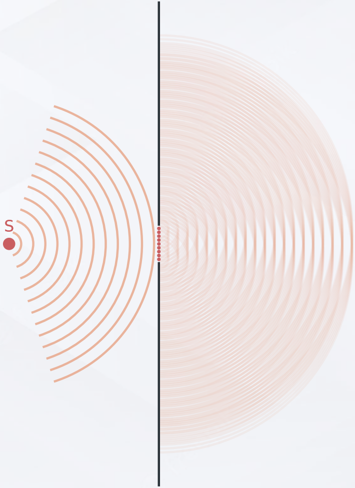
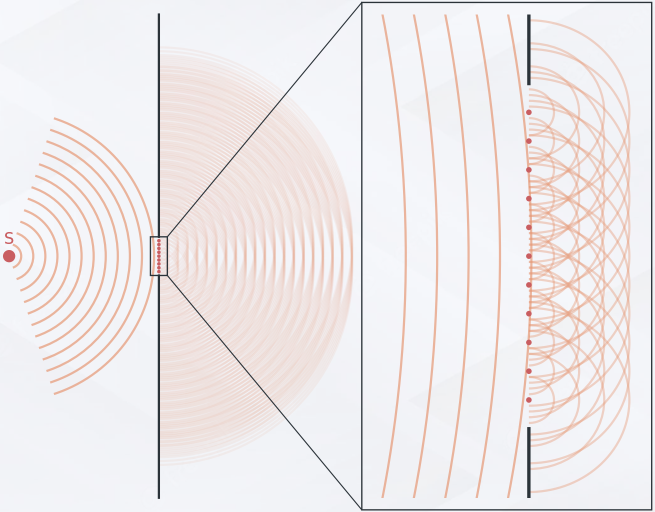
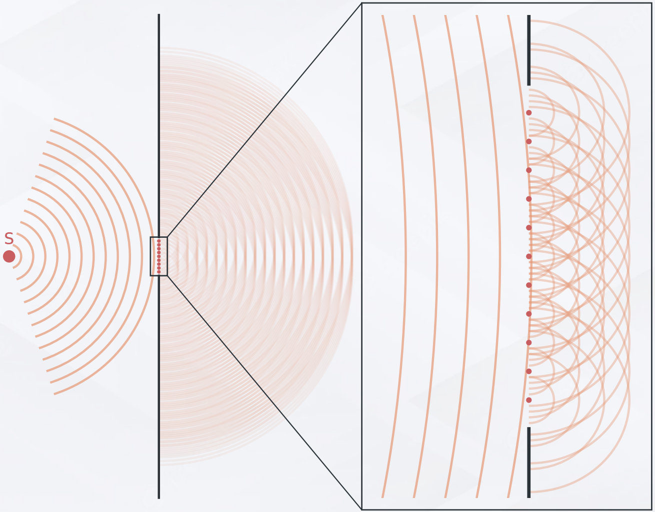
Hola

hablemos de difracción


S


S

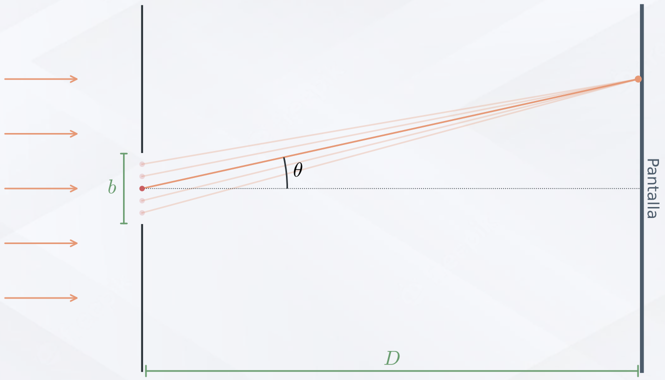

S



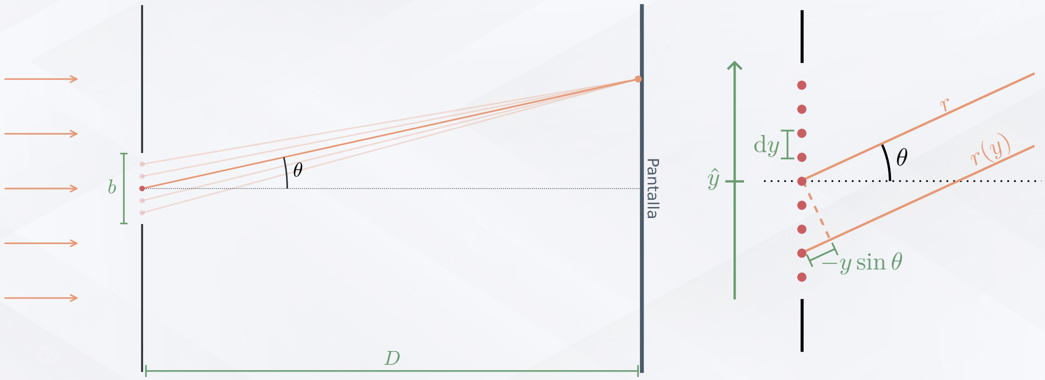
$$\mathbf{E} = \int_{-\infty}^{\infty} \mathrm{d}\mathbf{E}$$

$$\mathrm{d}\mathbf{E} = \frac{\mathbf{E_0}}{|r(y)|}\, e^{i[\mathbf{k}\cdot\mathbf{r}(y)-\omega\, t]}$$

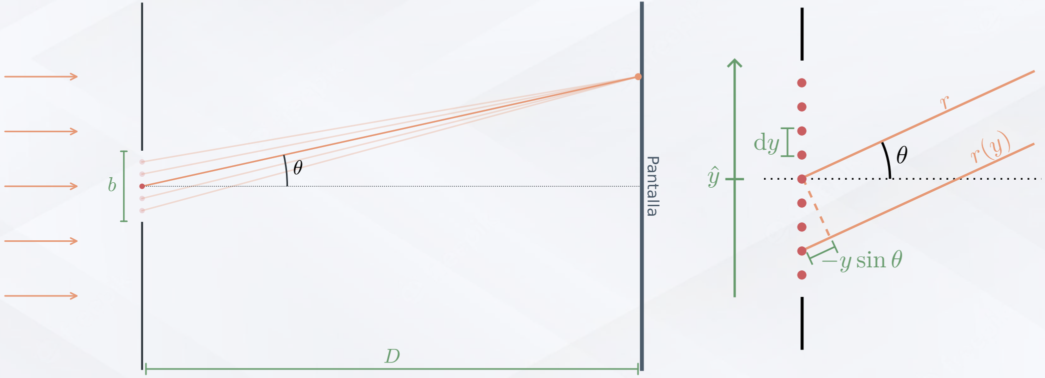
(donde está la rendija)



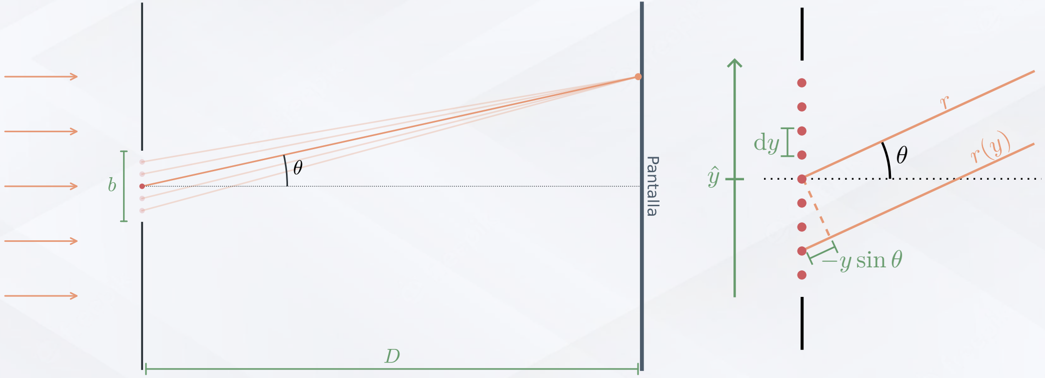
b
θ
Pantalla
D



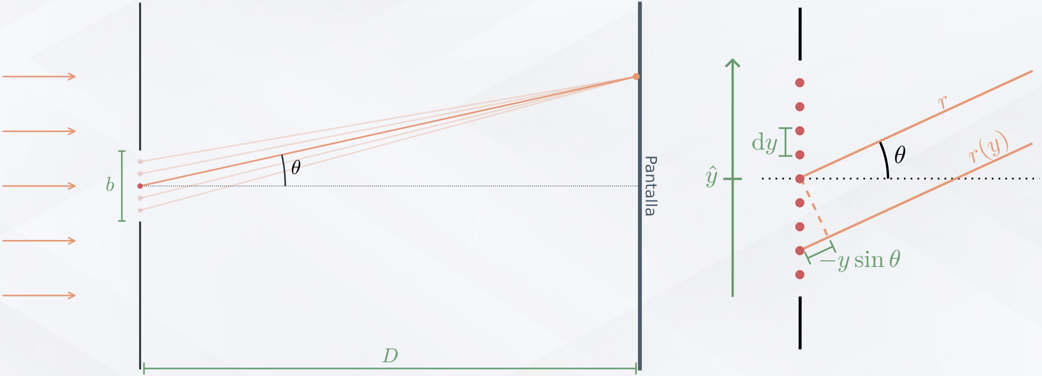
θ
b
Pantalla
D
$\hat{y}$
$\mathrm{d}y$
r
θ
$r(y)$
$-y\sin\theta$



$$d\mathbf{E} = \frac{\mathbf{E_0}}{|r(y)|}\, e^{i[\mathbf{k}\cdot\mathbf{r}(y) - \omega\, t]} \quad \text{con} \quad r(y) \simeq r - y\,\sin\,\theta \quad \text{y} \quad \frac{1}{|r(y)|} \simeq \frac{1}{r}$$



$$\mathrm{d}\mathbf{E} = \frac{\mathbf{E_0}}{r}\; e^{i[k(r - y\sin\theta) - \omega\, t]}$$



$$\mathrm{d}\mathbf{E} = \frac{\mathbf{E_0}}{r}\, e^{i[k(r - y\sin\theta) - \omega t]}, \quad \mathbf{E} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \mathrm{d}\mathbf{E} = \frac{\mathbf{E_0}}{r}\, e^{i(k\,r - \omega\,t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\,\mathrm{d}y$$

Un comentario antes de continuar...

$$\sin \theta$$

Un comentario antes de continuar...

$$\sin\theta \simeq \theta$$

Un comentario antes de continuar...

$$\sin\theta \simeq \theta \simeq \tan\theta$$

Un comentario antes de continuar...

$$\sin\theta \simeq \theta \simeq \tan\theta \underset{\uparrow}{\simeq} \frac{y}{D}$$

a veces

$$\mathbf{E} = \frac{\mathbf{E_0}}{r} e^{i(kr - \omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta} \, \mathrm{d}y$$

$$\mathbf{E} = \frac{\mathbf{E_0}}{r} e^{i(kr-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\,\mathrm{d}y = \frac{\mathbf{E_0}}{r}\, e^{i(kr-\omega t)} \left. \frac{e^{-iky\sin\theta}}{-ik\sin\theta} \right|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$\mathbf{E} = \frac{\mathbf{E_0}}{r} e^{i(kr-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\, \mathrm{d}y = \frac{\mathbf{E_0}}{r}\, e^{i(kr-\omega t)} \left. \frac{e^{-iky\sin\theta}}{-ik\sin\theta} \right|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$= \frac{\mathbf{E_0}}{r}\, e^{i(kr-\omega t)}\, \frac{e^{-ik\frac{b}{2}\sin\theta} - e^{ik\frac{b}{2}\sin\theta}}{-\ ik\ \ \sin\theta}$$

$$\mathbf{E} = \frac{\mathbf{E_0}}{r} e^{i(kr-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\,\mathrm{d}y = \frac{\mathbf{E_0}}{r}\, e^{i(kr-\omega t)}\ \frac{e^{-iky\sin\theta}}{-ik\sin\theta}\Bigg|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$= \frac{\mathbf{E_0}}{r}\, e^{i(kr-\omega t)}\ \frac{e^{-ik\frac{b}{2}\sin\theta} - e^{ik\frac{b}{2}\sin\theta}}{-2ik\frac{b}{2}\sin\theta}\, b$$

$$\mathbf{E} = \frac{\mathbf{E_0}}{r} e^{i(kr-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\,\mathrm{d}y = \frac{\mathbf{E_0}}{r}\, e^{i(kr-\omega t)} \left.\frac{e^{-iky\sin\theta}}{-ik\sin\theta}\right|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$= \frac{\mathbf{E_0}}{r}\, e^{i(kr-\omega t)}\, \frac{e^{-ik\frac{b}{2}\sin\theta} - e^{ik\frac{b}{2}\sin\theta}}{-2ik\frac{b}{2}\sin\theta}\, b = \frac{\mathbf{E_0}}{r}\, e^{i(kr-\omega t)}\, b\, \frac{\sin\left(k\frac{b}{2}\sin\theta\right)}{k\frac{b}{2}\sin\theta}$$

$$\mathbf{E} = \frac{\mathbf{E_0}}{r} e^{i(kr-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\, \mathrm{d}y = \frac{\mathbf{E_0}}{r} e^{i(kr-\omega t)} \left. \frac{e^{-iky\sin\theta}}{-ik\sin\theta} \right|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$= \frac{\mathbf{E_0}}{r} e^{i(kr-\omega t)} \frac{e^{-ik\frac{b}{2}\sin\theta} - e^{ik\frac{b}{2}\sin\theta}}{-2ik\frac{b}{2}\sin\theta}\, b = \frac{\mathbf{E_0}}{r} e^{i(kr-\omega t)}\, b\, \frac{\sin\left(k\frac{b}{2}\sin\theta\right)}{k\frac{b}{2}\sin\theta}$$

$$\implies \mathbf{E} = \frac{\mathbf{E_0}}{r}\, b\, e^{i(kr-\omega t)} \frac{\sin\beta}{\beta}, \quad \beta = \frac{bk}{2}\sin\theta$$

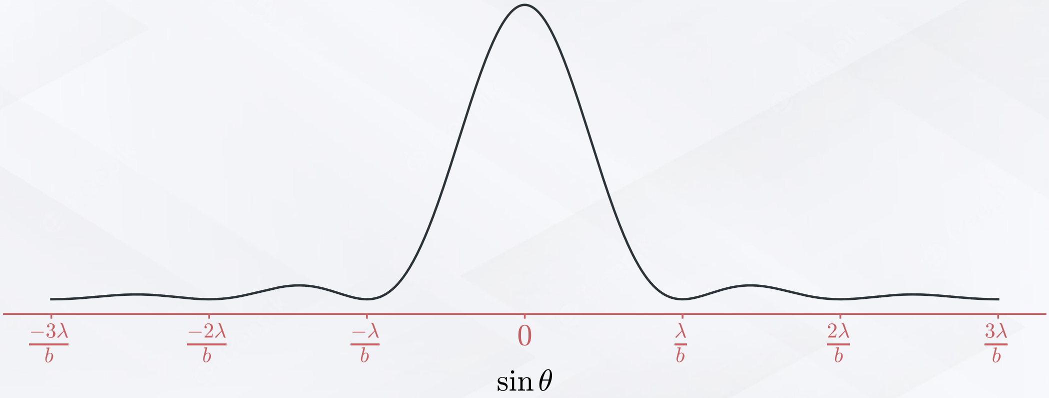
$$\mathbf{E} = \frac{\mathbf{E_0}}{r} e^{i(kr-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\,\mathrm{d}y = \frac{\mathbf{E_0}}{r}\, e^{i(kr-\omega t)} \left.\frac{e^{-iky\sin\theta}}{-ik\sin\theta}\right|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$= \frac{\mathbf{E_0}}{r}\, e^{i(kr-\omega t)} \frac{e^{-ik\frac{b}{2}\sin\theta} - e^{ik\frac{b}{2}\sin\theta}}{-2ik\frac{b}{2}\sin\theta}\, b = \frac{\mathbf{E_0}}{r}\, e^{i(kr-\omega t)}\, b\, \frac{\sin\left(k\frac{b}{2}\sin\theta\right)}{k\frac{b}{2}\sin\theta}$$

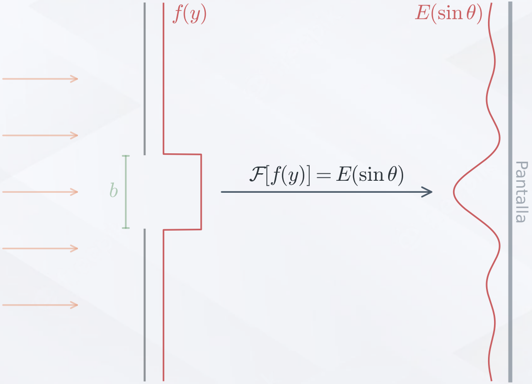
$$\implies \mathbf{E} = \frac{\mathbf{E_0}}{r}\, b\, e^{i(kr-\omega t)}\, \frac{\sin\beta}{\beta}, \quad \beta = \frac{bk}{2}\sin\theta$$

$$\implies I = \mathbf{E}^* \cdot \mathbf{E} = I_0\, \frac{\sin^2\beta}{\beta^2}, \quad I_0 = \left(\frac{2bE_0}{r}\right)^2$$

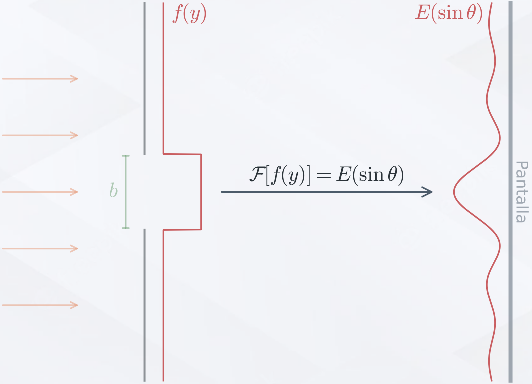
$$I = I_0 \;\text{sinc}^2\left(\frac{bk}{2}\,\sin\theta\right) = I_0 \;\text{sinc}^2\left(\frac{\pi\, b}{\lambda}\,\sin\theta\right)$$



b
Pantalla

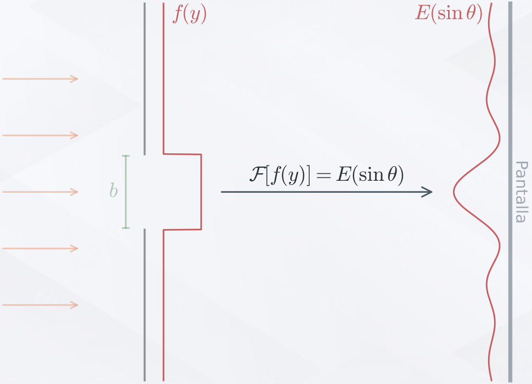


f(y)
E(sin θ)
b
𝓕[f(y)] = E(sin θ)
Pantalla



$$f(y) = \begin{cases} E_0 & \text{si } -\dfrac{b}{2} < y < \dfrac{b}{2} \\ 0 & \text{si no} \end{cases}$$

$$E = I_0 \operatorname{sinc}\left(\frac{kb}{2}\sin\theta\right)$$



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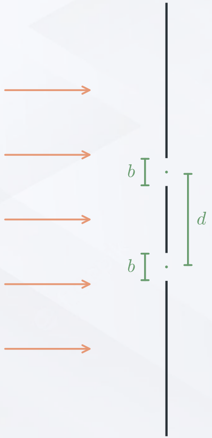
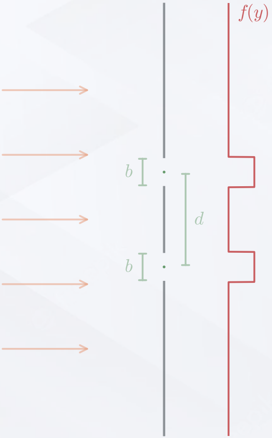
$$E = I_0 \operatorname{sinc}\left(\frac{kb}{2}\sin\theta\right)$$

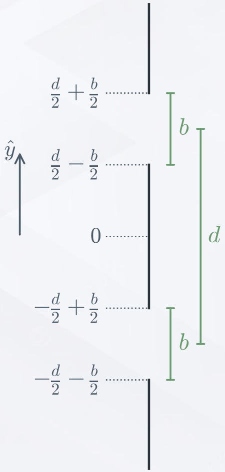
$$E = \frac{e^{i(kr-\omega t)}}{r}\int_{-\infty}^{\infty} f(y)\, e^{-iky\sin\theta}\, \mathrm{d}y$$



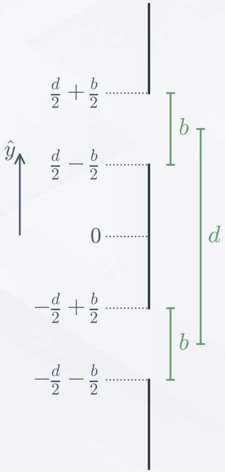
b
b
d



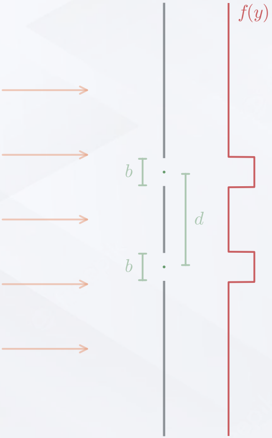
f(y)
b
d
b



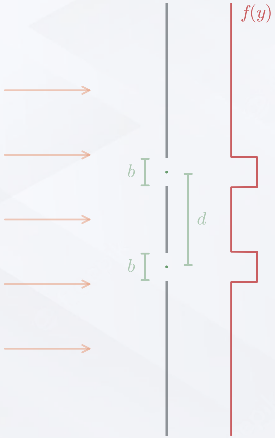
$\hat{y}$
$\frac{d}{2}+\frac{b}{2}$
$\frac{d}{2}-\frac{b}{2}$
0
$-\frac{d}{2}+\frac{b}{2}$
$-\frac{d}{2}-\frac{b}{2}$
b
d
b



$$f(y) = \begin{cases} E_0 & \text{si} \quad -\dfrac{d}{2}-\dfrac{b}{2} < y < -\dfrac{d}{2}+\dfrac{b}{2} \\ E_0 & \text{si} \quad \dfrac{d}{2}-\dfrac{b}{2} < y < \dfrac{d}{2}+\dfrac{b}{2} \\ 0 & \text{en otro caso} \end{cases}$$

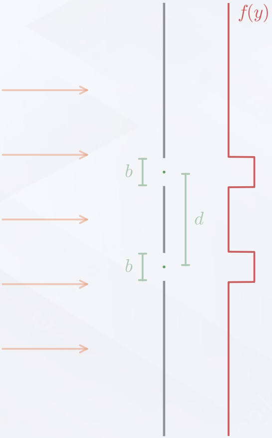


$$f(y) = \begin{cases} E_0 & \text{si} \quad -\frac{d}{2} - \frac{b}{2} < y < -\frac{d}{2} + \frac{b}{2} \\ E_0 & \text{si} \quad \frac{d}{2} - \frac{b}{2} < y < \frac{d}{2} + \frac{b}{2} \\ 0 & \text{en otro caso} \end{cases}$$



$$f(y) = \begin{cases} E_0 & \text{si} \quad -\dfrac{d}{2} - \dfrac{b}{2} < y < -\dfrac{d}{2} + \dfrac{b}{2} \\ E_0 & \text{si} \quad \dfrac{d}{2} - \dfrac{b}{2} < y < \dfrac{d}{2} + \dfrac{b}{2} \\ 0 & \text{en otro caso} \end{cases}$$

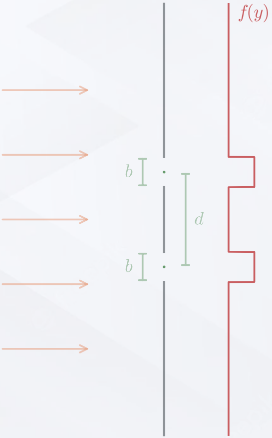
$$E = \frac{e^{i(kr-\omega t)}}{r} \int_{-\infty}^{\infty} f(y)\, e^{-iky\sin\theta}\, \mathrm{d}y$$



$$f(y) = \begin{cases} E_0 & \text{si} \quad -\frac{d}{2} - \frac{b}{2} < y < -\frac{d}{2} + \frac{b}{2} \\ E_0 & \text{si} \quad \frac{d}{2} - \frac{b}{2} < y < \frac{d}{2} + \frac{b}{2} \\ 0 & \text{en otro caso} \end{cases}$$

$$E = \frac{e^{i(kr-\omega t)}}{r} \int_{-\infty}^{\infty} f(y)\, e^{-iky\sin\theta}\, \mathrm{d}y$$

$$= \frac{e^{i(kr-\omega t)}}{r} \left[\int_{\frac{d}{2}-\frac{b}{2}}^{\frac{d}{2}+\frac{b}{2}} E_0\, e^{-iky\sin\theta}\, \mathrm{d}y + \int_{-\frac{d}{2}-\frac{b}{2}}^{-\frac{d}{2}+\frac{b}{2}} E_0\, e^{-iky\sin\theta}\, \mathrm{d}y \right]$$

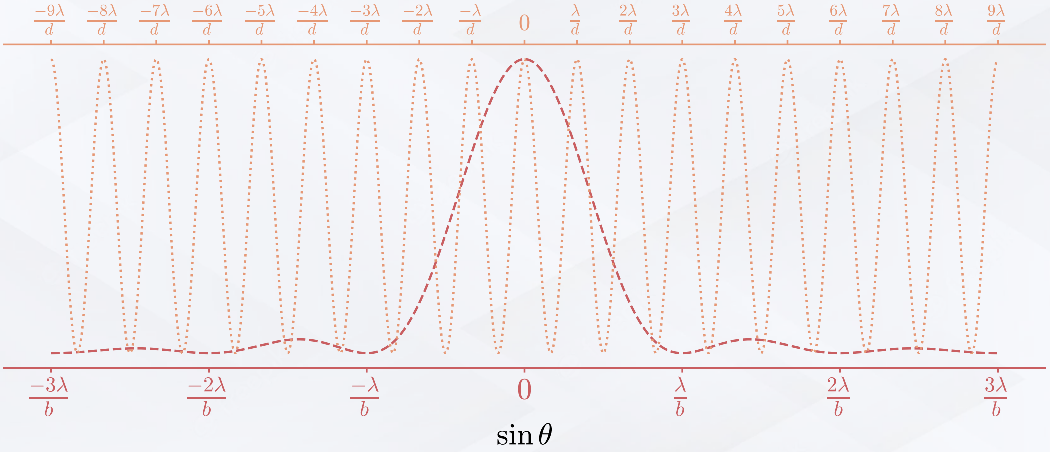


$\longleftarrow$ Seguimos allá

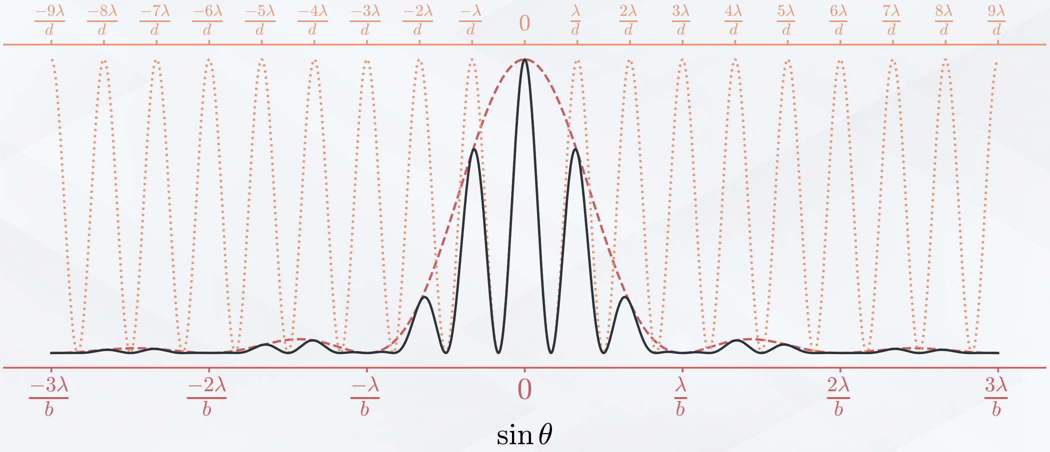
$$E = \frac{e^{i(kr-\omega t)}}{r} \int_{-\infty}^{\infty} f(y)\, e^{-iky\sin\theta}\, \mathrm{d}y$$

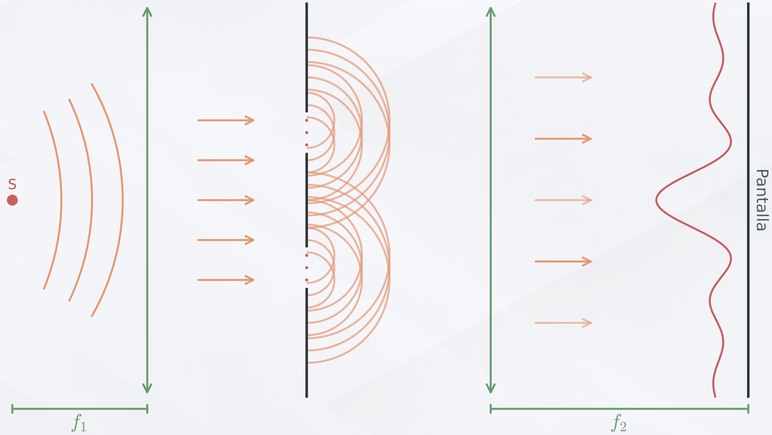
$$= \frac{e^{i(kr-\omega t)}}{r} \left[\int_{\frac{d}{2}-\frac{b}{2}}^{\frac{d}{2}+\frac{b}{2}} E_0\, e^{-iky\sin\theta}\, \mathrm{d}y + \int_{-\frac{d}{2}-\frac{b}{2}}^{-\frac{d}{2}+\frac{b}{2}} E_0\, e^{-iky\sin\theta}\, \mathrm{d}y \right]$$

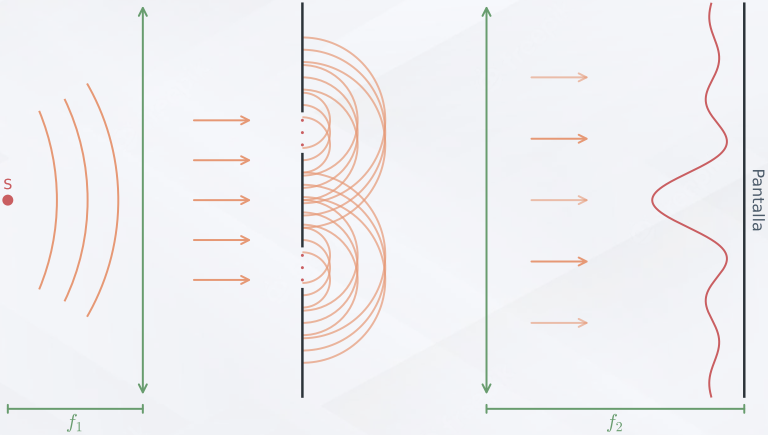
$$I = I_0 \operatorname{sinc}^2 \beta \cos^2 \alpha$$



$$I = I_0 \,\mathrm{sinc}^2\, \beta \cos^2 \alpha$$



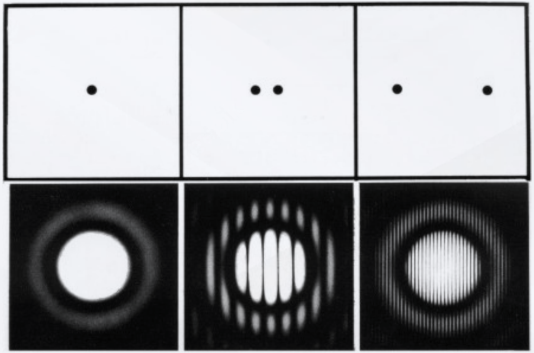

S
f_1
f_2
Pantalla



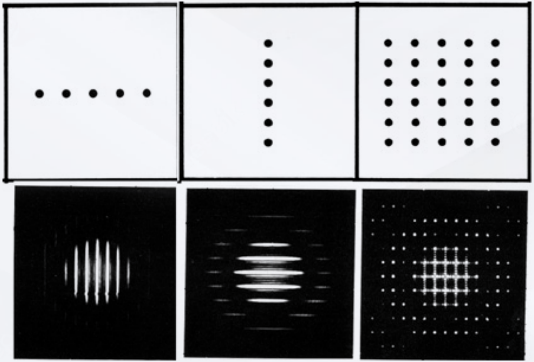
$$\sin\theta \simeq \theta \simeq \tan\theta \simeq \frac{y}{f_2}$$

↑ en este caso

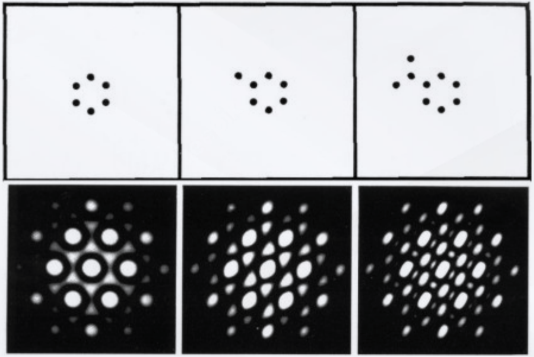
Algunos ejemplos en 2D



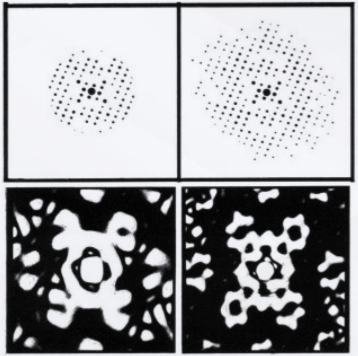
Algunos ejemplos en 2D



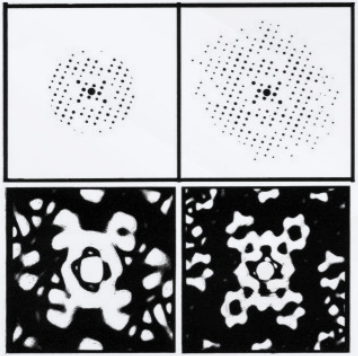
Algunos ejemplos en 2D

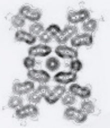


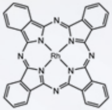
Algunos ejemplos en 2D



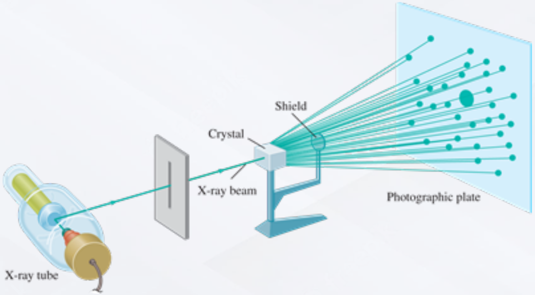
Algunos ejemplos en 2D







Difracción por Rayos X



Eso es todo.