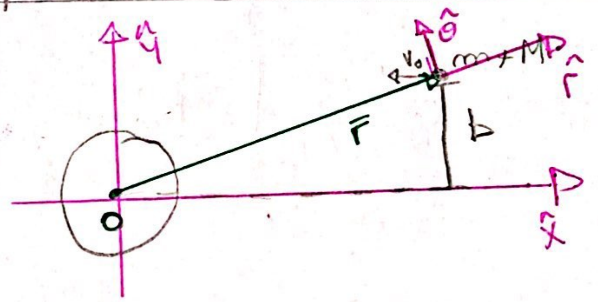
③ (a)

	ANTES	DESPUÉS
$\bar{P}$	$\sum F^{ext} = \bar{F}_g \neq 0$ ↳ $\boxed{\bar{p} \neq cte}$	$\sum F^{ext} = \bar{F}_g \neq 0$ ↳ $\boxed{\bar{p} \neq cte}$
$\bar{L}^o$	$\sum \bar{\tau}^{ext} = \bar{r} \wedge \bar{F}_g = r\hat{r} \wedge F_g \hat{r} = 0$ ↳ $\boxed{\bar{L}^o = cte}$	$\sum \bar{\tau}^{ext} = \bar{r} \wedge \bar{F}_g = r\hat{r} \wedge F_g \hat{r} = 0$ ↳ $\boxed{\bar{L}^o = cte}$
E_M	Fuerzas en juego ↳ F_g → CONS $\boxed{E_M = cte}$	Fuerzas en juego ↳ F_g → CONS $\boxed{E_M = cte}$



(b)

$\boxed{\text{En general antes de expulsión}}$ → $E_M = \frac{1}{2}(M+m) \cdot V^2 - \frac{G M_T (M+m)}{r}$

$\boxed{\text{En el } \infty}$ → $E_{M_i} = \frac{1}{2}(M+m) \cdot V_0^2$

• Conservación de $\bar{L}^o$ entre el ∞ y máx. acercamiento

$$\bar{L}^o_\infty = \bar{r} \wedge \bar{p} = (x_{(m+M)}\,\hat{x} + b\,\hat{y}) \wedge (-V_0\,\hat{x})$$

$$\bar{L}^o = V_0 . b\,\hat{z}$$

$$\bar{L}^{o}_{r_{MAX}} = (r_{MAX}\,\hat{r}) \wedge (\dot{r}_{MAX}\,\hat{r} + r_{MAX}\,\dot{\theta}_{r_{MAX}}\,\hat{\theta})$$

$$= r_{MAX}^{2}\,\dot{\theta}_{MAX}\,\hat{z}$$

$$\Rightarrow \quad V_0 . b = r_{MAX}^{2}\,\dot{\theta}_{MAX} \Rightarrow \boxed{\dot{\theta}_{MAX} = \frac{V_0\, b}{r_{MAX}^{2}}}$$

• Conservación de energía entre esos puntos

$$E_{Mi} = \frac{1}{2}(M+m)\,V_0^2$$

$$E_{M_{r_{MAX}}} = \frac{1}{2}.(M+m).(\dot{r}\,\hat{r} + r_{MAX}.\dot{\theta}_{MAX}\,\hat{\theta})^2 - \frac{G M_T (m+M)}{r_{MAX}}$$

OBS → MÁXIMO ACERCAMIENTO → $\boxed{\dot{r} = 0}$

$$E_{M_{r_{MAX}}} = \frac{1}{2}(M+m).\,r_{MAX}^{2}.\,\frac{V_0^2 b^2}{r_{MAX}^{4}} - \frac{G M_T (m+M)}{r_{MAX}}$$

$$\Rightarrow \frac{1}{2}\cancel{(M+m)}\,V_0^2 = \frac{1}{2}\cancel{(M+m)}\,\frac{V_0^2 b^2}{r_{MAX}^{2}} - \frac{G M_T \cancel{(m+M)}}{r_{MAX}}$$

Multiplico todo por r_{MAX}^{2}

$$V_0^2\, r_{MAX}^2 = V_0^2\, b^2 - 2 G M_T\, r_{MAX}$$

$$V_0^2\, r_{MAX}^2 + 2 G M_T\, r_{MAX} - V_0^2\, b^2 = 0$$

$$r_{MAX} = \frac{-2 G M_T \pm \sqrt{4 G^2 M_T^2 + 4.V_0^2.V_0^2 b^2}}{2 V_0^2}$$

$$r_{MAX} = \frac{-G.M_T \pm G M_T \sqrt{1 + \frac{4V_0^4 b^2}{G^2 M_T^2}}}{V_0^2}$$

$$1 + \frac{4V_0^4 b^2}{G M_T} > 1 \Rightarrow \sqrt{1 + \frac{4V_0^4 b^2}{G^2 M_T^2}} > 1$$

$$\Rightarrow -G M_T + G M_T . \sqrt{1 + \frac{4V_0^4 b^2}{G^2 M_T^2}} > 0$$

$$\rightarrow \boxed{r_{MAX} = \frac{G M_T \left(-1 + \sqrt{1 + \frac{4V_0^4 b^2}{G^2 M_T^3}}\right)}{V_0^2}}$$

$$\frac{m^4 m^2 s^4 kg^4}{kg^2 . m^2 s^4 m^4 kg^2}$$

Velocidad en ese punto

$$\bar{V} = \frac{r_{MAX} . V_0 b}{r^2_{MAX}} \hat{\theta} \Rightarrow \boxed{\bar{V} = \frac{V_0 b}{r_{MAX}} \hat{\theta}}$$

$$\frac{kg . m s^2 m^2 . kg}{s^2 \; kg^2 m^2}$$

(c) Durante la expulsión del combustible

$$\frac{d\bar{p}}{dt} = \sum F^{ext} \Rightarrow d\bar{p} = \sum F^{ext} . \underbrace{dt}_{dt \ll 1} \rightarrow \boxed{d\bar{p} = 0}$$

$$\bar{P}_{\substack{ANTES\\EXPUL}} = (m+M) . \frac{V_0 b}{r_{MAX}} \hat{\theta}$$

$$\bar{P}_{\substack{DESPUÉS\\EXPUL}} = m.(-V_m \hat{r}) + M . (V_r \hat{r} + V_0 \hat{\theta})$$

$$\Rightarrow (m+M) \frac{V_0 b}{r_{MAX}} \hat{\theta} = -m V_m \hat{r} + M(V_r \hat{r} + V_0 \hat{\theta})$$

$(\hat{r})\quad 0 = -m V_m + M V_r \quad \hat{r}$

$$\Rightarrow \boxed{V_r = \frac{m}{M} V_m}$$

$(\hat{\theta})\quad (m+M)\dfrac{V_0 b}{r_{MAX}} = M V_\theta\ \hat{\theta}$

$$\boxed{V_\theta = \frac{(M+m)}{M}\frac{V_0 b}{r_{MAX}}}$$

$$\boxed{\bar{V}_{\substack{\text{DESPUÉS}\\ \text{EXPULSIÓN}}} = \frac{m}{M} V_m \hat{r} + \frac{(M+m)}{M}\frac{V_0 b}{r_{MAX}}}$$

(d) $\boxed{\text{Energía Mecánica Nave post expulsión}}$

$$\rightarrow E_{M_i} = \frac{1}{2} M \left(\frac{m}{M} V_m \hat{r} + \frac{(m+M)}{M}\frac{V_0 b \hat{\theta}}{r_{MAX}}\right)^2 - \frac{G M_T M}{r_{MAX}}$$

Conservación de $\bar{L}^o$ post expulsión

$$\bar{L}_i^o = (r_{MAX}\ \hat{r}) \wedge \left(\frac{m}{M} V_m \hat{r} + \frac{(M+m)}{M}\frac{V_0 b}{r_{MAX}}\hat{\theta}\right)$$

$$= \frac{(M+m)}{M} \cdot V_0 b \hat{z} = \ell\ \hat{z}$$

$$\bar{L}^o_{general} = (r\ \hat{r}) \wedge (\dot{r}\ \hat{r} + r\dot{\theta} \cdot \hat{\theta})$$

$$= r^2 \dot{\theta}\ \hat{z}$$

Conservación

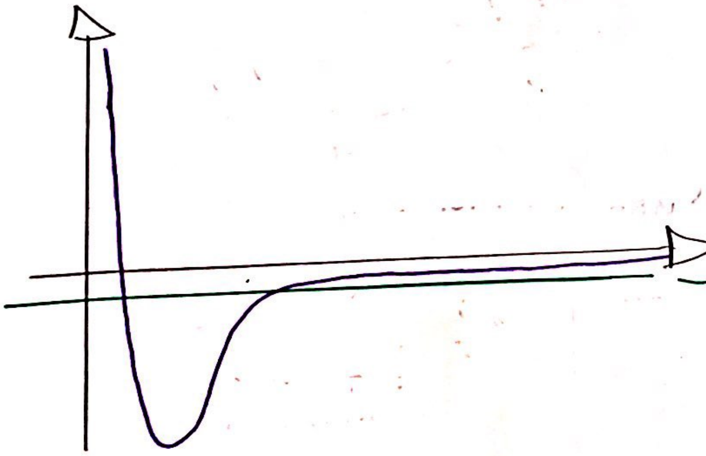
$$\ell = r^2\dot{\theta} \Rightarrow \boxed{\dot{\theta} = \frac{\ell}{r^2}}$$

$$\Rightarrow E_M = \frac{1}{2} M\left(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}\right)^2 - \frac{G M_T M}{r}$$

$$E_M = \frac{1}{2} M \dot{r}^2 + \frac{1}{2} M \frac{r^2 \ell^2}{r^4} - \frac{G M_T M}{r}$$

$$E_M = \frac{1}{2} M \dot{r}^2 + \underbrace{\frac{1}{2}\frac{M\ell^2}{r^2} - \frac{G M_T M}{r}}_{V_{eff}}$$

(el mismo de siempre)



Ligado $\Rightarrow \boxed{E_M \leq 0}$

$$\Rightarrow E_{Mi} \leq 0 \Rightarrow \frac{1}{2} M\left(\frac{m^2}{M^2} v_m^2 + \frac{(m+M)^2}{M^2}\frac{v_0^2 b^2}{r_{MAX}^2}\right) - G M_T M / r_{MAX} \leq 0 \rightarrow$$

despeja v_m.