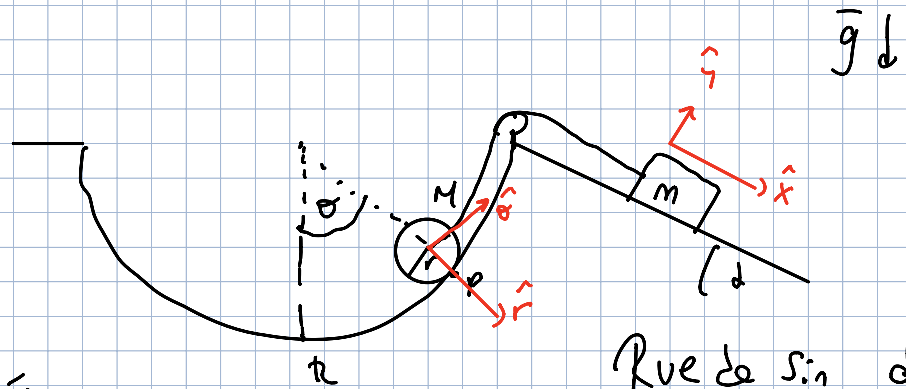
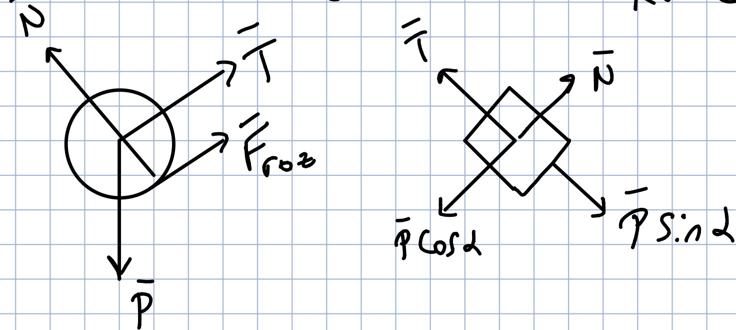


Rueda sin deslizar.



Vínculos: $\bar{r}_{cm} = (R-r)\hat{r}$ $\quad \dot{r}_{cm} = 0 = \ddot{r}_{cm}$ $\quad (R-r) = \tilde{R}$

$\dot{y} = \ddot{y} = 0$ $\quad L_{soga} = \tilde{R}\left(\frac{\pi}{2} - \theta\right) + x - x_p \Rightarrow \boxed{\begin{array}{l} \tilde{R}\dot{\theta} = \dot{x} \\ \tilde{R}\ddot{\theta} = \ddot{x} \end{array}}$

Newton: (1) $M\tilde{R}\ddot{\theta} = T + F_{roz} - Mg\sin\theta \quad (\hat{\theta})$

$-M\tilde{R}\dot{\theta}^2 = Mg\cos\theta - N \quad (\hat{r})$

(2) $m\ddot{x} = mg\sin\alpha - T$

$m\ddot{y} = N - mg\cos\alpha = 0$

Condición de Rigidez: $\bar{V}_p = \bar{V}_{cm} + \bar{\Omega} \times (\bar{r}_p - \bar{r}_{cm})$

Elijo el punto P que esté en contacto con el piso.

$\bar{V}_p = 0 = \bar{V}_{cm} + \bar{\Omega} \times r\hat{r}$ $\qquad$ $\bar{\Omega} = \Omega \hat{z}$

$\bar{V}_{cm} = -\Omega r \hat{\theta}$ $\qquad$ $\bar{V}_{cm} = \tilde{R}\dot{\theta}\hat{\theta}$

$\Rightarrow \tilde{R}\dot{\theta} = -\Omega r$ $\qquad$ $\boxed{\Omega = \frac{(R-r)}{r}\dot{\theta}}$ Esto pedía el item b.

Hago (1) + (2) usando $\ddot{x} = \tilde{R}\ddot{\theta}$

$$(M+m)\tilde{R}\ddot{\theta} = mg\sin\alpha - Mg\sin\theta + F_{roz} \quad (3)$$

Para tener la ec. de movimiento me falta saber F_{roz}.

Ec. de torques: $\mathbb{I}\dot{\Omega} = \sum \bar{\tau}$ $\qquad$ $\mathbb{I}^{cm} = M\frac{r^2}{2}$

Quiero ver los torques desde el CM.

$\bar{P}$ y $\bar{T}$ no hacen torque porque actuan en el CM.

$\bar{\tau}_{\bar{N}} = r\hat{r} \times N\hat{r} = 0$ $\qquad$ $\bar{\tau}_{F_{roz}} = r\hat{r} \times F_{roz}\hat{\theta} = rF_{roz}\hat{z}$

$M\frac{r^2}{2}\dot{\Omega} = rF_{roz}$ $\qquad$ $\boxed{F_{roz} = M\frac{r}{2}\dot{\Omega}}$

Tenemos $\Omega = -\frac{\tilde{R}}{r}\dot{\theta} \Rightarrow \dot{\Omega} = -\frac{\tilde{R}}{r}\ddot{\theta}$

$\Rightarrow \boxed{F_{roz} = -M\frac{\tilde{R}}{2}\ddot{\theta}}$ $\qquad$ Reemplazo en (3)

$$(M+m)\tilde{R}\ddot{\theta} = mg\sin\alpha - Mg\sin\theta - M\frac{\tilde{R}}{2}\ddot{\theta}$$

$$\Rightarrow \left(\frac{3}{2}M+m\right)\tilde{R}\ddot{\theta} = g\,(m\sin\alpha - M\sin\theta)$$

$$\boxed{\ddot{\theta} = \frac{g(m\sin\alpha - M\sin\theta)}{\tilde{R}\left(\frac{3}{2}M+m\right)}}$$

Equilibrio: $\ddot{\theta} = 0 = \dfrac{g(m\sin\alpha - M\sin\theta)}{\tilde{R}\left(\frac{3}{2}M+m\right)} \Rightarrow \boxed{\sin\theta_{eq} = \dfrac{m}{M}\sin\alpha}$

Estabilidad: $\left.\dfrac{d\ddot{\theta}}{d\theta}\right|_{\theta_{eq}} = \dfrac{-Mg\cos\theta_{eq}}{\tilde{R}\left(\frac{3}{2}M+m\right)}$

$$\sin\theta_{eq} = \frac{m}{M}\sin\alpha \Rightarrow \quad 0 \leq \sin\theta_{eq} \leq 1 \Rightarrow \theta_{eq} \in \left[0, \frac{\pi}{2}\right]$$

$$\text{Si } \theta_{eq} \in \left[0, \frac{\pi}{2}\right] \Rightarrow \cos\theta_{eq} \in [0, 1] \Rightarrow \cos\theta_{eq} \geq 0$$

$$\Rightarrow \left.\frac{d\ddot{\theta}}{d\theta}\right|_{\theta_{eq}} = \frac{-Mg\cos\theta_{eq}}{\tilde{R}\left(\frac{3}{2}M+m\right)} \leq 0 \quad \text{Estable}$$