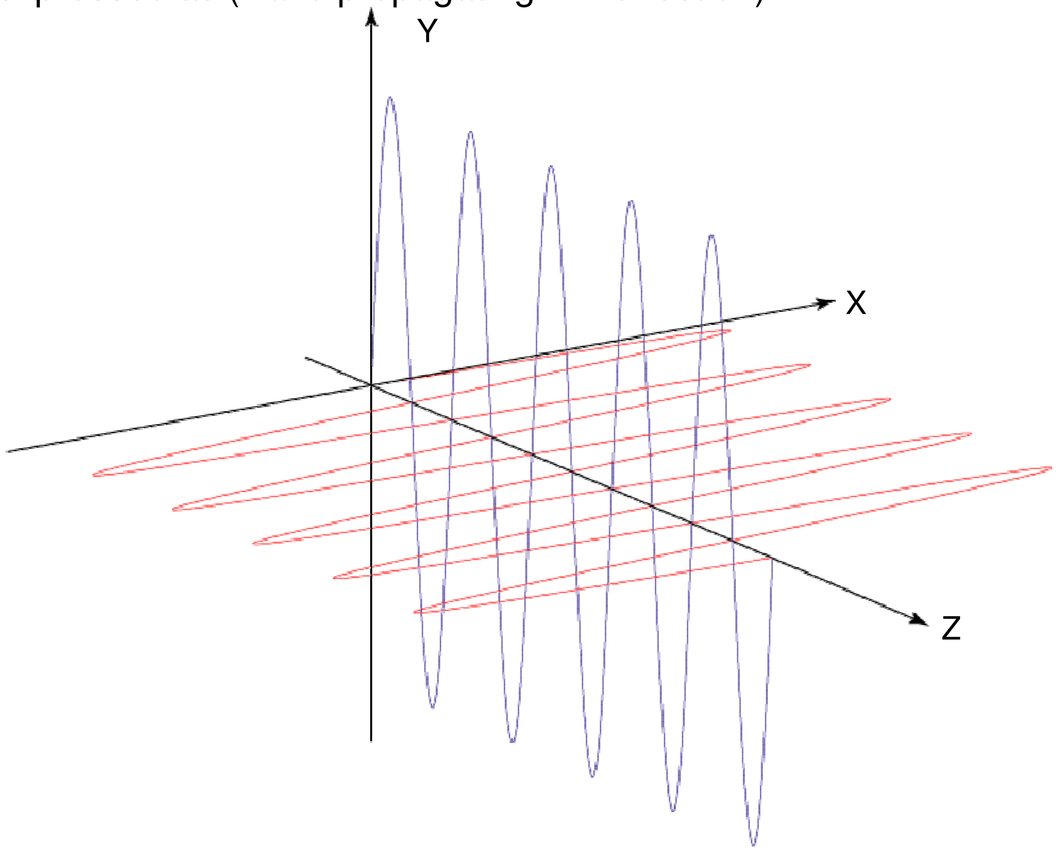
- Circa1670 Christian Huygens first suggested the vectoral nature of light to explain its behavior propagating through crystals.
- In the words of Newton (~1672), it appeared as though light had "sides…"
- In 1818 Fresnel and Arago were able to explain Young's interference experiment with a with a light wave that consists of two transverse components (oscillating perpendicular to the direction of propagation), and no longitudinal component.
- Initially seen as a "defect" in Fresnel's theory, solving Maxwell's equations in free space verified only transverse components arise.
- These transverse components can be expressed as (wave propagating in z direction):

$$E_x(z,t) = E_{0x} \cos(\tau + \delta_x)$$

$$E_y(z,t) = E_{0y} \cos(\tau + \delta_y)$$



➢ The propagation of E_x and E_y give rise to a vector describing a locus of points in space that generates a curve whose form can be derived using the previous equations.

$$E_x(z,t) = E_{0x}\cos(\tau + \delta_x) \quad and \quad E_y(z,t) = E_{0y}\cos(\tau + \delta_y)$$

➢ We start by dividing out by the amplitude and applying the double angle formulae to the argument of the cosine function:

$$\frac{E_x}{E_{0x}} = \cos(\tau)\cos(\delta_x) - \sin(\tau)\sin(\delta_y) \quad and \quad \frac{E_y}{E_{0y}} = \cos(\tau)\cos(\delta_y) - \sin(\tau)\sin(\delta_y)$$

➢ Re-arranging terms, subtracting, and using the double angle formula again, the following two expressions can be written:

$$\frac{E_x}{E_{0x}}\sin(\delta_y) - \frac{E_y}{E_{0y}}\sin(\delta_x) = \cos(\tau)\sin(\delta_y - \delta_x) \qquad \frac{E_x}{E_{0x}}\cos(\delta_y) - \frac{E_y}{E_{0y}}\cos(\delta_x) = \sin(\tau)\sin(\delta_y - \delta_x)$$

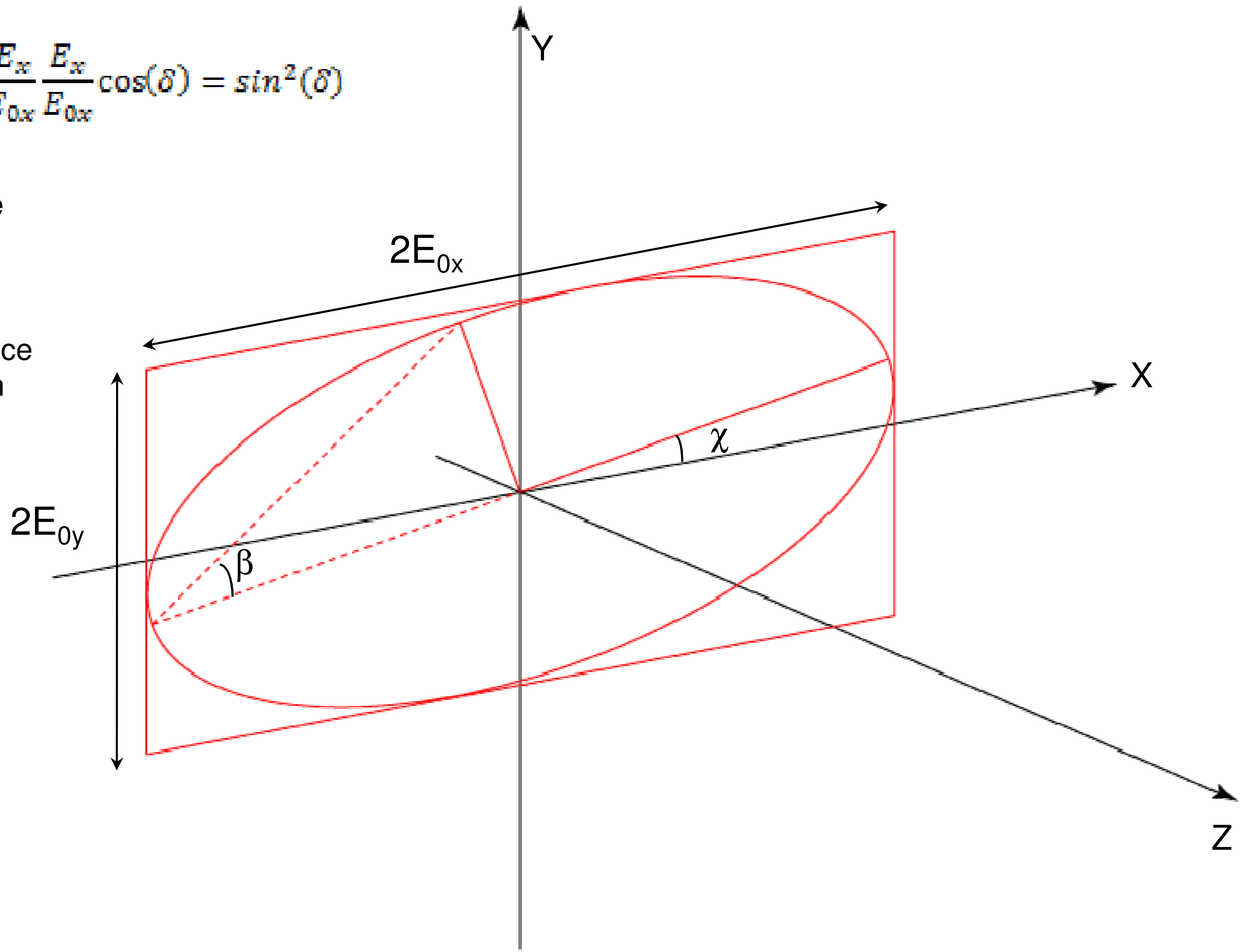
➢ Squaring the above two expressions, and adding them together yields ($\delta = \delta_y - \delta_x$):

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = sin^2(\delta)$$

➢ This is the equation for an ellipse. What it shows is that at any instant of time the locus of points described by the propagation of E_x and E_y will trace out this curve.

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = sin^2(\delta)$$

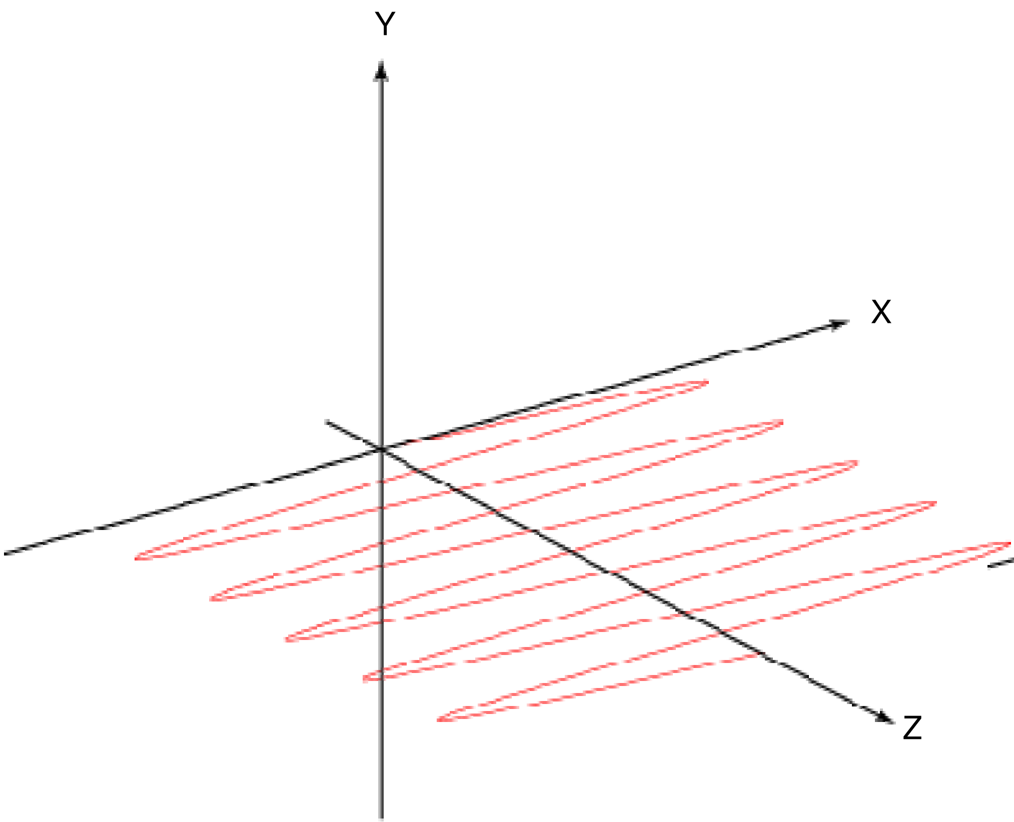
- β is measure of the of the ellipticity
- χ is rotation of the ellipse (consequence of the cross term in above equation)



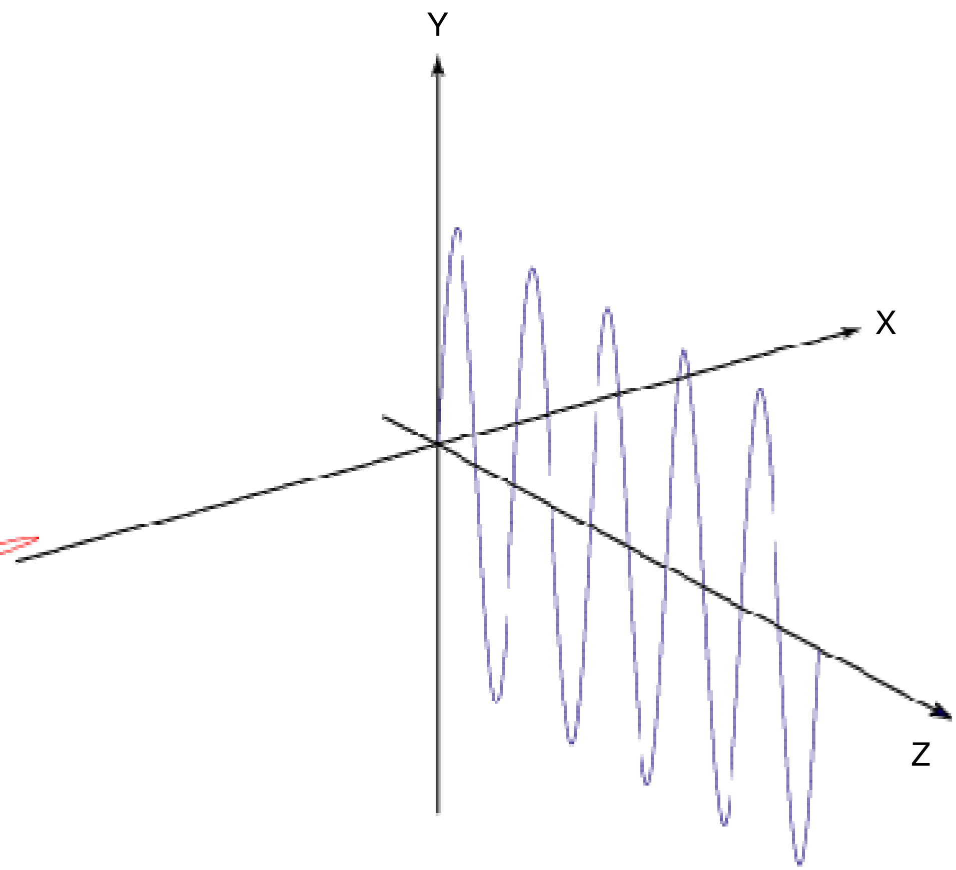
- For specific values of E_{0x}, E_{0y}, and δ, the polarization ellipse reduces to 4 specialized forms:

- Case I: Linear horizontal, or linear vertical polarization

$E_{0y} = 0$ (Linear Horizontal)



$E_{0x} = 0$ (Linear Vertical)



➢ Case II: Linear ±45° polarization ($\delta = 0$ or π, and $E_{0x} = E_{0y}$)

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = sin^2(\delta) \rightarrow \left(\frac{E_x}{E_{0x}} \pm \frac{E_y}{E_{0y}}\right)^2 = 0 \rightrightarrows E_y = \pm E_x$$

- The slope of the resultant vector is either positive or negative depending upon the relative phase shift between the two waves.

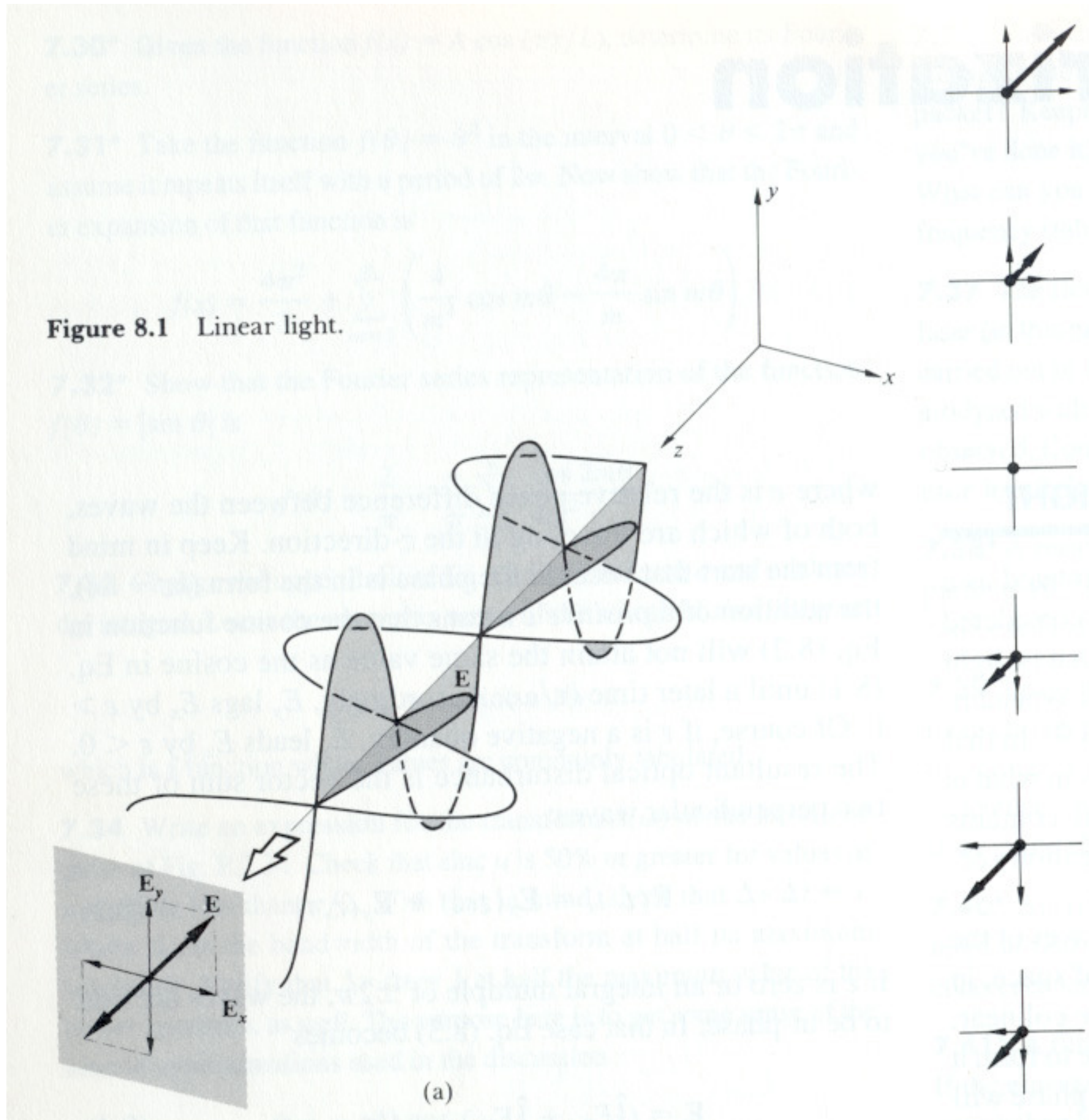
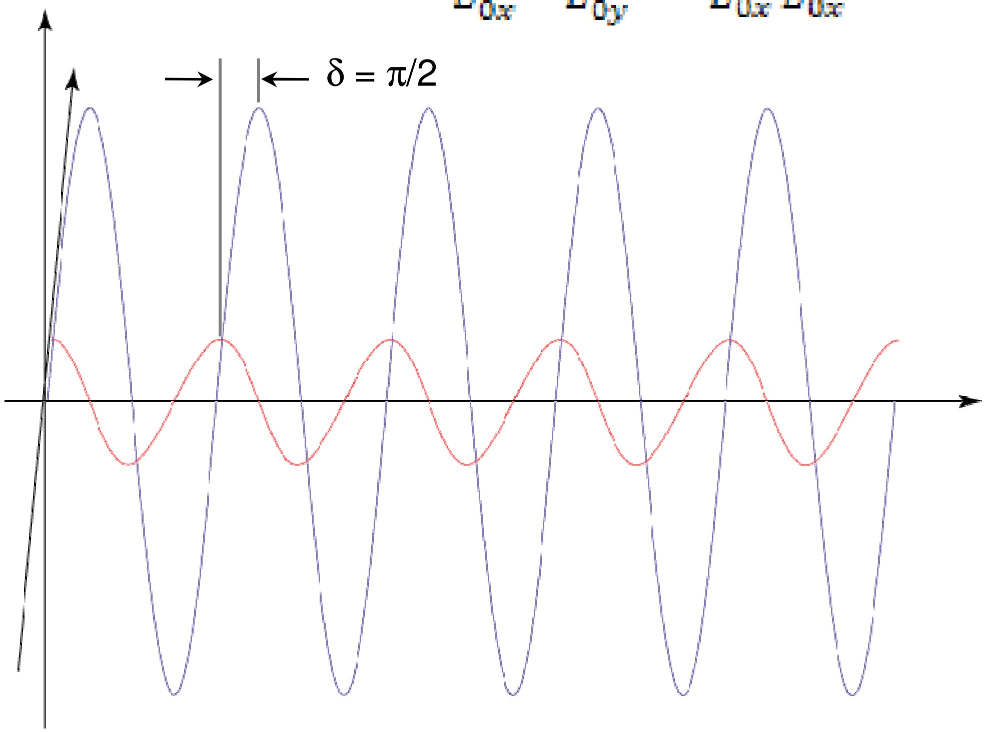
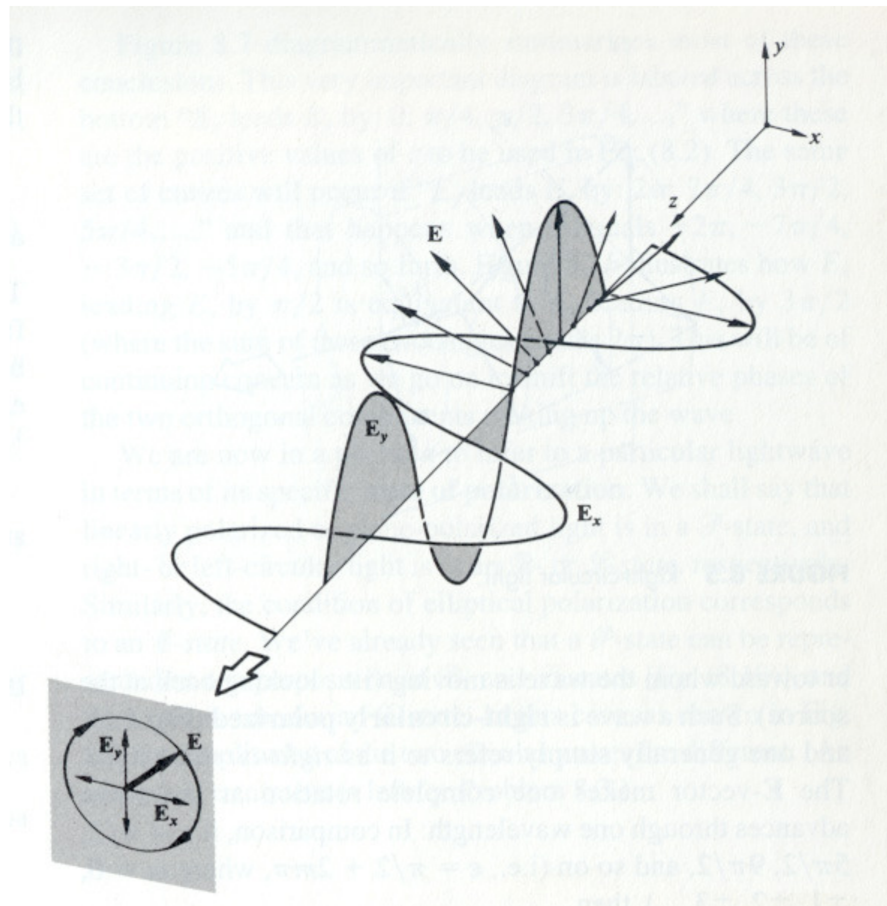


Figure 8.1 Linear light.

- Case III: Right or Left Circular Polarization ($\delta = \pi/2$ or $3\pi/2$, and $E_{0x} = E_{0y} = E_0$)

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = sin^2(\delta) \rightarrow \frac{E_x^2}{E_0^2} + \frac{E_y^2}{E_0^2} = 1$$

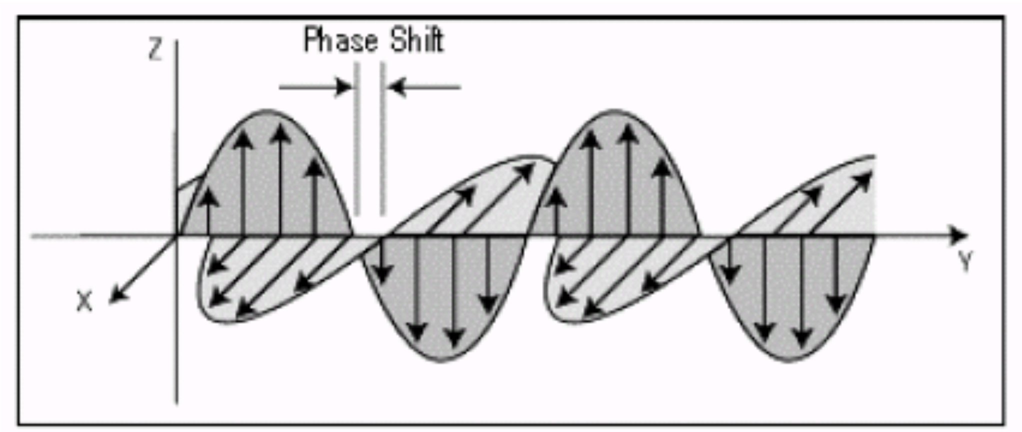




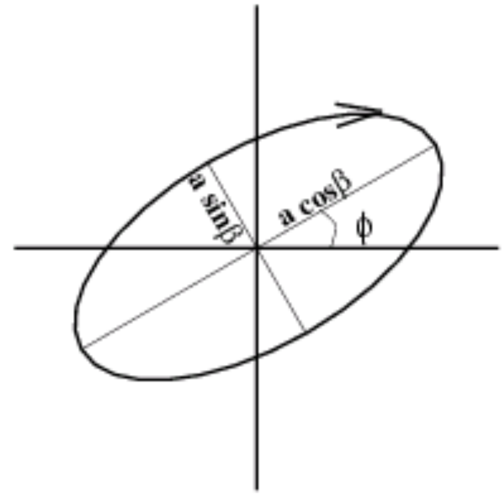
- Right circular occurs when E_y is advanced in phase by $\lambda/4$.

- Left circular occurs when E_x is advanced in phase by $\lambda/4$.

➢ Case IV: Elliptical Polarization (Linear + Circular = Elliptical)



D. Elliptically Polarized Light



➢ When δ = π/2 or 3π/2 the polarization ellipse reduces to the standard equation of an ellipse:

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = sin^2(\delta) \rightarrow \frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} = 1$$

➢ Illustration Aids:

- http://webphysics.davidson.edu/physlet_resources/dav_optics/examples/polarization.html
- http://www.ub.es/javaoptics/index-en.html
- http://www.colorado.edu/physics/2000/applets/polarized.html

- The next few slides will discuss the relationship between the parameters of the polarization ellipse.

- Recall the form of the polarization ellipse (again, $\delta = \delta_y - \delta_x$):

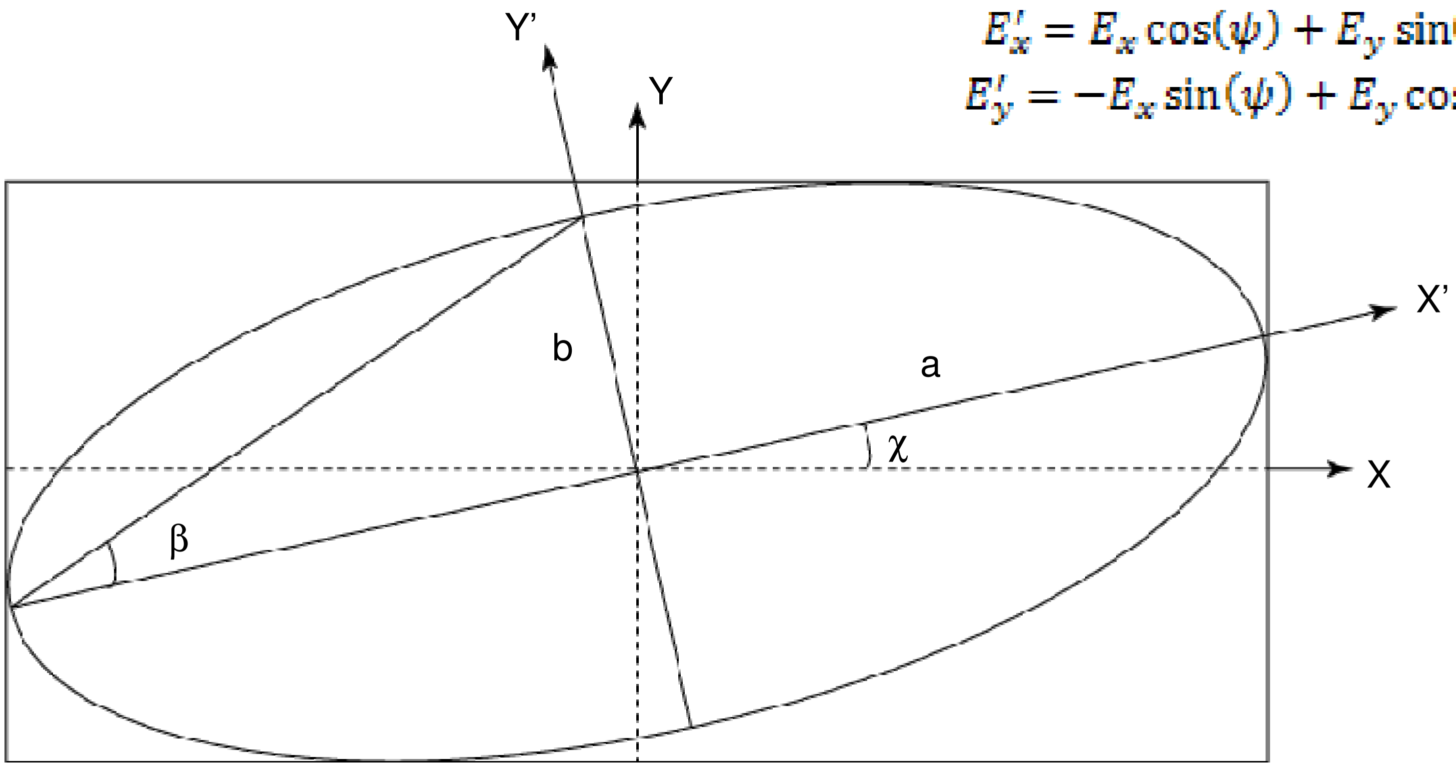
$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = sin^2(\delta)$$

- Due to the cross term, the ellipse is rotated relative to the x and y directions.

- The purpose of the next couple slides is to show the mathematical relations between polarization ellipse, E_{0x}, E_{0y}, δ and the angle of rotation χ, and β the ellipticity angle.

- From the figure below, the transformation between the primed and unprimed coordinates:

$$E_x' = E_x\cos(\psi) + E_y\sin(\psi)$$
$$E_y' = -E_x\sin(\psi) + E_y\cos(\psi)$$



- Using the rotated components:

$$E'_x = E_x \cos(\psi) + E_y \sin(\psi)$$
$$E'_y = -E_x \sin(\psi) + E_y \cos(\psi)$$

- The equation of the ellipse in terms of primed coordinates:

$$E'_x = a\cos(\tau + \delta')$$
$$E'_y = \pm b sin(\tau + \delta')$$

- The original equations for the optical field:

$$\frac{E_x}{E_{0x}} = \cos(\tau + \delta_x)$$
$$\frac{E_y}{E_{0y}} = \cos(\tau + \delta_y)$$

- And a whole lot of algebra, the following can be derived:

$$\tan(2\chi) = \frac{2E_{0x}E_{0y}\cos(\delta)}{E_{0x}^2 + E_{0y}^2}$$
$$\tan(\beta) = \pm\frac{b}{a}$$