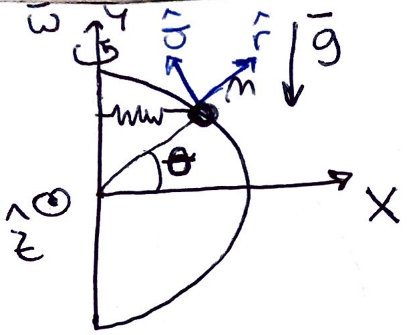


$\cancel{\bar{F}_{centrífuga}}$

$$\bar{F}_{cent.} = -m\bar{\omega}\times(\bar{\omega}\times\bar{r})$$

$$\bar{F}_{cor} = -2m\bar{\omega}\times\dot{\bar{r}}$$

$$\bar{F}_{cent} = -m\bar{\omega}\times(\bar{\omega}\times(R\cos\theta\,\hat{x} + R\sin\theta\,\hat{y}))$$

$$= -m\omega\hat{y}\times(\omega\hat{y}\times(R\cos\theta\,\hat{x} + R\sin\theta\,\hat{y}))$$

$$= -m\omega\,\hat{y}\times(\omega R\cos\theta(-\hat{z})) = m\omega^2 R\cos\theta\,\hat{x}$$

$$\boxed{\bar{F}_{cent} = m\omega^2 R\cos\theta\,\hat{x}}$$

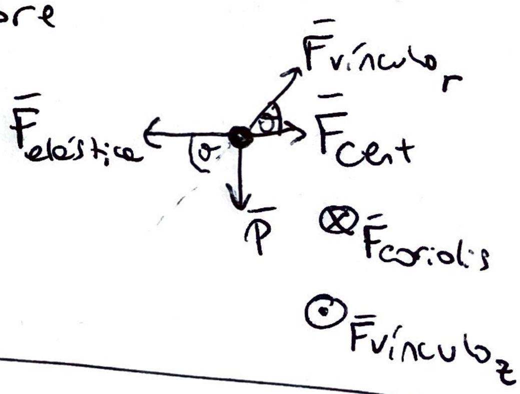
$$\bar{\omega} = \omega\hat{y} \qquad \hat{y} = \sin\theta\,\hat{r} + \cos\theta\,\hat{\theta} \Rightarrow \bar{\omega} = \omega(\sin\theta\,\hat{r} + \cos\theta\,\hat{\theta})$$

$$\bar{F}_{cor} = -2m\,\omega(\sin\theta\,\hat{r} + \cos\theta\,\hat{\theta})\times\underbrace{R\dot{\theta}\,\hat{\theta}}_{\dot{\bar{r}}}$$

$$\bar{F}_{cor} = -2m\omega\sin\theta R\dot{\theta}\underbrace{(\hat{r}\times\hat{\theta})}_{\hat{z}} \qquad \boxed{\bar{F}_{cor} = -2m\omega R\dot{\theta}\sin\theta\,\hat{z}}$$

$$\dot{\omega} = 0 \Rightarrow \boxed{\bar{F}_{arrastre} = 0}$$

Diagrama de cuerpo libre



Vínculos: $r = R \quad \dot{r} = \ddot{r} = 0$

$z = 0 \quad \dot{z} = \ddot{z} = 0$

(solidario al riel)

~~Newton:~~

$\cancel{-mR\dot{\theta}^2 = F_v - mg\sin\theta - kR\cos\theta}$

$$\bar{F}_{el} = -kx\,\hat{x} = -kR\cos\theta\,\hat{x} \qquad \hat{x} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

$$\bar{F}_{el} = -kR\cos\theta\,(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$$

Newton: $\hat{r})\ -mR\dot{\theta}^2 = -kR\cos^2\theta + F_{v_r} + m\omega^2 R\cos^2\theta - mg\sin\theta$

$\hat{\theta})\ mR\ddot{\theta} = kR\cos\theta\sin\theta - m\omega^2 R\cos\theta\sin\theta - mg\cos\theta$

$\hat{z})\ m\ddot{z} = 0 = -2m\omega R\dot{\theta}\sin\theta + F_{v_z} \Rightarrow \boxed{F_{v_z} = 2m\omega R\dot{\theta}\sin\theta}$

Para los equilibrios estudiamos la ecuación de $\hat{\theta}$.

$$mR\ddot{\theta} = kR\cos\theta\sin\theta - m\omega^2 R\cos\theta\sin\theta - mg\cos\theta$$

$$\ddot{\theta} = \frac{k}{m}\cos\theta\sin\theta - \omega^2\cos\theta\sin\theta - \cancel{m}g\cos\theta/R$$

$$\boxed{\ddot{\theta} = \cos\theta\left[\left(\frac{k}{m} - \omega^2\right)\sin\theta - \cancel{m}g/R\right]}$$

En los puntos de equilibrio $\ddot{\theta}\big|_{\theta eq} = 0$

$$0 = \cos\theta_{eq}\left[\left(\frac{k}{m} - \omega^2\right)\sin\theta_{eq} - \frac{\cancel{m}g}{R}\right] \Rightarrow \boxed{\theta^1_{eq} = \frac{\pi}{2} \quad \theta^2_{eq} = -\frac{\pi}{2}}$$

Si $\cos\theta_{eq} \neq 0 \Rightarrow \left(\frac{k}{m} - \omega^2\right)\sin\theta_{eq} - \cancel{m}g/R = 0$

$$\sin\theta_{eq} = \frac{\cancel{m}g/R}{\frac{k}{m} - \omega^2} \qquad \boxed{\sin\theta_{eq} = \frac{g/R}{\frac{k}{m} - \omega^2}} \Rightarrow \text{Este punto existe si:}$$

$$-1 \leq \frac{g/R}{\frac{k}{m} - \omega^2} \leq 1$$

Ahora usamos que $\omega = \sqrt{\frac{2k}{m}}$

$$\sin\vartheta_{eq} = \frac{g}{R\left(\frac{k}{m}-\frac{2k}{m}\right)} = \frac{-g}{R\frac{k}{m}} = \frac{-mg}{Rk} \qquad \vartheta_{eq} = \arcsin\left(-\frac{mg}{kR}\right)$$

Estabilidad:

$$\ddot\vartheta = \cos\vartheta\left[\left(\frac{k}{m}-\frac{2k}{m}\right)\sin\vartheta - \frac{g}{R}\right]$$

$$\ddot\vartheta = \cos\vartheta\left[-\frac{k}{m}\sin\vartheta - \frac{g}{R}\right]$$

$$\frac{d\ddot\vartheta}{d\vartheta} = -\sin\vartheta\left[-\frac{k}{m}\sin\vartheta - \frac{g}{R}\right] + \cos\vartheta\left[-\frac{k}{m}\cos\vartheta\right]$$

$$= \frac{k}{m}\left(\sin^2\vartheta - \cos^2\vartheta\right) + \frac{g}{R}\sin\vartheta$$

$$= \frac{k}{m}\left(2\sin^2\vartheta - 1\right) + \frac{g}{R}\sin\vartheta$$

$$\left.\frac{d\ddot\vartheta}{d\vartheta}\right|_{\frac{\pi}{2}} = \frac{k}{m}(2-1) + \frac{g}{R} = \frac{k}{m} + \frac{g}{R} > 0 \Rightarrow \text{inestable}$$

$$\left.\frac{d\ddot\vartheta}{d\vartheta}\right|_{-\frac{\pi}{2}} = \frac{k}{m}(2-1) - \frac{g}{R} = \frac{k}{m} - \frac{g}{R} \Rightarrow \text{si } \frac{k}{m} > \frac{g}{R} \text{ es inestable}$$

$$\text{si } \frac{g}{R} > \frac{k}{m} \text{ es estable}$$

$$\left.\frac{d\ddot\vartheta}{d\vartheta}\right|_{\arcsin\left(-\frac{mg}{kR}\right)} = \frac{k}{m}\left(2\frac{m^2g^2}{k^2R^2} - 1\right) - \frac{g}{R}\frac{mg}{kR} =$$

$$= 2\frac{g^2m}{R^2k} - \frac{k}{m} - \frac{g^2m}{R^2k} = \frac{g^2m}{R^2k} - \frac{k}{m} = \frac{m}{k}\left(\frac{g^2}{R^2} - \frac{k^2}{m^2}\right)$$

$$\Rightarrow \text{Si } \frac{g}{R} > \frac{k}{m} \text{ es inestable}$$

$$\text{Si } \frac{g}{R} < \frac{k}{m} \text{ es estable}$$

c) $\frac{mg}{kR} < 1 \Rightarrow \frac{g}{R} < \frac{k}{m} \Rightarrow$ El único equilibrio estable es $\sin\theta_{eq} = -\frac{mg}{kR}$

Buscamos la ecuación de movimiento alrededor de ese punto.

$$\ddot{\theta} = \cos\theta\left[-\frac{k}{m}\sin\theta - \frac{g}{R}\right]$$

$$\theta = \theta_{eq} + \varepsilon \qquad \dot{\theta} = \dot{\varepsilon} \qquad \ddot{\theta} = \ddot{\varepsilon}$$

$$\varepsilon \ll 1$$

$$\cos(\theta_{eq}+\varepsilon) \cong \cos\theta_{eq} - \sin\theta_{eq}\,\varepsilon \qquad \text{(TAYLOR)}$$

$$\sin(\theta_{eq}+\varepsilon) \cong \sin\theta_{eq} + \cos\theta_{eq}\,\varepsilon$$

$$\ddot{\varepsilon} = (\cos\theta_{eq} - \sin\theta_{eq}\,\varepsilon)\left[-\frac{k}{m}(\sin\theta_{eq} + \cos\theta_{eq}\,\varepsilon) - \frac{g}{R}\right]$$

$$\ddot{\varepsilon} = \boxed{\cos\theta_{eq}\left[-\frac{k}{m}\sin\theta_{eq} - \frac{g}{R}\right]} - \cos^2\theta_{eq}\frac{k}{m}\varepsilon + \frac{k}{m}\sin^2\theta_{eq}\,\varepsilon + \frac{g}{R}\sin\theta_{eq}\,\varepsilon + \sin\theta_{eq}\cos\theta_{eq}\frac{k}{m}\varepsilon^2$$

Es la ec. de movimiento evaluada en θ_{eq}. Entonces vale 0.

como estoy viendo pequeños apartamientos desprecio este término ($\varepsilon^2 \ll \varepsilon \ll 1$)

Entonces la ecuación de movimiento que nos queda es:

$$\ddot{\varepsilon} = \left[\frac{k}{m}(\sin^2\theta_{eq} - \cos^2\theta_{eq}) + \frac{g}{R}\sin\theta_{eq}\right]\varepsilon$$

$$\ddot{\varepsilon} = \left[\frac{k}{m}\left(2\sin^2\vartheta_{eq} - 1\right) + \frac{g}{R}\sin\vartheta_{eq}\right]\varepsilon \qquad \sin\vartheta_{eq} = \frac{-mg}{kR}$$

$$\ddot{\varepsilon} = \left[\frac{k}{m}\left(\frac{2m^2g^2}{k^2R^2} - 1\right) - \frac{mg^2}{kR^2}\right]\varepsilon$$

$$\ddot{\varepsilon} = \left[\frac{2mg^2}{kR^2} - \frac{mg^2}{kR^2} - \frac{k}{m}\right]\varepsilon$$

$$\boxed{\ddot{\varepsilon} = \left(\frac{mg^2}{kR^2} - \frac{k}{m}\right)\varepsilon}$$

Ecuación de oscilador armónico con frecuencia

$$\omega = \sqrt{\left(\frac{mg^2}{kR^2} - \frac{k}{m}\right)}$$

$$\Rightarrow T = \frac{2\pi}{\omega} \qquad \boxed{T = \frac{2\pi}{\sqrt{\left(\frac{mg^2}{kR^2} - \frac{k}{m}\right)}}}$$