SISTEMAS DE COORDENADAS ORTOGONALES

SUSTITUCIONES PARA TRANSFORMAR CAMPOS ESCALARES

	A coordenadas cartesianas	A coordenadas cilíndricas	A coordenadas esféricas
De coordenadas cartesianas	$x = x$ $y = y$ $z = z$	$x = \rho\,cos(\phi)$ $y = \rho\,sin(\phi)$ $z = z$	$x = r\,sin(\theta)cos(\phi)$ $y = r\,sin(\theta)sin(\phi)$ $z = r\,cos(\theta)$
De coordenadas cilíndricas	$\rho = \sqrt{x^2 + y^2}$ $\phi = tan^{-1}(y/x)$ $z = z$	$\rho = \rho$ $\phi = \phi$ $z = z$	$\rho = r\,sin(\theta)$ $\phi = \phi$ $z = r\,cos(\theta)$
De coordenadas esféricas	$r = \sqrt{x^2 + y^2 + z^2}$ $\phi = tan^{-1}(y/x)$ $\theta = cos^{-1}\left(z/\sqrt{x^2 + y^2 + z^2}\right)$	$r = \sqrt{\rho^2 + z^2}$ $\phi = \phi$ $\theta = tan^{-1}(\rho/z)$	$r = r$ $\phi = \phi$ $\theta = \theta$

DIFERENCIALES DE LONGITUD

Sistema de coordenadas	Coordenada que varía sobre la trayectoria	dl	$d\vec{l}$
Cartesianas	x y z	dx dy dz	$\hat{x}\,dx$ $\hat{y}\,dy$ $\hat{z}\,dz$
Cilíndricas	ρ ϕ z	$d\rho$ $\rho\,d\phi$ dz	$\hat{\rho}\,d\rho$ $\hat{\phi}\,\rho\,d\phi$ $\hat{z}\,dz$
Esféricas	r ϕ θ	dr $r\,sin(\theta)d\phi$ $rd\theta$	$\hat{r}dr$ $\hat{\phi}\,r\,sin(\theta)d\phi$ $\hat{\theta}\,rd\theta$

DIFERENCIALES DE ÁREA

Sistema de coordenadas	Coordenada que se mantiene constante sobre la superficie	dS	$d\vec{S}$
Cartesianas	x	$dy\,dz$	$\hat{x}\,dy\,dz$
	y	$dx\,dz$	$\hat{y}\,dx\,dz$
	z	$dx\,dy$	$\hat{z}\,dx\,dy$
Cilíndricas	ρ	$\rho\,d\phi\,dz$	$\hat{\rho}\,\rho\,d\phi\,dz$
	ϕ	$d\rho\,dz$	$\hat{\phi}\,d\rho\,dz$
	z	$\rho\,d\phi\,d\rho$	$\hat{z}\rho\,d\phi\,d\rho$
Esféricas	r	$r^2\,sin(\theta)\,d\theta\,d\phi$	$\hat{r}\,r^2\,sin(\theta)\,d\theta\,d\phi$
	ϕ	$r\,d\theta\,dr$	$\hat{\phi}\,r\,d\theta\,dr$
	θ	$r\,sin(\theta)\,dr\,d\phi$	$\hat{\theta}r\,sin(\theta)\,dr\,d\phi$

DIFERENCIALES DE VOLUMEN

Sistema de coordenadas	Diferencial de volumen
Cartesiano	$dV = dx\,dy\,dz$
Cilíndrico	$dV = \rho\,d\rho\,d\phi\,dz$
Esférico	$dV = r^2\,sin(\theta)\,dr\,d\phi\,d\theta$

TRANSFORMACIÓN DE VECTORES UNITARIOS (VERSORES)

	A coordenadas cartesianas	A coordenadas cilíndricas
De coordenadas cartesianas		$\hat{x} = \hat{\rho}\cos(\phi) - \hat{\phi}\sin(\phi)$ $\hat{y} = \hat{\rho}\sin(\phi) - \hat{\phi}\cos(\phi)$ $\hat{z} = \hat{z}$
De coordenadas cilíndricas	$\hat{\rho} = \hat{x}\cos(\phi) + \hat{y}\sin(\phi)$ $\hat{\phi} = -\hat{x}\sin(\phi) + \hat{y}\cos(\phi)$ $\hat{z} = \hat{z}$	

	A coordenadas cartesianas	A coordenadas esféricas
De coordenadas cartesianas		$\hat{x} = \hat{r}\sin(\theta)\cos(\phi) + \hat{\theta}\cos(\theta)\cos(\phi)$ $\quad - \hat{\phi}\sin(\phi)$ $\hat{y} = \hat{r}\sin(\theta)\sin(\phi) + \hat{\theta}\cos(\theta)\sin(\phi)$ $\quad + \hat{\phi}\cos(\phi)$ $\hat{z} = \hat{r}\cos(\theta) - \hat{\theta}\sin(\theta)$
De coordenadas esféricas	$\hat{r} = \hat{x}\sin(\theta)\cos(\phi) + \hat{y}\sin(\theta)\sin(\phi)$ $\quad + \hat{z}\cos(\theta)$ $\hat{\theta} = \hat{x}\cos(\theta)\cos(\phi) + \hat{y}\cos(\theta)\sin(\phi)$ $\quad + \hat{z}\sin(\theta)$ $\hat{\phi} = -\hat{x}\sin(\phi) + \hat{y}\cos(\phi)$	

	A coordenadas cilíndricas	A coordenadas esféricas
De coordenadas cilíndricas		$\rho = \hat{r}\sin(\theta) + \hat{\theta}\cos(\theta)$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{r}\cos(\theta) - \hat{\theta}\sin(\theta)$
De coordenadas esféricas	$\hat{r} = \hat{\rho}\,sin(\theta) + \hat{z}\,cos(\theta)$ $\hat{\theta} = \hat{\rho}\,cos(\theta) - \hat{z}\,sin(\theta)$ $\hat{\phi} = \hat{\phi}$	

Fórmulas del gradiente en distintos sistemas de coordenadas

Cartesianas: $\quad \vec{\nabla}g(\vec{r}) = \left(\dfrac{\partial g(\vec{r})}{\partial x}, \dfrac{\partial g(\vec{r})}{\partial y}, \dfrac{\partial g(\vec{r})}{\partial z} \right) = \dfrac{\partial g(\vec{r})}{\partial x}\,\breve{e}_x + \dfrac{\partial g(\vec{r})}{\partial y}\,\breve{e}_y + \dfrac{\partial g(\vec{r})}{\partial z}\breve{e}_z$

Cilíndricas: $\quad \vec{\nabla}g(\vec{r}) = \dfrac{\partial g(\vec{r})}{\partial \rho}\,\breve{e}_\rho + \dfrac{1}{\rho}\dfrac{\partial g(\vec{r})}{\partial \phi}\,\breve{e}_\phi + \dfrac{\partial g(\vec{r})}{\partial z}\breve{e}_z$

Esféricas: $\quad \vec{\nabla}g(\vec{r}) = \dfrac{\partial g(\vec{r})}{\partial r}\,\breve{e}_r + \dfrac{1}{r\,sen\vartheta}\dfrac{\partial g(\vec{r})}{\partial \phi}\,\breve{e}_\phi + \dfrac{1}{r}\dfrac{\partial g(\vec{r})}{\partial \vartheta}\breve{e}_\vartheta$

Fórmulas de la divergencia en distintos sistemas de coordenadas

Cartesianas:
$$\vec{\nabla} \bullet \vec{F}(\vec{r}) = \frac{\partial}{\partial x} F_x(\vec{r}) + \frac{\partial}{\partial y} F_y(\vec{r}) + \frac{\partial}{\partial z} F_z(\vec{r})$$

Cilíndricas:
$$\vec{\nabla} \bullet \vec{F}(\vec{r}) = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left[\rho\, F_\rho(\vec{r})\right] + \frac{1}{\rho} \frac{\partial}{\partial \phi} F_\phi(\vec{r}) + \frac{\partial}{\partial z} F_z(\vec{r})$$

Esféricas:
$$\vec{\nabla} \bullet \vec{F}(\vec{r}) = \frac{1}{r^2} \frac{\partial}{\partial r}\left[r^2\, F_r(\vec{r})\right] + \frac{1}{r\,sen\vartheta} \frac{\partial}{\partial \phi} F_\phi(\vec{r}) + \frac{1}{r\,sen\vartheta} \frac{\partial}{\partial \vartheta}\left[sen\vartheta\, F_\vartheta(\vec{r})\right]$$

Fórmulas del rotor en distintos sistemas de coordenadas

Cartesianas:
$$\vec{\nabla} \times \vec{F}(\vec{r}) = \left\{ \left[\frac{\partial F_z(\vec{r})}{\partial y} - \frac{\partial F_y(\vec{r})}{\partial z}\right] \breve{e}_x + \left[\frac{\partial F_x(\vec{r})}{\partial z} - \frac{\partial F_z(\vec{r})}{\partial x}\right] \breve{e}_y + \left[\frac{\partial F_y(\vec{r})}{\partial x} - \frac{\partial F_x(\vec{r})}{\partial y}\right] \breve{e}_z \right\} =$$

$$= \begin{vmatrix} \breve{e}_x & \breve{e}_x & \breve{e}_x \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

Cilíndricas:
$$\vec{\nabla} \times \vec{F}(\vec{r}) = \left\{ \left[\frac{1}{\rho}\frac{\partial F_z(\vec{r})}{\partial \phi} - \frac{\partial F_\phi(\vec{r})}{\partial z}\right] \breve{e}_\rho + \left[\frac{\partial F_\rho(\vec{r})}{\partial z} - \frac{\partial F_z(\vec{r})}{\partial \rho}\right] \breve{e}_\phi + \frac{1}{\rho}\left[\frac{\partial\left[\rho\, F_\phi(\vec{r})\right]}{\partial \rho} - \frac{\partial F_\rho(\vec{r})}{\partial \phi}\right] \breve{e}_z \right\}$$

Esféricas:
$$\vec{\nabla} \times \vec{F}(\vec{r}) = \left\{ \frac{1}{r\,sen\vartheta}\left[\frac{\partial\left[F_\phi(\vec{r})sen\vartheta\right]}{\partial \vartheta} - \frac{\partial F_\vartheta(\vec{r})}{\partial \phi}\right] \breve{e}_r + \frac{1}{r}\left[\frac{\partial\left[r\, F_\vartheta(\vec{r})\right]}{\partial r} - \frac{\partial F_r(\vec{r})}{\partial \vartheta}\right] \breve{e}_\phi + \right.$$

$$\left. + \frac{1}{r}\left[\frac{1}{sen\vartheta}\frac{\partial F_r(\vec{r})}{\partial \phi} - \frac{\partial\left[r\, F_\phi(\vec{r})\right]}{\partial r}\right] \breve{e}_\vartheta \right\}$$