Oscilador armónico

$$\ddot{x} + \omega^2 x = 0 \qquad \left(\omega^2 = \frac{k}{m}\right)$$

$$x(t) = A\cos(\omega t + \phi)$$

A, ϕ dependían de x_0, v_0

$$x(0) = A.\cos(\phi) = x_0$$

$$\dot{x}(0) = -A\omega\,\text{sen}(\phi) = v_0 \Rightarrow \frac{v_0}{\omega} = -A\,\text{sen}(\phi)$$

$$A^2\cos^2\phi + A^2\text{sen}^2\phi = x_0^2 + \frac{v_0^2}{\omega^2}$$

$$A^2\underbrace{(\cos^2\phi + \text{sen}^2\phi)}_{=1.} = x_0^2 + \frac{v_0^2}{\omega^2}$$

$$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$$

$$x(t) = A\cos(\omega t + \phi)$$

$$x(t) = \underbrace{A\,\text{sen}\phi}_{C}\cos(\omega t) + \underbrace{A\cos\phi}_{D}\,\text{sen}(\omega t)$$

$$x(t) = C\cos(\omega t) + D\,\text{sen}(\omega t)$$

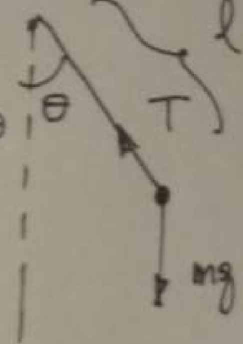
$$x(0) = \underline{x_0 = C.}$$

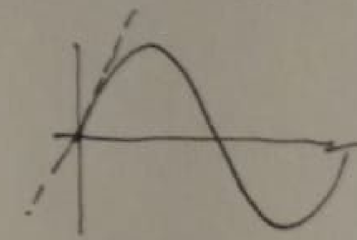
$$\dot{x}(t) = -C.\omega\,\text{sen}(\omega t) + D\omega\cos(\omega t)$$

$$\dot{x}(0) = v_0 = D.\omega \Rightarrow D = \frac{v_0}{\omega}$$

$$x(t) = x_0\cos(\omega t) + \frac{v_0}{\omega}\text{sen}(\omega t)$$

Pequeñas oscilaciones

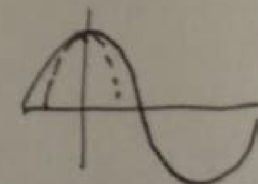




$$\hat{\theta}: \quad m\left(l\ddot{\theta} + \underbrace{\cancel{2\dot{r}\dot{\theta}}}_{=0}\right) = -mg\,\mathrm{sen}\,\theta$$

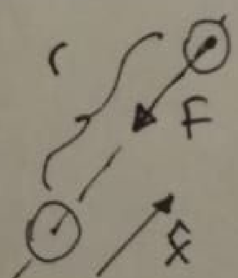
$$\ddot{\theta} = -\frac{g}{l}\,\mathrm{sen}\,\theta$$

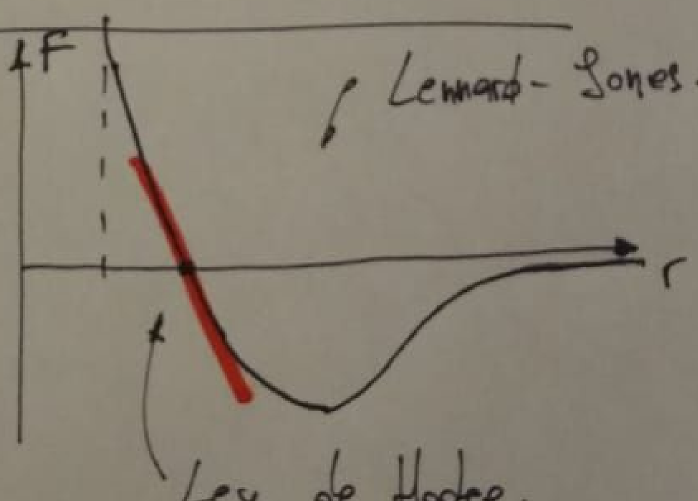
Aprox. pequeños oscilaciones $\qquad \mathrm{sen}\,x \simeq x + \dots \quad x \approx 0.$

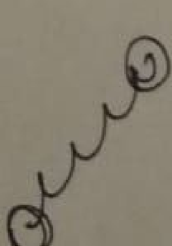


$$\ddot{\theta} = \frac{-g}{l}\cdot\theta$$

$$\ddot{\theta} + \frac{g}{l}\theta = 0 \qquad \frac{g}{l} \to \omega^2$$





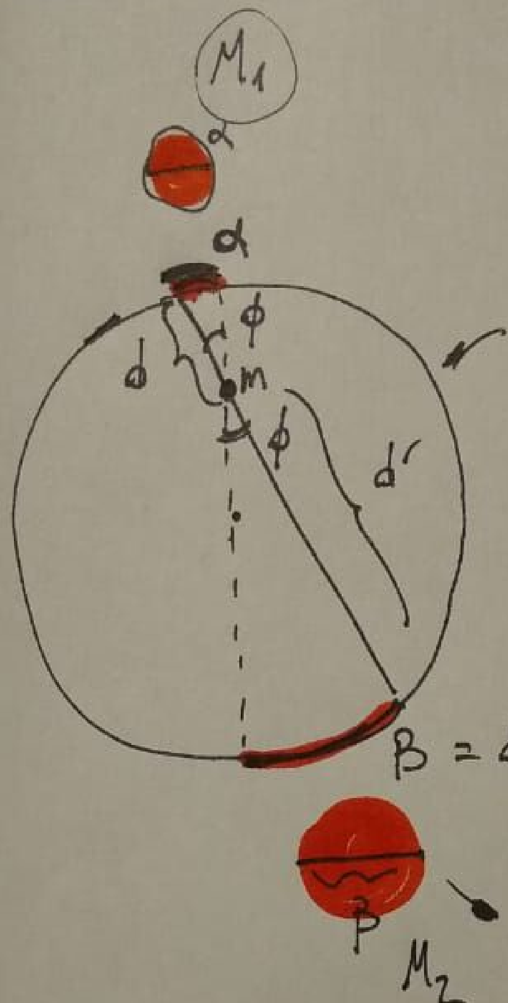


Tren gravitatorio

"Total Recall."

→ Arnold. ✓

→ Colin Farrell. ✗



Esfera hueca.

Densidad δ

$\alpha = d\phi$

$\beta = d'\phi$

$$M_1 = \pi\left(\frac{\alpha}{2}\right)^2$$

$$M_2 = \pi\left(\frac{\beta}{2}\right)^2$$

$$M_1 = \frac{\pi}{4} d^2 \phi^2$$

$$M_2 = \frac{\pi}{4} d'^2 \phi^2$$

$$F_T = \frac{G \cdot M_1 m}{d^2} - \frac{G M_2 m}{d'^2}$$

$$= \frac{G \cdot \pi/4\, \cancel{d^2} \phi^2 \cdot m}{\cancel{d^2}} - G\frac{\pi/4\, \cancel{d'^2} \phi^2 m}{\cancel{d'^2}} = 0.$$

Fuerza sobre la masa es 0.

$$\frac{4}{3}\pi . x^3 \delta = M$$

$$F = -\frac{GmM}{x^2} = \frac{4}{3}\pi x^{\cancel{3}}\delta . \left(-\frac{G.m}{\cancel{x^2}}\right)$$

$$F = -Gm\frac{4}{3}\pi\delta x$$

$$\cancel{m}\ddot{x} = -\underbrace{G\cancel{m}\frac{4}{3}\pi\delta}_{\omega^2} x$$

$$T = \frac{2\pi}{\omega}$$

$$\ddot{x} + \omega^2 x = 0$$

$$t_f = \frac{T}{2} = \frac{\pi}{\omega} \cong 42\ \text{min}$$

(38 min)

$$x(t) = R\cos(\omega t)$$

$$\dot{x}(t) = -R\omega\, \text{sen}(\omega t).$$

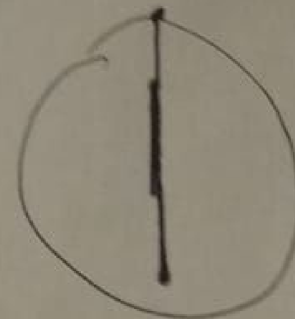
$V_{máx} = 7600$ m/s.

$V_{tierra} = 30.000$ m/s.

$V_{747} = 250$ m/s

$V_{sonido} = 340$ m/s

Oscilador armónico amortiguado

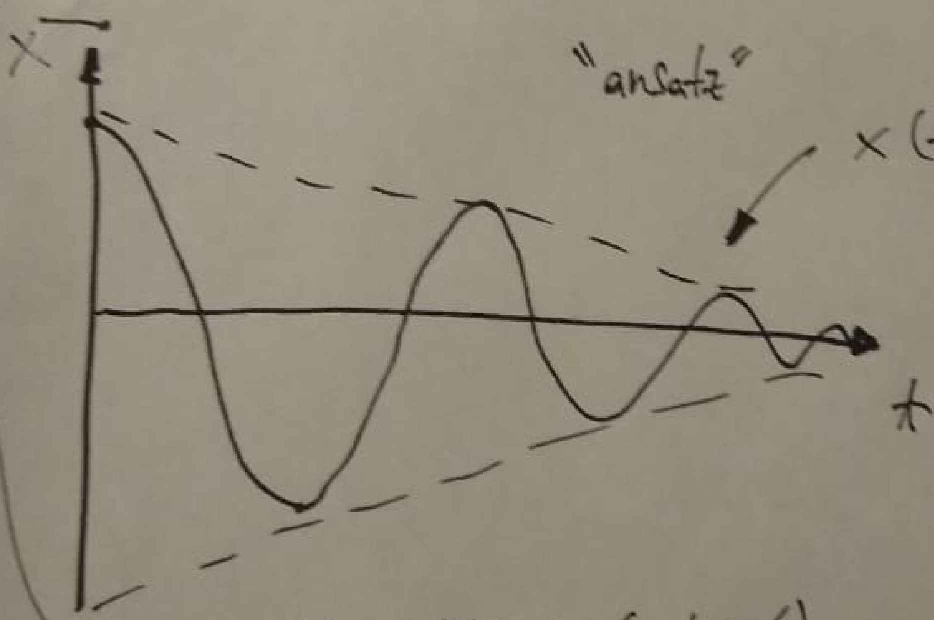


$$\vec{F}_v = -\lambda \vec{v}$$

$m\ddot{x} = -kx - \lambda v$, Ec. osc. armónico amortiguado.

$$m\ddot{x} + kx + \lambda\dot{x} = 0.$$

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0. \qquad \omega_0^2 = \frac{k}{m} \qquad 2\gamma = \frac{\lambda}{m}$$



"ansatz"

$x(t) = A(t)\,\mathrm{sen}(\omega t + \phi)$

$$x(t) = A(t)\,\mathrm{sen}(\omega t + \phi)$$

$$\begin{cases} \dot{x}(t) = \underbrace{\dot{A}\,\mathrm{sen}(\omega t + \phi)} + A.\omega.\cos(\omega t + \phi) \\ \ddot{x}(t) = \ddot{A}\,\mathrm{sen}(\omega t + \phi) + \dot{A}\omega\cos(\omega t + \phi) \end{cases}$$

$$+ \dot{A}\omega\cos(\omega t + \phi) - A\omega^2\,\mathrm{sen}(\omega t + \phi)$$

⑥

Reemplazo el "ansatz" en la ecuación

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0 \leftarrow x(t) = A(t)\,\text{sen}(\omega t+\phi)$$

$$\ddot{A}\,\text{sen}(\omega t+\phi) + \dot{A}\omega\cos(\omega t+\phi) + \dot{A}\omega\cos(\omega t+\phi)$$
$$-A\omega^2\,\text{sen}(\omega t+\phi) + 2\gamma\left[\dot{A}\,\text{sen}(\omega t+\phi) + A\omega\cos(\omega t+\phi)\right]$$
$$+\,\omega_0^2 A\,\text{sen}(\omega t+\phi) = 0$$

Y

$$\left\{\ddot{A} \underbrace{-A\omega^2 + 2\gamma\dot{A} + \omega_0^2 A}_{=0}\right\}\text{sen}(\omega t+\phi) +$$

$A = A(t)$

$$\left\{\underbrace{2\dot{A}\omega + 2\gamma A\omega}_{=0}\right\}\cos(\omega t+\phi) = 0.$$

W

$$\begin{cases} \boxed{\ddot{A} - A\omega^2 + 2\gamma\dot{A} + \omega_0^2 A = 0} \\ 2\dot{A}\omega + 2\gamma A\omega = 0 \end{cases}$$

$$\gamma = \frac{\lambda}{2m}$$

$$2\dot{A} + 2\gamma A = 0.$$

$$\dot{A} + \gamma A = 0.$$

$$\frac{dA}{dt} = -\gamma A$$

$$\int_{A_0}^{A} \frac{dA}{A} = \int_0^t -\gamma\,dt$$

$$\ln(A) - \ln(A_0) = \boxed{\ln\left(\frac{A}{A_0}\right) = -\gamma t}$$

$$\boxed{A(t) = A_0 e^{-\gamma t}}$$

$$x(t) = A_0 e^{-\gamma t} \operatorname{sen}(\omega t + \phi)$$

$$\dot{A}(t) = -\gamma A_0 e^{-\gamma t}$$

$$\ddot{A}(t) = \gamma^2 A_0 e^{-\gamma t}.$$

$$\gamma^2 A_0 e^{-\gamma t} + A_0 e^{-\gamma t}(\omega_0^2 - \omega^2) - 2\gamma^2 A_0 e^{-\gamma t} = 0$$

$$\gamma^2 + \omega_0^2 - \omega^2 - 2\gamma^2 = 0.$$

$$\boxed{\omega^2 = \omega_0^2 - \gamma^2}$$

< 0

$\omega_0^2 = \frac{k}{m}$

$\gamma = \frac{\lambda}{2m}$

sobreamortiguado

$$\omega = \sqrt{\omega_0^2 - \gamma^2}$$

$$x(t) = A_0 e^{-\gamma t} \operatorname{sen}(\omega t + \phi)$$

A_0, ϕ = constantes