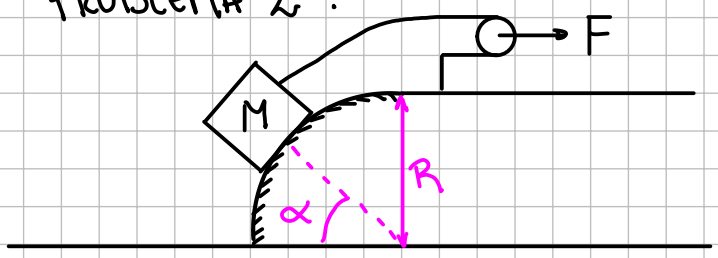
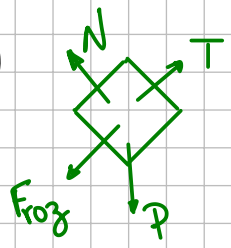
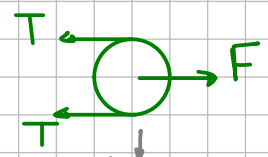
PROBLEMA 2:



a)





$0 = m_p \ddot{x}_p = F - 2T \Rightarrow T = \frac{F}{2}$

Escribo las ecs. de Newton para M:

$\hat{r}$) $M(\ddot{r} - r\dot{\alpha}^2) = -MR\dot{\alpha}^2 = N - Mg\,\mathrm{sen}\,\alpha$

$\hat{\theta}$) $M(r\ddot{\alpha} + 2\dot{r}\dot{\alpha}) \overset{r = R = cte}{=} MR\ddot{\alpha} = T - Froz - Mg\cos\alpha$

$$\Rightarrow \begin{cases} -MR\dot{\alpha}^2 = N - Mg\,\mathrm{sen}\,\alpha \\ \boxed{MR\ddot{\alpha} = \frac{F}{2} - Froz - Mg\cos\alpha} \end{cases}$$

↝ EC. de Movimiento.

b) F_{MIN} y F_{MAX} tales que M no se mueva.

Sabemos que $|Froz| \le \mu_e N$. En este caso además hay que considerar que la tensión no puede ser negativa, pues la soga no puede empujar. ⇒ F tampoco puede ser negativa

Entonces, $Froz = \frac{F}{2} - Mg\cos\alpha$ y $N = Mg\,\mathrm{sen}\,\alpha$

$$-Mg\,\mathrm{sen}\,\alpha \le \frac{F}{2} - Mg\cos\alpha \le Mg\,\mathrm{sen}\,\alpha \Rightarrow Mg\cos\alpha - \mu_e Mg\,\mathrm{sen}\,\alpha \le \frac{F}{2} \le Mg\cos\alpha + \mu_e Mg\,\mathrm{sen}\,\alpha$$

$$\Rightarrow 2Mg(\cos\alpha - \mu_e\,\mathrm{sen}\,\alpha) \le F \le 2Mg(\cos\alpha + \mu_e\,\mathrm{sen}\,\alpha) \Rightarrow \boxed{F_{max} = 2Mg(\cos\alpha + \mu_e\,\mathrm{sen}\,\alpha)}$$

$(\cos\alpha - \mu_e\,\mathrm{sen}\,\alpha)$ tiene que ser positivo

y $\boxed{F_{min} = \begin{cases} 2Mg(\cos\alpha - \mu_e\,\mathrm{sen}\,\alpha) & \text{si } \cos\alpha > \mu_e\,\mathrm{sen}\,\alpha \\ 0 & \text{si no} \end{cases}}$

c) $N = N(\alpha)$.

Tenemos $N = Mg\,\mathrm{sen}\,\alpha - MR\dot{\alpha}^2$

$$\ddot{\alpha} = \frac{d\dot{\alpha}}{d\alpha}\dot{\alpha} = \frac{F}{2MR} - \frac{g}{R}\cos\alpha$$

$$\Rightarrow \int_{0=\dot{\alpha}_0}^{\dot{\alpha}} d\dot{\alpha}'\,\dot{\alpha}' = \int_{\alpha_0}^{\alpha} d\alpha' \left(\frac{F}{2MR} - \frac{g}{R}\cos\alpha\right)$$

$$\frac{\dot{\alpha}^2}{2} = \frac{F}{2MR}(\alpha - \alpha_0) - \frac{g}{R}(\mathrm{sen}\,\alpha - \mathrm{sen}\,\alpha_0)$$

$$\Rightarrow N = Mg\,\mathrm{sen}\,\alpha - MR\left[\frac{F}{MR}(\alpha - \alpha_0) - \frac{2g}{2}(\mathrm{sen}\,\alpha - \mathrm{sen}\,\alpha_0)\right]$$

$$\boxed{N(\alpha) = Mg(3\,\mathrm{sen}\,\alpha - 2\,\mathrm{sen}\,\alpha_0) - F(\alpha - \alpha_0)}$$