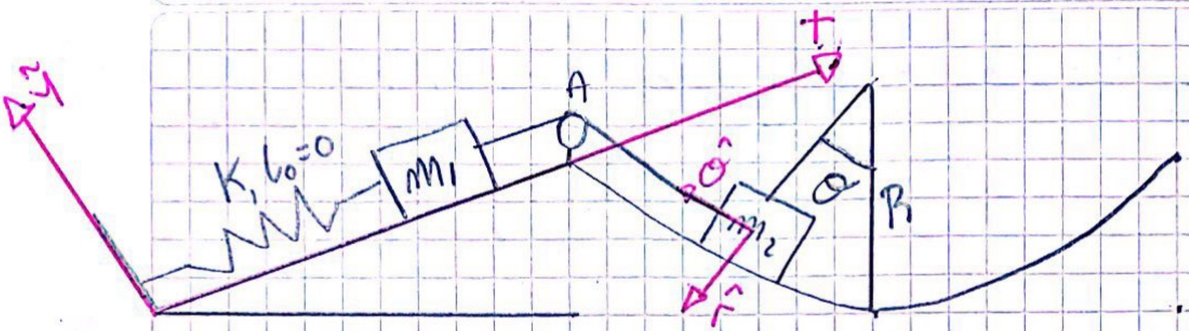
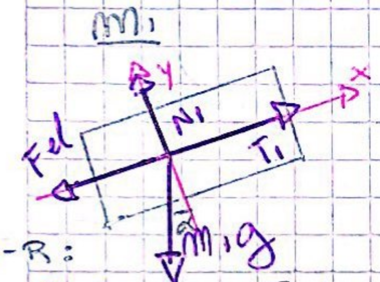
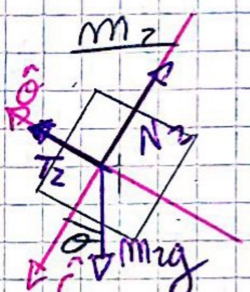
PROBLEMA 1



(a) DCL





PAR A-R:

$\bar{F}^{el} \rightarrow -\bar{F}^{el}$ en el resorte

$m_1\bar{g} \rightarrow -m_1\bar{g}$ en el centro de la Tierra

$\bar{N}_1 \rightarrow -\bar{N}_1$ en el plano inclinado

$\bar{T}_1 \rightarrow -\bar{T}_1$ en la soga

$m_2\bar{g} \rightarrow -m_2\bar{g}$ en la Tierra

$\bar{T}_2 \rightarrow -\bar{T}_2$ en la soga

$\bar{N}_2 \rightarrow -\bar{N}_2$ en la semicircunferencia

obs: $T_1 \quad T_2$

$T_1 = T_2 = T$

Vínculos

$\dot{r} = \ddot{r} = 0 \qquad r = R \qquad \dot{y}_1 = \ddot{y}_1 = 0$

$$L_{SOGA} = L - x_1 + R\left(\frac{\pi}{2} - \theta\right) \longrightarrow \ddot{x}_1 = -R\ddot{\theta}$$

Newton

$$m_1 \begin{cases} (\hat{x}) \; -Kx_1 + T - m_1 g \,.\, \operatorname{sen}\alpha = m_1 \ddot{x}_1 \quad (1) \\ (\hat{y}) \; N_1 - m_1 g \cos\alpha = 0 \end{cases}$$

$$m_2 \begin{cases} (\hat{r}) \; m_2 g \,.\, \cos\theta - N_2 = -m_2 R\dot{\theta}^2 \\ (\hat{\theta}) \; T - m_2 g \operatorname{sen}\theta = m_2 R\ddot{\theta} \quad (2) \end{cases}$$

(b) (1) - (2) :

$$-Kx_1 - m_1 g \operatorname{sen}\alpha + m_2 g \operatorname{sen}\theta = m_1 \ddot{x}_1 - m_2 \ddot{\theta}.R$$

+ vínculo $\rightarrow$ $\ddot{x}_1 = -R\ddot{\theta}$ y $x_1 = R\frac{\pi}{2} - R\theta$

$$-K\cancel{R}\frac{\pi}{2} + \cancel{R}K\theta - \frac{m_1 g \operatorname{sen}\alpha}{R} + \frac{m_2 g \operatorname{sen}\theta}{R} = -m_1\cancel{R}\ddot{\theta} - m_2\ddot{\theta}\cancel{R}$$

$$(m_1+m_2)\ddot{\theta} = \frac{m_1 g \operatorname{sen}\alpha}{R} - \frac{m_2 g \operatorname{sen}\theta}{R} + K\left(\frac{\pi}{2} - \theta\right)$$

$$\ddot{\theta} = \frac{-K\theta}{(m_1+m_2)} - \frac{m_2 g}{(m_1+m_2)R}\operatorname{sen}\theta + \frac{K}{(m_1+m_2)}\frac{\pi}{2} + \frac{m_1 g \operatorname{s}\alpha}{R(m_1+m_2)}$$

$$\boxed{\ddot{\theta} = \frac{-K\theta}{(m_1+m_2)} - \frac{m_2 g}{(m_1+m_2)R}\operatorname{sen}\theta + \frac{K}{(m_1+m_2)}\frac{\pi}{2} + \frac{m_1 g \operatorname{sen}\alpha}{R(m_1+m_2)}}$$

de (2) :

$$T = m_2 g \operatorname{sen}\theta + m_2 R\ddot{\theta}$$

meto $\ddot{\theta}$ ahí

(c) Llamo: $A = \frac{K}{(m_1+m_2)}$ $B = \frac{m_2 g}{(m_1+m_2)R}$

$$C = \frac{K}{(m_1+m_2)}\frac{\pi}{2} + \frac{m_1 g \operatorname{sen}\alpha}{R(m_1+m_2)}$$

A, B, C constantes positivas.

$$\Rightarrow \ddot{\theta} = -A\theta - B\operatorname{sen}\theta + C$$

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta} \cdot \frac{d\theta}{dt} = \frac{d\dot{\theta}}{d\theta} \cdot \dot{\theta}$$

$$\Rightarrow \dot{\theta}\frac{d\dot{\theta}}{d\theta} = -A\theta - B\,\text{sen}\,\theta + C$$

$$\int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \int_0^{\theta} -A\theta - B\,\text{sen}\,\theta + C \; d\theta$$

$$\frac{\dot{\theta}^2}{2} = -A\frac{\theta^2}{2} + B\cos\theta + C\theta \Big|_0^{\theta}$$

$$\frac{\dot{\theta}^2}{2} = -A\frac{\theta^2}{2} + B(\cos\theta - 1) + C\theta$$

$$\dot{\theta}^2 = -A\theta^2 + 2B(\cos\theta - 1) + 2C\theta$$

$$\dot{\theta} = \sqrt{-A\theta^2 + 2B(\cos\theta - 1) + 2C\cdot\theta}$$

Quiero que $\dot{\theta}$ sea ≥ 0 en $\theta = \pi/2$

$$\dot{\theta}\left(\frac{\pi}{2}\right) = \sqrt{-A\frac{\pi^2}{4} + 2B(-1) + \cancel{2}C\frac{\pi}{\cancel{2}}} \geq 0$$

$$\Leftrightarrow \frac{-K}{m_1+m_2}\cdot\frac{\pi^2}{4} - \frac{2m_2 g}{(m_1+m_2)R} + \frac{K}{(m_1+m_2)}\frac{\pi^2}{2} + \frac{m_1 g\, s\alpha\, \pi}{R(m_1+m_2)} \geq 0$$

$$\frac{2m_2 g}{(m_1+m_2)R} \leq \frac{K\pi^2}{2(m_1+m_2)} + \frac{m_1 g\,\text{sen}\,\alpha\,\pi}{R(m_1+m_2)} - \frac{K\pi^2}{4(m_1+m_2)}$$

$$\frac{2m_2 g}{R} \leq \frac{K\pi^2}{2} + \frac{m_1 g \operatorname{sen}\alpha \pi}{R} - \frac{K\pi^2}{4}$$

$$m_2 \leq \frac{RK\pi^2}{4g} + \frac{m_1 \operatorname{sen}\alpha \pi}{2} - \frac{K\pi^2 R}{8g}$$

¿unidades?

$$\frac{[K][R]}{[g]} = \frac{\cancel{m} N\, s^2}{m \cdot m} = \frac{kg \cdot \cancel{m}\, s^2\, \cancel{m}}{s^2 \cdot \cancel{m} \cdot \cancel{m}} = kg \checkmark$$

$$[m_1] = kg \checkmark \qquad \frac{[K] \cdot [R]}{[g]} = kg \checkmark$$

$$m_2 \leq \frac{RK\pi^2}{g}\left(\frac{1}{4} - \frac{1}{8}\right) + \frac{m_1 \operatorname{sen}\alpha \pi}{2}$$

$$m_2 \leq \frac{RK\pi^2}{8g} + \frac{m_1 \operatorname{sen}\alpha \pi}{2} > 0 \checkmark$$

Velocidad en ese momento

$\rightarrow R \cdot \dot{\theta}\left(\frac{\pi}{2}\right)$ Reemplazar...

Posición y velocidad m_1

$$\rightarrow \dot{x}_1 = -R\dot{\theta}\left(\frac{\pi}{2}\right)$$

$$\rightarrow X_1 = L - L_{SOGA} + R\frac{\pi}{2} - R\frac{\pi}{2} = L - L_{SOGA} = 0$$