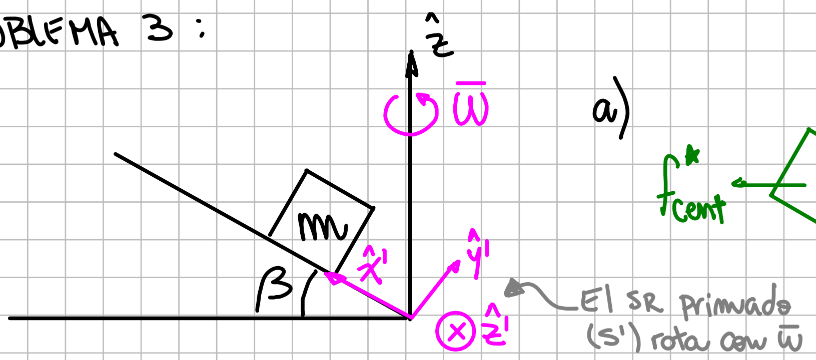
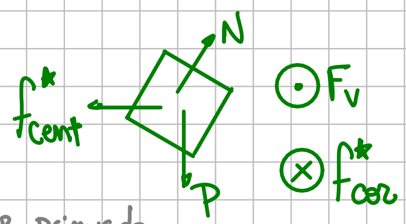
PROBLEMA 3 :



a)



$\bar{r}' = x'\,\hat{x}'$

$\bar{v}' = \dot{x}'\,\hat{x}'$

$\hat{z} = \operatorname{sen}\beta\,\hat{x}' + \cos\beta\,\hat{y}'$

Escribo las pseudofuerzas :

- $f^*_{cor} = -2m\,\bar{\omega}\times\bar{v}' = -2m\omega\,\hat{z}\times\dot{x}'\,\hat{x}' = -2m\omega\dot{x}\,(\operatorname{sen}\beta\,\hat{x}' + \cos\beta\,\hat{y}')\times\hat{x}'$
  $= -2m\omega\dot{x}\cos\beta\,(-\hat{z}') = 2m\omega\dot{x}\cos\beta\,\hat{z}'$

- $f^*_{cent} = -m\,\bar{\omega}\times(\bar{\omega}\times\bar{r}') = -m\omega\,\hat{z}\times[\omega(\operatorname{sen}\beta\,\hat{x}' + \cos\beta\,\hat{y}')\times x'\hat{x}'] =$
  $= -m\omega\,\hat{z}\times\omega\cos\beta\, x'(-\hat{z}') = m\omega^2 x'\cos\beta\,[(\operatorname{sen}\beta\,\hat{x}' + \cos\beta\,\hat{y}')\times\hat{z}']$
  $= m\omega^2 x'\cos\beta\,[\operatorname{sen}\beta\,(-\hat{y}') + \cos\beta\,\hat{x}']$

Ahora sí, escribo las ec. de Newton en S' :

$$\begin{cases} m\ddot{x}' = -mg\operatorname{sen}\beta + m\omega^2\cos^2\beta\, x' & (1)\\ m\ddot{y}' = -m\omega^2\operatorname{sen}\beta\cos\beta\, x' + N = 0 \quad \text{pues } \ddot{y}' = 0 & (2)\\ m\ddot{z}' = 2m\omega\cos\beta\,\dot{x} - F_v = 0 \quad \text{pues } \ddot{z}' = 0 & (3)\end{cases}$$

b) Equilibrio : $\ddot{x}'(x'_{eq}) = 0 \Rightarrow m\omega^2\cos^2\beta\, x'_{eq} = mg\operatorname{sen}\beta$ (por (1))

$$\Rightarrow \boxed{x'_{eq} = \frac{g}{\omega^2}\frac{\operatorname{sen}\beta}{\cos^2\beta}}$$

Para estudiar la estabilidad miro $\left.\frac{dF}{dx'}\right|_{x'_{eq}}$:

$$\left.\frac{dF}{dx'}\right|_{x'_{eq}} = \left.\frac{d(m\ddot{x}')}{dx'}\right|_{x'_{eq}} = m\omega^2\cos^2\beta \geq 0 \Rightarrow \text{El equilibrio es INESTABLE.}$$

c) v_0 / $\dot{x}'(x'_{eq}) = 0$

$$\ddot{x}' = \frac{d\dot{x}'}{dx'}\dot{x}' \underset{Ec(1)}{=} \omega^2\cos^2\beta\, x' - g\operatorname{sen}\beta \Rightarrow \int_{v_0}^{0} d\dot{x}'\,\dot{x}' = \int_0^{x'_{eq}} dx'\,(\omega^2\cos^2\beta\, x' - g\operatorname{sen}\beta)$$

$$\Rightarrow -\frac{v_0^2}{2} = \omega^2\cos^2\beta\,\frac{x'^2_{eq}}{2} - g\operatorname{sen}\beta\, x'_{eq} = \omega^2\cos^2\beta\,\frac{1}{2}\left(\frac{g}{\omega^2}\frac{\operatorname{sen}\beta}{\cos^2\beta}\right)^2 - g\operatorname{sen}\beta\left(\frac{g}{\omega^2}\frac{\operatorname{sen}\beta}{\cos^2\beta}\right) =$$

$$= \frac{1}{2}\frac{g^2}{\omega^2}\operatorname{tg}^2\beta - \frac{g^2}{\omega^2}\operatorname{tg}^2\beta = -\frac{1}{2}\frac{g^2}{\omega^2}\operatorname{tg}^2\beta$$

$$\Rightarrow \boxed{v_0 = \frac{g}{\omega}\operatorname{tg}\beta}$$

Chequeo unidades : $\left[\frac{g}{\omega}\operatorname{tg}\beta\right] = \frac{m/s^2}{1/s} = \frac{m}{s}$ ✓

d) $F_v(x')$ y $N(x')$

De (2) : $\boxed{N = m w^2 \cos\beta \,\text{sen}\beta\, x'}$

De (3) : $F_v = 2 m w \cos\beta\, \dot{x}'$ ← Necesito $\dot{x}'(x')$

De nuevo, $\ddot{x}' = \frac{d\dot{x}'}{dx'}\dot{x}' = w^2\cos^2\beta\, x - g\,\text{sen}\beta \implies \int_{v_0}^{\dot{x}} d\dot{x}'\,\dot{x}' = \int_0^{x} dx'\,(w^2\cos^2\beta\, x' - g\,\text{sen}\beta)$

$\implies \frac{\dot{x}'}{2} - \frac{v_0^2}{2} = w^2\cos^2\beta\,\frac{x'^2}{2} - g\,\text{sen}\beta\, x' \implies \dot{x}'^2 = w^2\cos^2\beta\, x'^2 - 2g\,\text{sen}\beta\, x' + v_0^2$

Finalmente,

$$\boxed{F_v = 2 m w \cos\beta \left[w^2\cos\beta\, x'^2 - 2g\,\text{sen}\beta\, x' + v_0^2 \right]^{1/2}}$$