Matrices tridiagonales

Autovalores de matrices de $n \times n$

Caso 1

$$M = \begin{pmatrix} b & 1 & 0 & ... & 0 & 0 & 0 \\ 1 & b & 1 & ... & 0 & 0 & 0 \\ 0 & 1 & b & ... & 0 & 0 & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & 0 & 0 & ... & 1 & b & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & b \end{pmatrix} \tag{1}$$

$$\lambda_k = b + 2\,\cos\left(\frac{k\,\pi}{n+1}\right) \quad , \quad k = 1, ..., n \tag{2}$$

Caso 2

$$M = \begin{pmatrix} b+1 & 1 & 0 & ... & 0 & 0 & 0 \\ 1 & b & 1 & ... & 0 & 0 & 0 \\ 0 & 1 & b & ... & 0 & 0 & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & 0 & 0 & ... & 1 & b & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & b+1 \end{pmatrix} \tag{3}$$

$$\lambda_k = b + 2\,\cos\left(\frac{(k-1)\,\pi}{n}\right) \quad , \quad k = 1, ..., n \tag{4}$$

Caso 3

$$M = \begin{pmatrix} b-1 & 1 & 0 & ... & 0 & 0 & 0 \\ 1 & b & 1 & ... & 0 & 0 & 0 \\ 0 & 1 & b & ... & 0 & 0 & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & 0 & 0 & ... & 1 & b & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & b-1 \end{pmatrix} \tag{5}$$

$$\lambda_k = b + 2\,\cos\left(\frac{k\,\pi}{n}\right) \quad , \quad k = 1, ..., n \tag{6}$$

Caso 4

$$M = \begin{pmatrix} b & 1 & 0 & ... & 0 & 0 & 0 \\ 1 & b & 1 & ... & 0 & 0 & 0 \\ 0 & 1 & b & ... & 0 & 0 & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & 0 & 0 & ... & 1 & b & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & b+1 \end{pmatrix} \quad , \quad \lambda_k = b + 2\,\cos\left(\frac{(2k-1)\,\pi}{2n+1}\right) \tag{7}$$

$$M = \begin{pmatrix} b & 1 & 0 & ... & 0 & 0 & 0 \\ 1 & b & 1 & ... & 0 & 0 & 0 \\ 0 & 1 & b & ... & 0 & 0 & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & 0 & 0 & ... & 1 & b & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & b-1 \end{pmatrix} \quad , \quad \lambda_k = b + 2\,\cos\left(\frac{2k\,\pi}{2n+1}\right) \tag{8}$$

Caso 5

$$M = \begin{pmatrix} b+1 & 1 & 0 & ... & 0 & 0 & 0 \\ 1 & b & 1 & ... & 0 & 0 & 0 \\ 0 & 1 & b & ... & 0 & 0 & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & 0 & 0 & ... & 1 & b & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & b \end{pmatrix} \quad , \quad \lambda_k = b + 2\,\cos\left(\frac{(2k-1)\,\pi}{2n+1}\right) \tag{9}$$

$$M = \begin{pmatrix} b-1 & 1 & 0 & ... & 0 & 0 & 0 \\ 1 & b & 1 & ... & 0 & 0 & 0 \\ 0 & 1 & b & ... & 0 & 0 & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & 0 & 0 & ... & 1 & b & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & b \end{pmatrix} \quad , \quad \lambda_k = b + 2\,\cos\left(\frac{2k\,\pi}{2n+1}\right) \tag{10}$$

Caso 6

$$M = \begin{pmatrix} b \pm 1 & 1 & 0 & ... & 0 & 0 & 0 \\ 1 & b & 1 & ... & 0 & 0 & 0 \\ 0 & 1 & b & ... & 0 & 0 & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & 0 & 0 & ... & 1 & b & 1 \\ 0 & 0 & 0 & ... & 0 & 1 & b \mp 1 \end{pmatrix} \tag{11}$$

$$\lambda_k = b + 2\,\cos\left(\frac{(2k-1)\,\pi}{2n}\right) \quad , \quad k = 1, ..., n \tag{12}$$