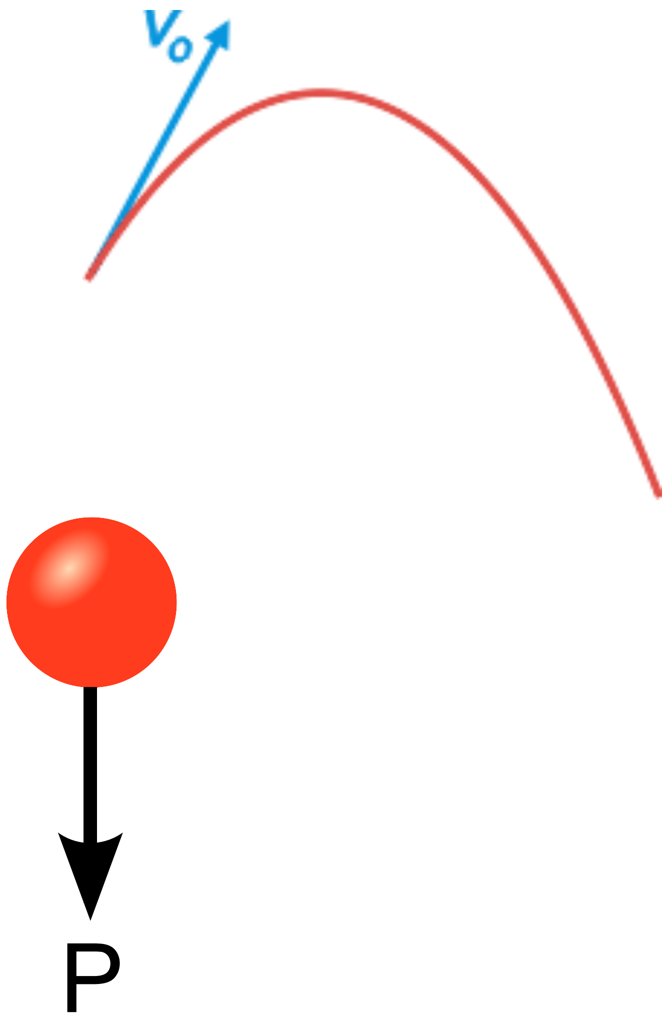
Velocidad límite, viscosidad y modelo de Stokes

http://materias.df.uba.ar/f1qa2020c1/laboratorios/

Física 1 (Q): Tiro Oblicuo



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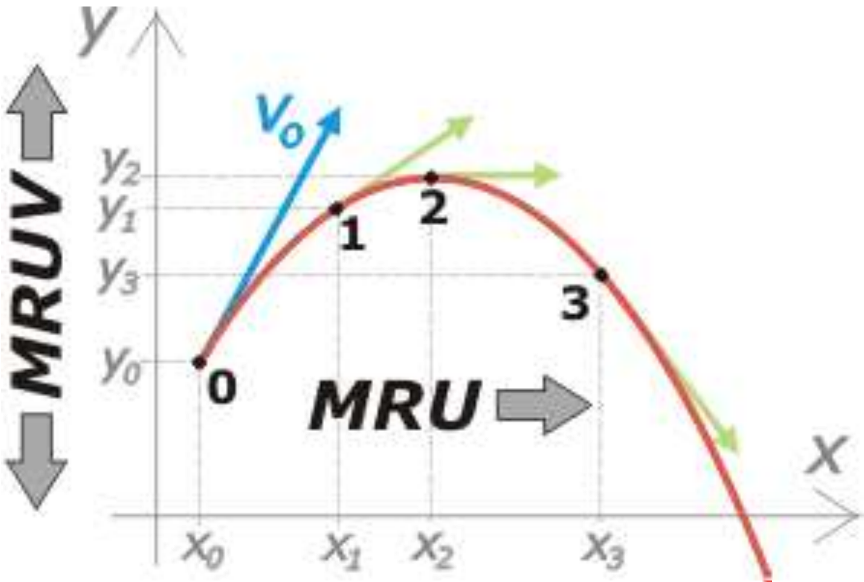

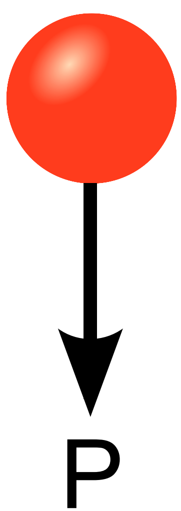

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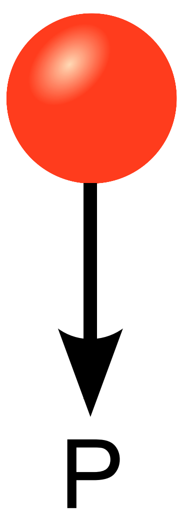
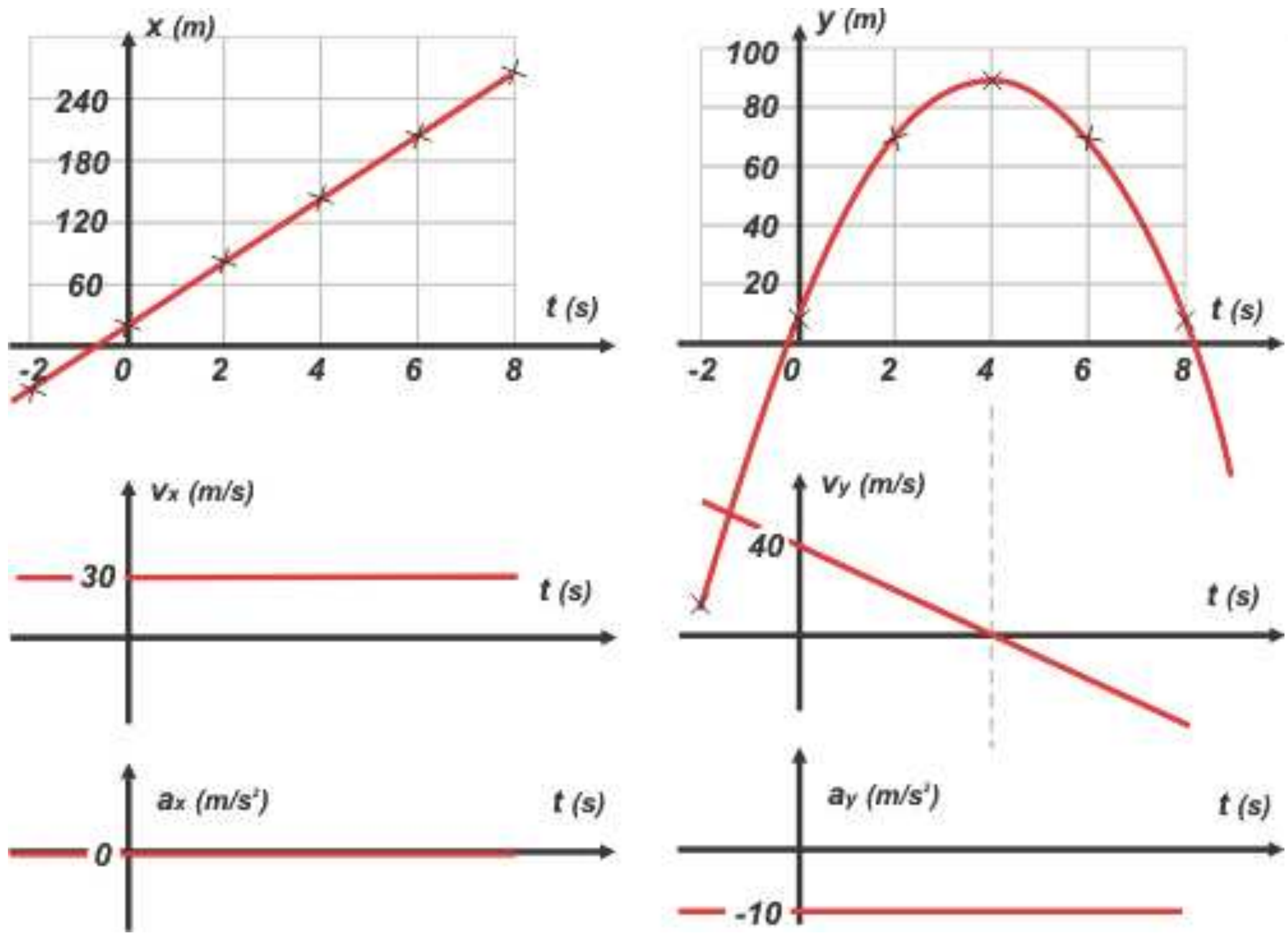
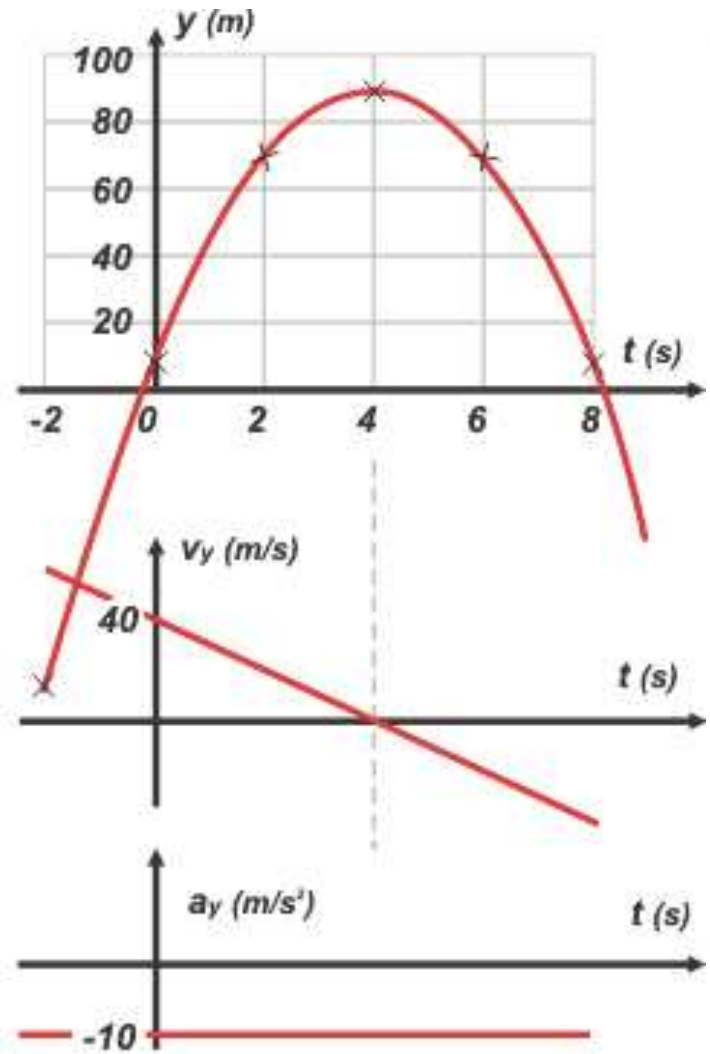




Física 1 (Q): Tiro Oblicuo

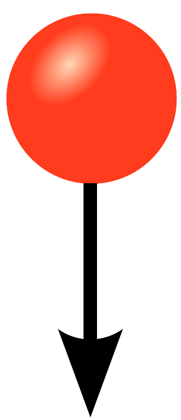
→ Hipótesis del modelo matemático

- → Movimiento en el vacío
- → Masa Puntual
- → No hay obstáculos



Ecuación de movimiento

$$m\frac{d^2y}{dt^2} = P$$



P

Solución

$$y(t) = y_0 + v_0 \cdot t + a \cdot \frac{t^2}{2}$$

$$\dot{y}(t) = v_y(t) = v_0 + a \cdot t$$

Física 1 (Q): Tiro Oblicuo

→ Hipótesis del modelo matemático

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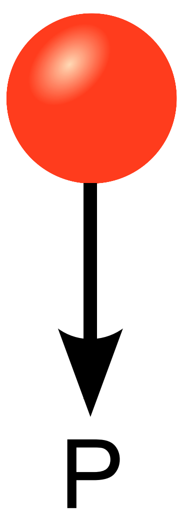

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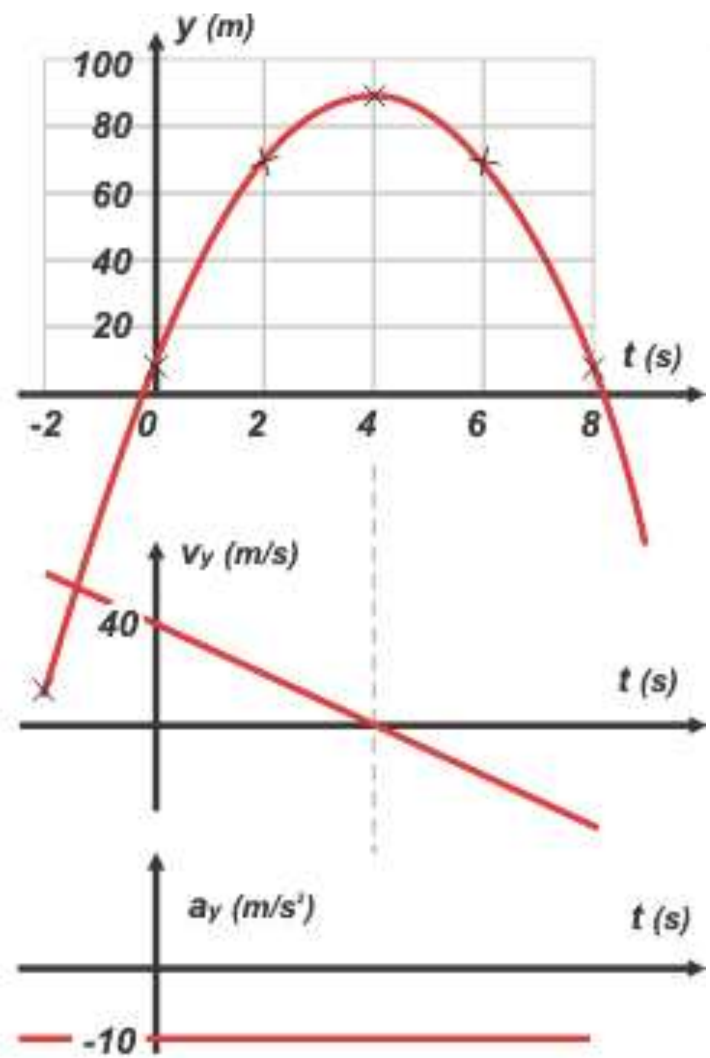
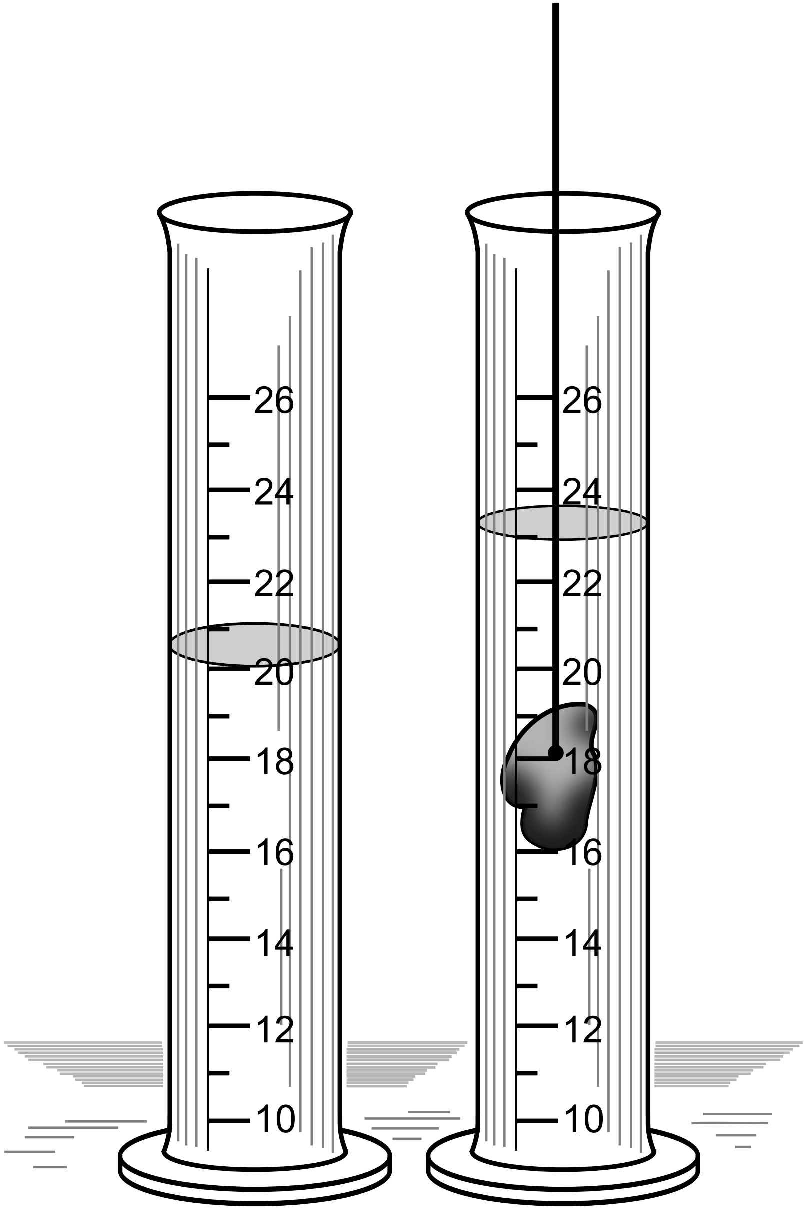
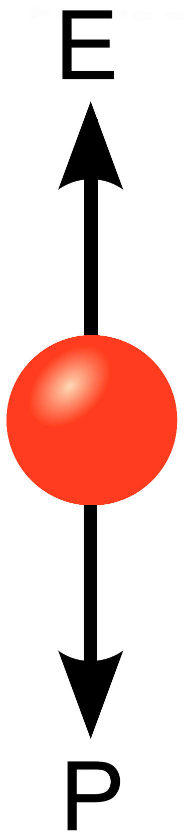
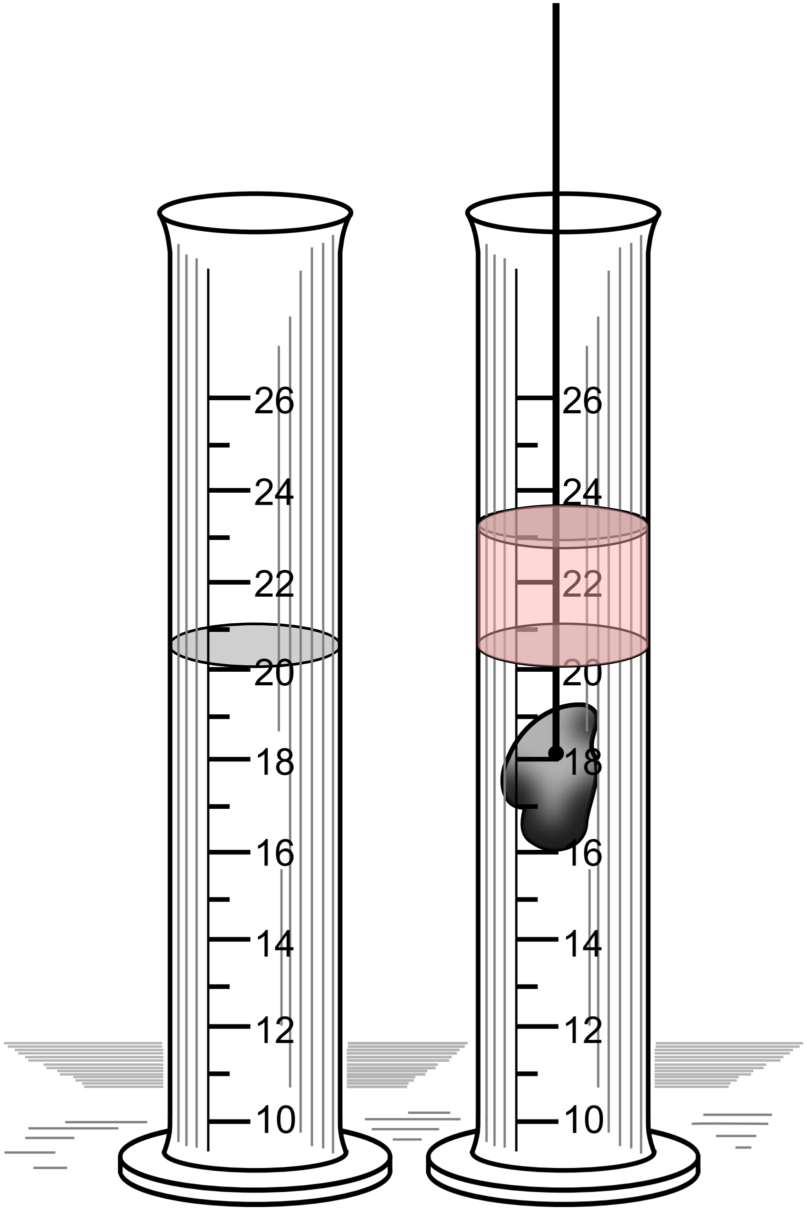




Principio de Arquímedes



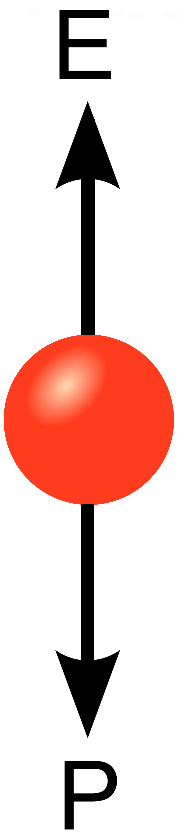
Física 1 (Q): Empuje



Principio de Arquímedes

Empuje igual al peso del
volumen de líquido
desplazado

Modelo de volumen:
ESFERA



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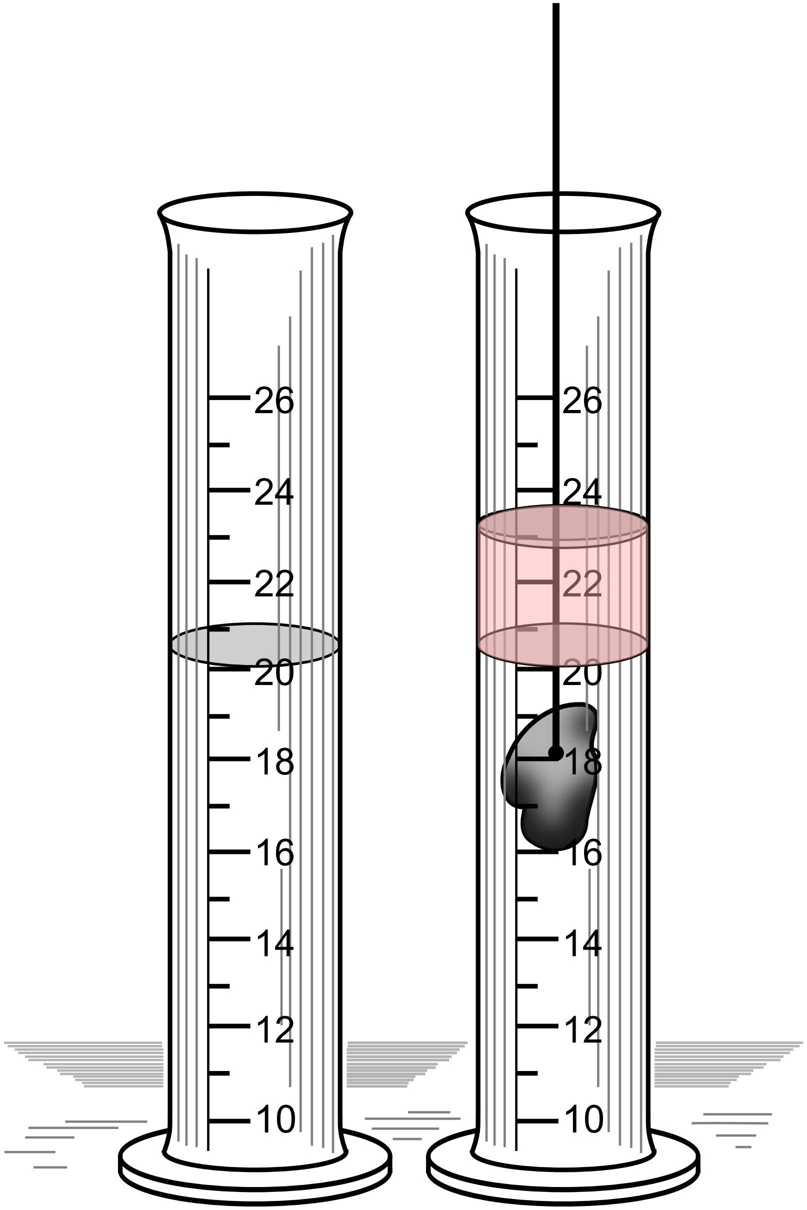


Principio de Arquímedes

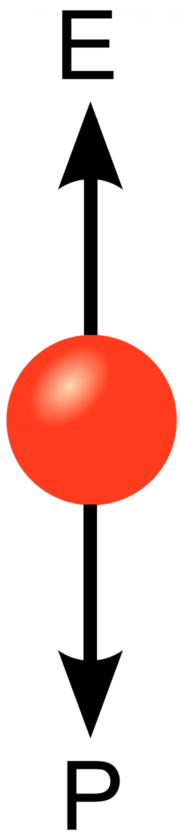
Volumen

$$E = \frac{4\pi}{3} \cdot r^3 \cdot \rho_L \cdot g$$

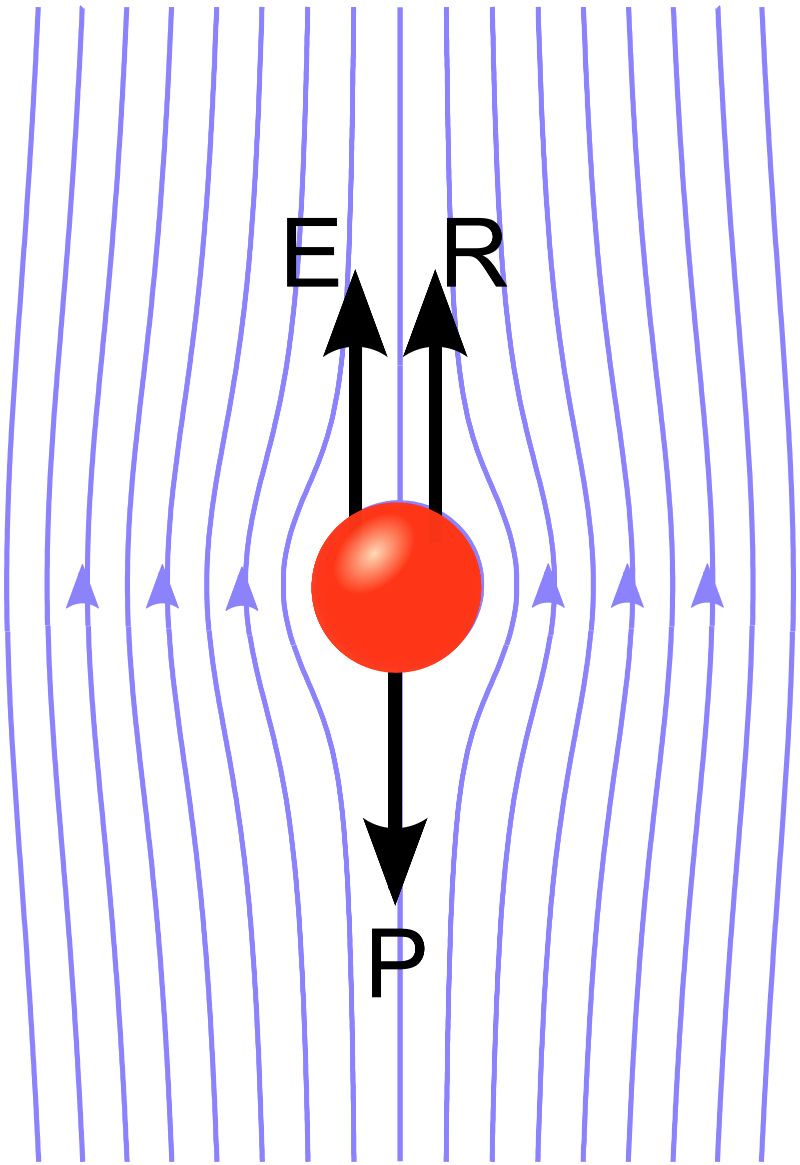
Densidad

Ecuación de movimiento

$$m\frac{d^2y}{dt^2} = P - E$$



Física 1 (Q): Fricción



Fricción del fluido: Viscosidad

R = ?

¿Cómo lo modelamos?

Paréntesis: (Navier-Stokes)

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \eta\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial P}{\partial y} + \eta\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) + \rho g_y$$

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Paréntesis: (Navier-Stokes)

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{\partial P}{\partial x}+\eta\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

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Coordenada y

Paréntesis: (Navier-Stokes)

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Coordenada y

Terminos convectivos (bajo Re)

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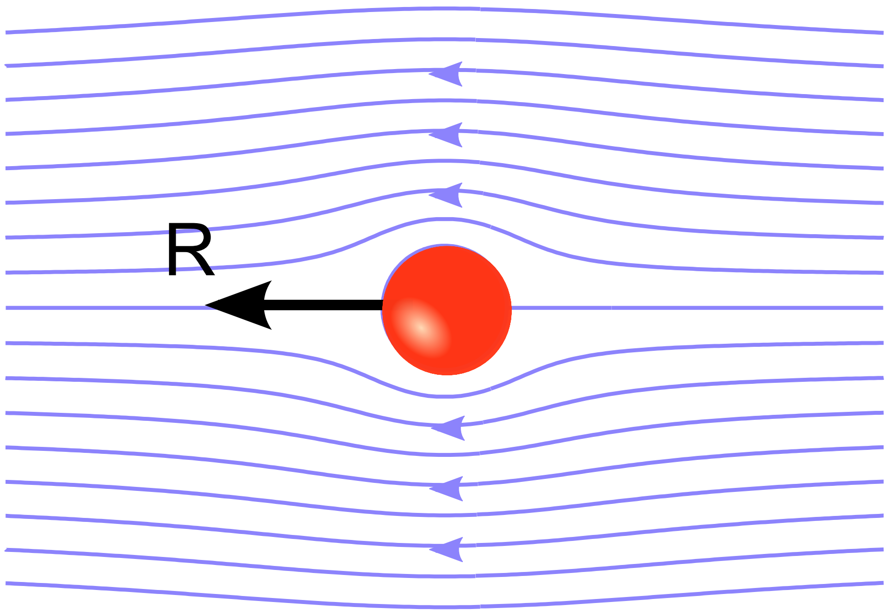
Coordenada y

Terminos convectivos (bajo Re)

Estacionario (velocidad no cambia)

Paréntesis: (Navier-Stokes)

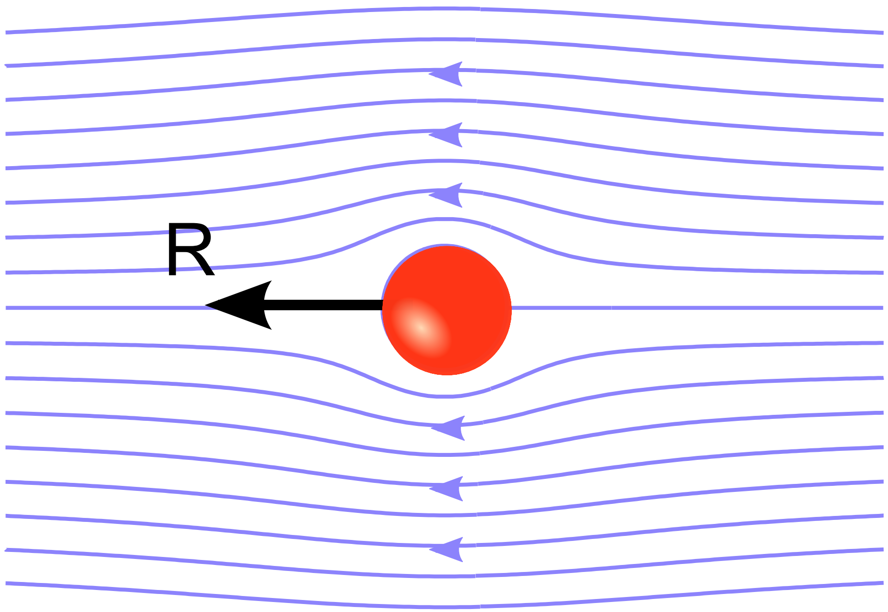
$$0 = -\frac{\partial P}{\partial y} + \eta\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right)$$



Coordenada y
Terminos convectivos (bajo Re)
Estacionario (velocidad no cambia)
Objeto esférico en fluido
de extensión infinita

Ley de Stokes

$$R = 6 \cdot \pi \cdot r \cdot \eta \cdot v$$



Coordenada y
Terminos convectivos (bajo Re)
Estacionario (velocidad no cambia)
Objeto esférico en fluido
de extensión infinita

Ley de Stockes y velocidad límite

Ecuación de movimiento

$$m\frac{d^2y}{dt^2} = P - E - R$$

$$P = m\,g = \frac{4}{3}\pi \cdot r^3 \cdot \rho_E \cdot g$$

$$E = \frac{4\pi}{3} \cdot r^3 \cdot \rho_L \cdot g$$

$$R = 6 \cdot \pi \cdot r \cdot \eta \cdot v$$

Ley de Stockes y velocidad límite

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$$R = 6 \cdot \pi \cdot r \cdot \eta \cdot v$$

Soluciones

$$m\frac{d^2y}{dt^2} = P - E - R = 0 \Rightarrow v_{lim} = \frac{2}{9} \cdot \frac{r^2 g}{\eta} \cdot (\rho_E - \rho_L)$$

$$m\frac{d^2y}{dt^2} = P - E - 6 \cdot \pi \cdot r \cdot \eta \cdot \frac{dy}{dt}$$

$$\Rightarrow \dot{y}(t) = v_y(t) = v_{lim}(1 - exp(-\alpha t))$$

Ley de Stockes y velocidad límite

Modelo para trabajar

Terminos convectivos (bajo Re)

Estacionario (velocidad no cambia)

Objeto esférico en fluido de extensión infinita

Velocidad límite:

$$v_{lim} = \frac{2}{9} \cdot \frac{r^2 g}{\eta} \cdot (\rho_E - \rho_L)$$

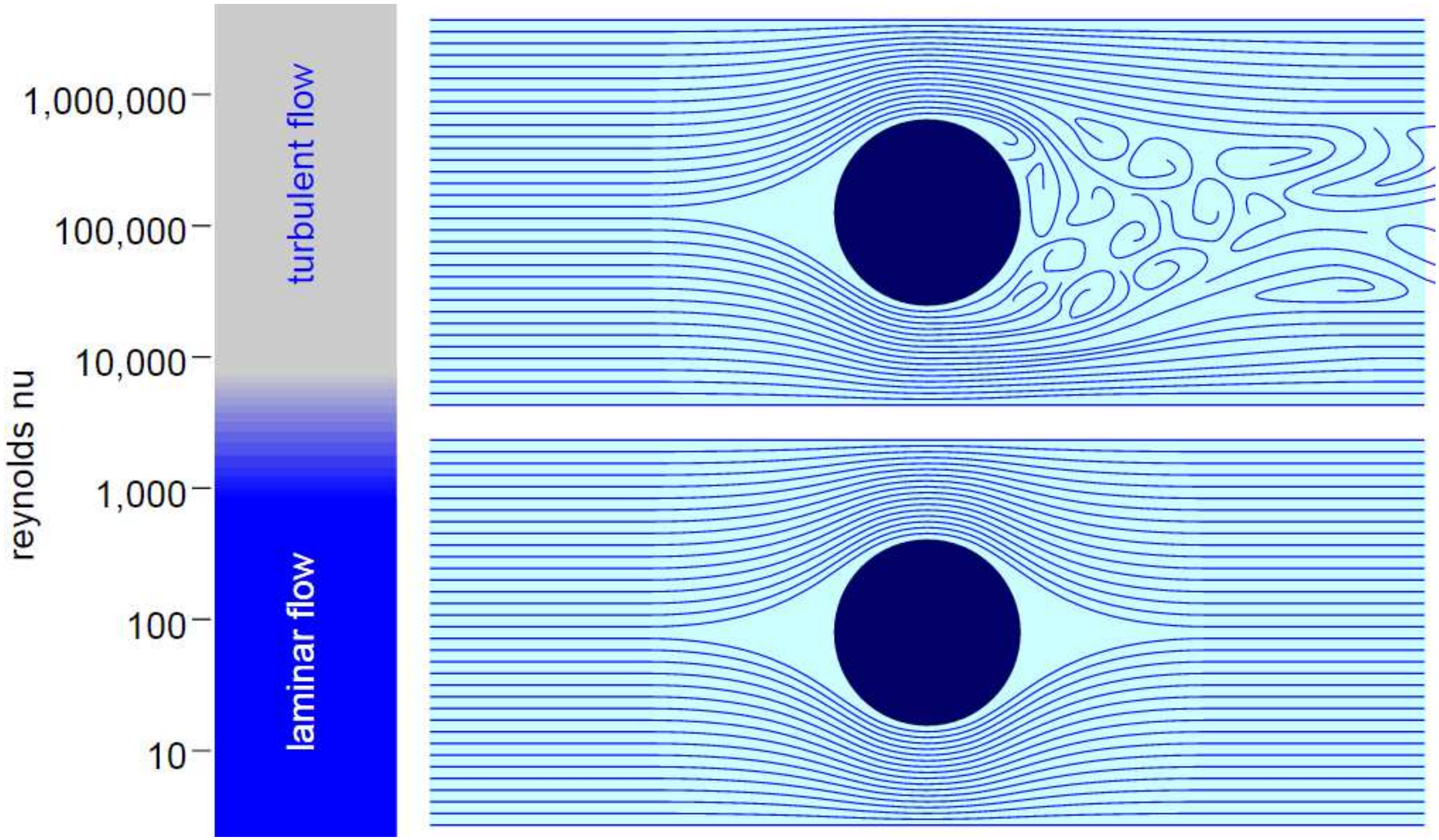
Velocidad en el tiempo:

$$v_y(t) = v_{lim}(1 - exp(-\alpha t))$$

Aproximaciones: Reynolds

Terminos convectivos (bajo Re)

$$Re = \frac{\rho_L \cdot v_{lim} \cdot d}{\eta}$$

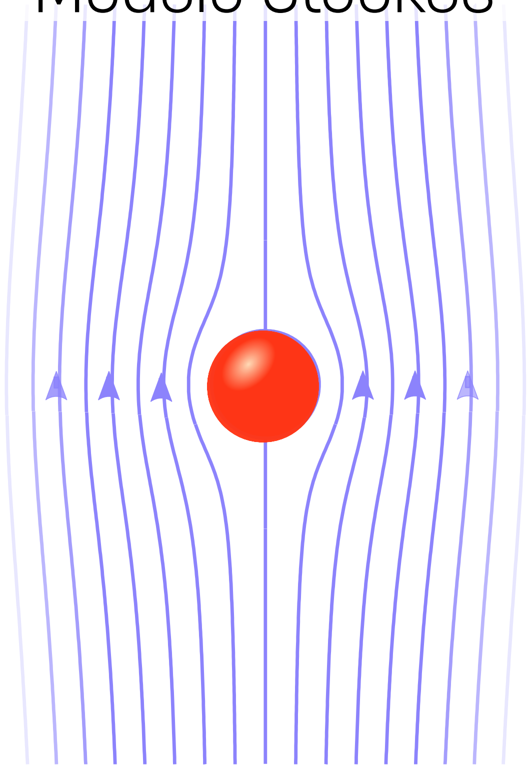


Corrección de Ladenburg

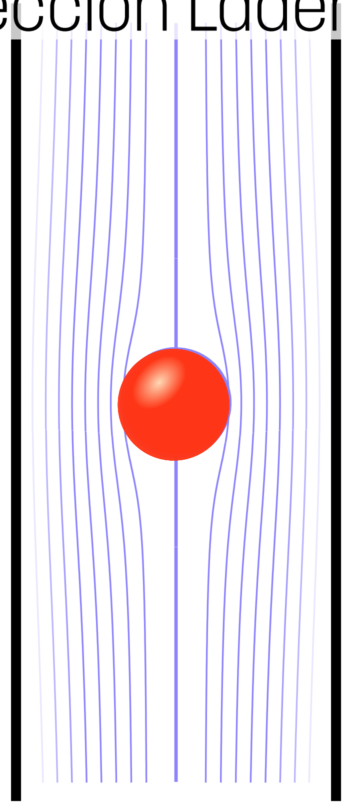
Objeto esférico en fluido de extensión infinita

$$\lambda = 1 + 2.4\frac{r}{r_{tubo}} \quad \Rightarrow \quad v_{lim} = \lambda \cdot v_{medida}$$

Modelo Stockes



Corrección Ladenburg



Presentación de la actividad

Física 1 (Q)

- **Cuaderno de laboratorio**
- **Dos series de videos**
  - Caída libre en un fluido
- **Papers y datos**

Física 1 (Q)

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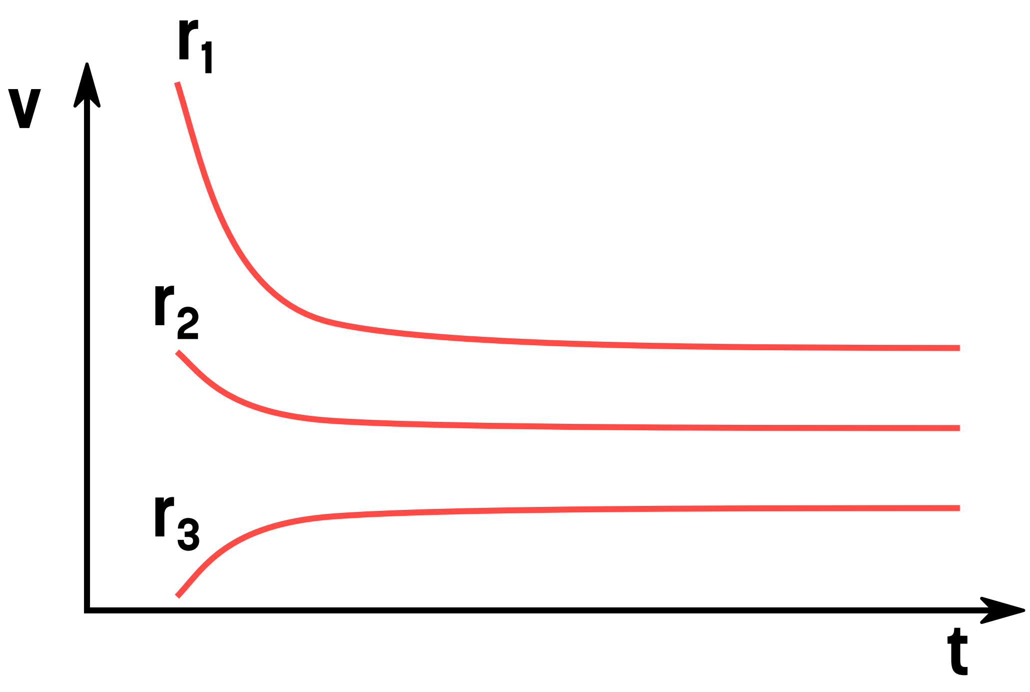
  Caída libre en un fluido
- **Papers y datos**

r	v_{final}
r_1	
r_2	
r_3	



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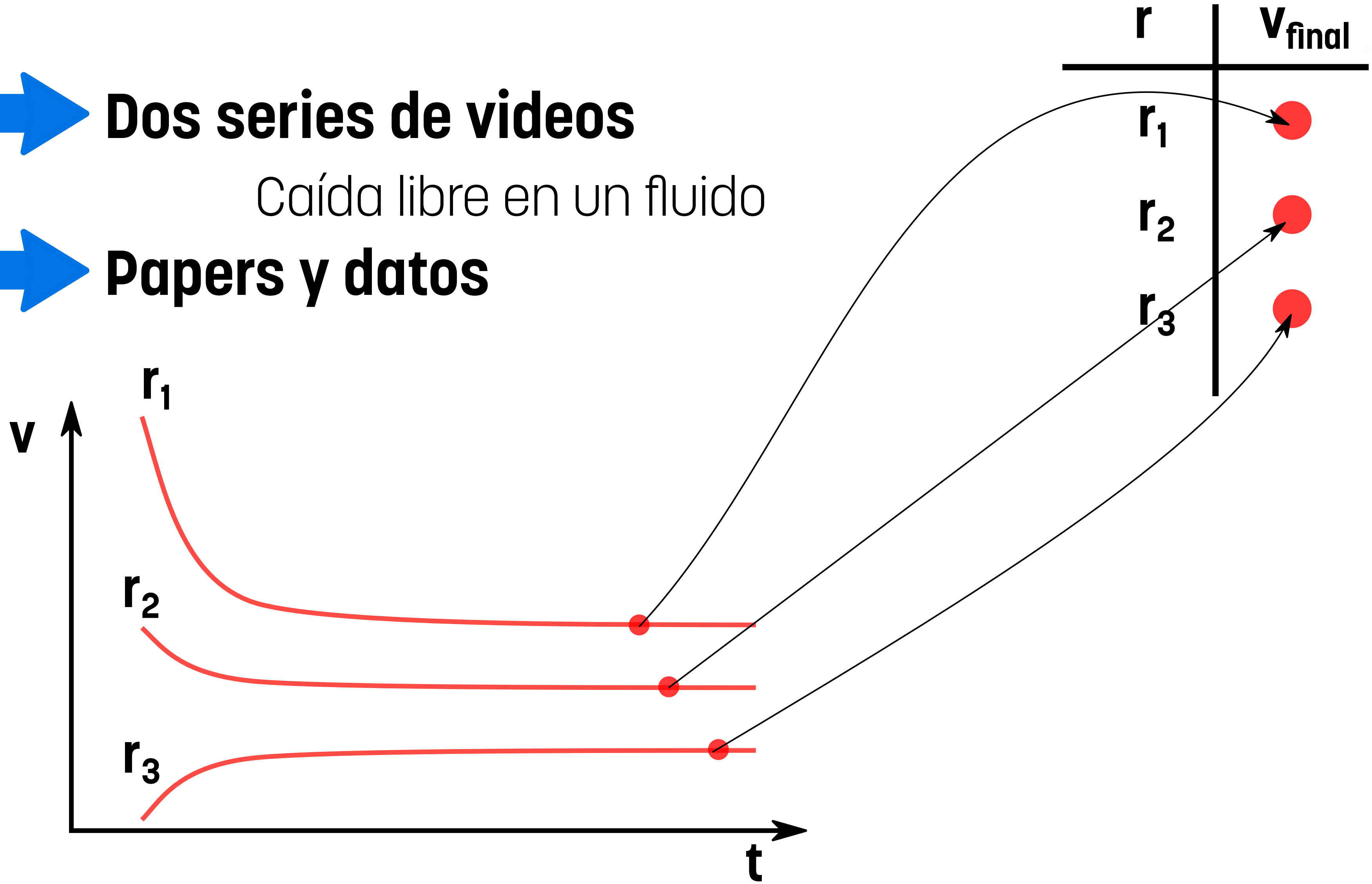

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r_1	
r_2	
r_3	

$$\lambda = 1 + 2.4 \frac{r}{r_{tubo}}$$

$$v_{lim} = \lambda \cdot v_{medida}$$

$$v_{lim} = \frac{2}{9} \cdot \frac{r^2 g}{\eta} \cdot (\rho_E - \rho_L)$$

Cuaderno de laboratorio

Dos series de videos

Caída libre en un fluido

Papers y datos

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r_1	
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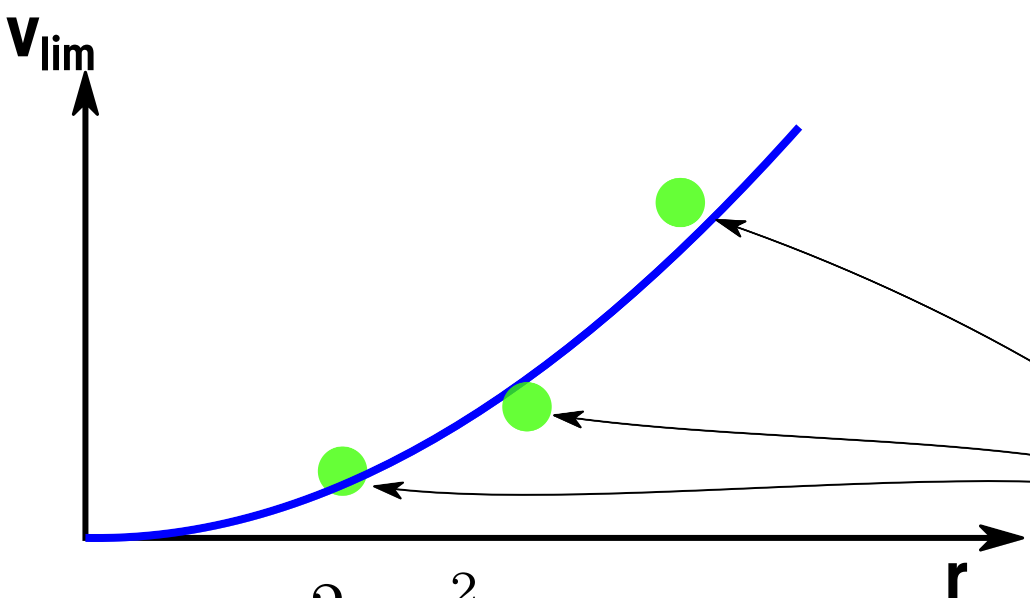

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- **Cuaderno de laboratorio**
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  Caída libre en un fluido
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