U_C(t) = \frac{L(Q_0 \omega \sin(\omega t + \varphi))^2}{2} + \frac{(Q_0 \cos(\omega t + \varphi))^2}{2C}$$

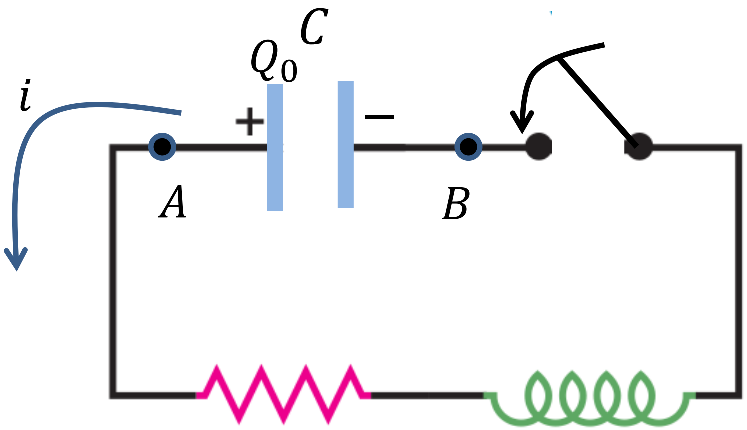
$$\frac{1}{LC}$$

$$= \frac{L{Q_0}^2 \omega^2}{2} (\sin(\omega t + \varphi))^2 + \frac{Q_0}{2C} (\cos(\omega t + \varphi))^2$$

$$= \frac{Q_0}{2C} [(\sin(\omega t + \varphi))^2 + (\cos(\omega t + \varphi))^2] = \frac{Q_0}{2C}$$

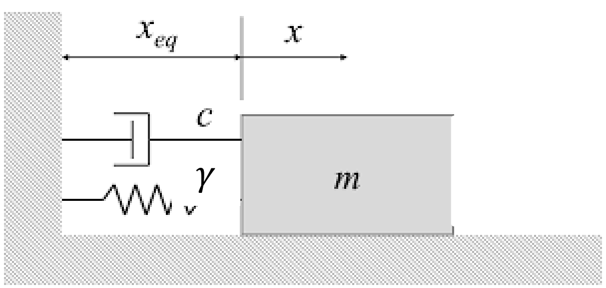
Energía inicial
del sistema

Circuito RLC

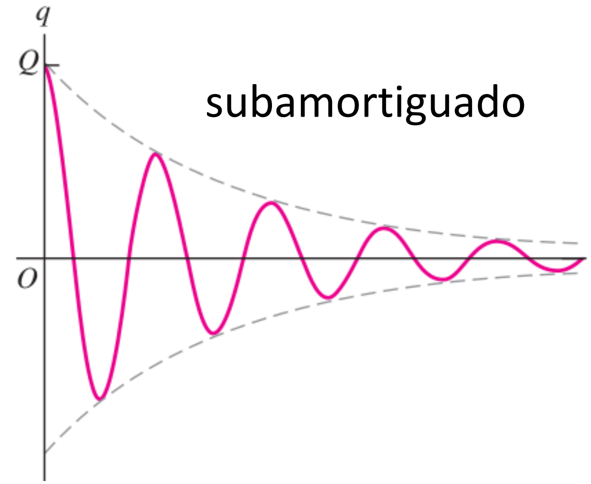


$$L\frac{d^2q(t)}{dt^2}+\frac{dq(t)}{dt}R+\frac{q(t)}{C}=0$$

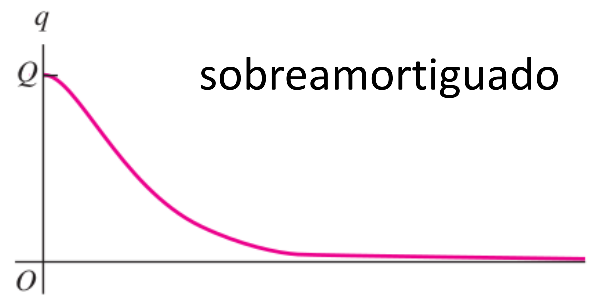
$$m\frac{d^2x(t)}{dt^2}+\gamma\frac{dx}{dt}+kx=0$$



$$R<\sqrt{\frac{4L}{C}}$$



$$R>\sqrt{\frac{4L}{C}}$$



$$R=\sqrt{\frac{4L}{C}}$$