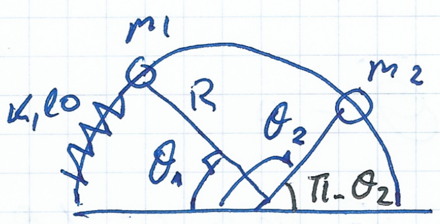
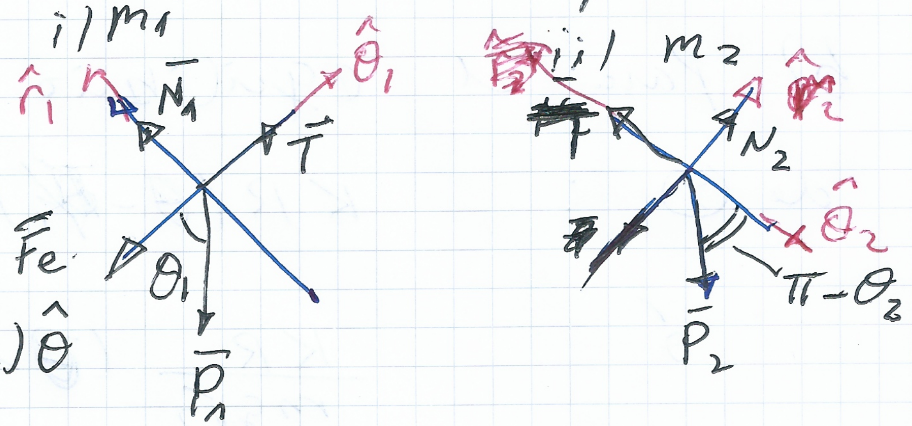
P21



• Diagrama de cuerpo libre:



$$\vec{F}_e = -k(l-l_0)\hat{\theta}$$

con $\begin{cases} l = R\theta_1 \\ l_0 = R\frac{\pi}{4} \end{cases}$

Usaremos:

$$\cos(\pi-\theta_2) = \sin\theta$$

• Vínculo: $R(\theta_2-\theta_1) = \cancel{l_0} = \frac{\pi}{2}R$

$$\Rightarrow \boxed{\theta_2 = \frac{\pi}{2} + \theta_1} \qquad (\theta_1 = \theta)$$

$$\therefore \quad \pi - \theta_2 = \frac{\pi}{2} - \theta_1$$

• Newton: (polares)

m1

$$\hat{\theta}_1 : \quad T - kR\left(\theta - \frac{\pi}{4}\right) - mg\cos\theta = mR\ddot{\theta} \quad (1)$$

$$\hat{r}_1 : \quad N_1 - mg\sin\theta = -mR\dot{\theta}^2 \quad (2)$$

m2

$$\hat{\theta}_2 : \quad -T + mg\sin\theta = mR\ddot{\theta} \quad (3)$$

$$\hat{r}_2 : \quad N_2 - mg\cos\theta = -mR\dot{\theta}^2 \quad (4)$$

(1)+(3):

$$-kR\left(\theta-\frac{\pi}{4}\right) - mg\underbrace{(\cos\theta - \sin\theta)}_{-\sqrt{2}\sin\left(\theta-\frac{\pi}{4}\right)} = 2mR\ddot{\theta} \quad (5)$$

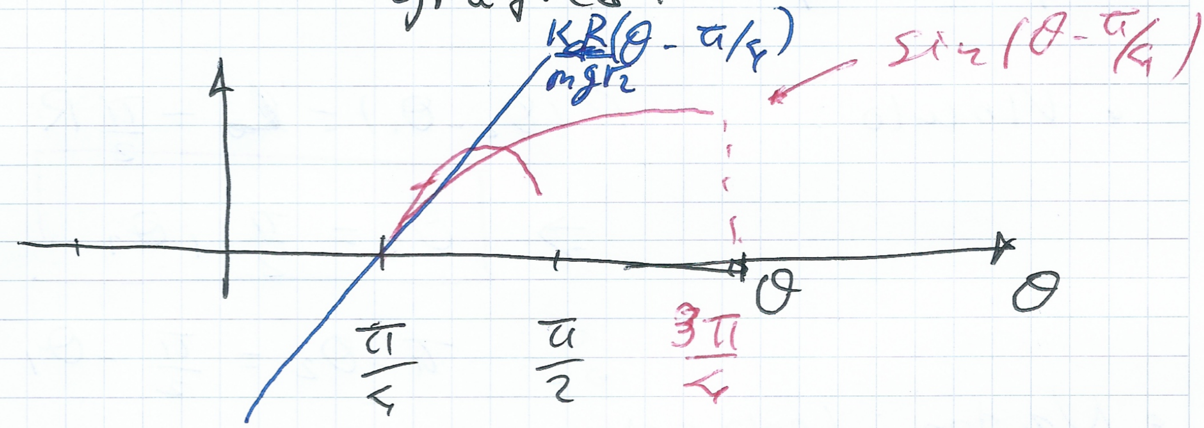
a) ⑤ es la ec. de movimiento para $\theta(t)$
(sólo involucra a θ, $\ddot{\theta}$ y
datos)

b) Para el equilibrio $\ddot{\theta} = 0$

de ⑤ : $KR(\theta - \pi/4) = \sqrt{2}\, mg \sin(\theta - \frac{\pi}{4})$

ó $\frac{KR}{mg\sqrt{2}}(\theta - \pi/4) = \sin(\theta - \frac{\pi}{4})$

Las raíces se hallan en forma
gráfica:



Si $\frac{KR}{mg\sqrt{2}} > 1$ la pendiente
de la recta
es mayor a la pendiente
inicial del $\sin(\theta - \pi/4)$
y se cortan en un solo
punto : $\theta - \pi/4 = 0$

$\therefore$ Hay un solo equilibrio $\theta_{eq} = \frac{\pi}{4}$

Estabilidad:

$$F_\theta = -KR\left(\theta-\frac{\pi}{4}\right)+\sqrt{2}\,mg\sin\left(\theta-\frac{\pi}{4}\right)$$

$$\left.\frac{dF_\theta}{d\theta}\right|_{\theta_{eq}} = -KR+\sqrt{2}\,mg\cos\left(\theta-\pi/4\right)\Big|_{eq}$$

$$\left.\frac{dF_\theta}{d\theta}\right|_{\theta_{eq}} = -KR+\sqrt{2}\,mg < 0$$

$\therefore$ equilibrio estable si $KR > \sqrt{2}\,mg$

que es el caso.

c) Pequeños apartamientos del equilibrio estable.

Linealizamos F_θ:

$$F_\theta \simeq \underbrace{F_{\theta_{eq}}}_{0} + \left.\frac{dF}{d\theta}\right|_{\theta_{eq}}(\theta-\theta_{eq})$$

$$F_\theta \simeq -(KR-\sqrt{2}\,mg)(\theta-\pi/4)$$

en (5): $$-(KR-\sqrt{2}\,mg)\left(\theta-\frac{\pi}{4}\right) = 2mR\ddot{\theta}$$

$\mu=\theta-\frac{\pi}{4}$ $$-(KR-\sqrt{2}\,mg)\mu = 2m\ddot{\mu}$$

$$\Rightarrow \boxed{\theta-\frac{\pi}{4} = A\cos(\omega t+\varphi)}$$

con $$\boxed{\omega^2 = \frac{KR-\sqrt{2}\,mg}{2Rm}}$$