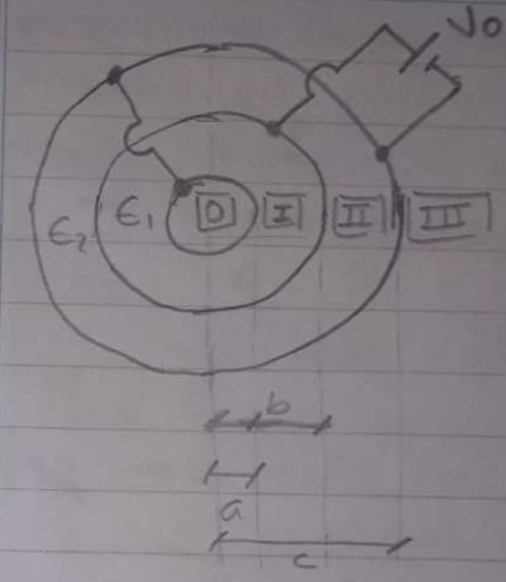
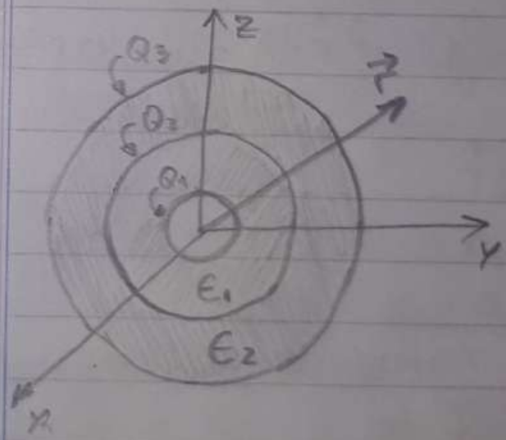




COORDENADAS ESPÉRICAS

DEBIDO A LA CONEXIÓN DE LA BATERÍA SE DISTRIBUIRÁN CARGAS LIBRES Q_1, Q_2 Y Q_3 EN LA SUPERFICIE DE LOS CONDUCTORES.
SOBRE LOS MATERIALES ϵ_1 Y ϵ_2 SE INDUCIRÁN CARGAS DE POLARIZACIÓN.

DEBIDO A LA SIMETRÍA ESFÉRICA Y A QUE LOS MATERIALES SON LINEALES, ISÓTROPOS Y HOMOGÉNEOS, PUEDO USAR LA "LEY DE GAUSS" PARA $\vec{D}$

$$\oiint \vec{D}\cdot\hat{n}\,dA = Q_{\text{ENC LIBRE}}$$

LAS SUPERFICIES GAUSSIANAS SERÁN ESFERAS DE RADIO r

$$\oiint \vec{D}\cdot\hat{n}\,dA = \oiint D(r)\,\hat{r}\cdot\hat{r}\,dA = D(r)\,4\pi r^2 = Q_{\text{ENC LIB}}$$

$$\vec{D} = D(r)\,\hat{r}$$

SIM. ESFÉRICA

REGIÓN $\boxed{0}$ (INTERIOR DEL CONDUCTOR)

$$\vec{D}_0 = \vec{E}_0 = \vec{P}_0 = 0$$

REGIÓN $\boxed{I}$ $a<r<b$

$$D_I(r)\,4\pi r^2 = Q_1 \rightarrow \vec{D}_I = \frac{Q_1}{4\pi r^2}\,\hat{r}$$

COMO EL MATERIAL ES LIH

$$\vec{D}_I = \epsilon_1 \vec{E}_I$$

$$\vec{E}_I = \frac{Q_1}{4\pi r^2 \epsilon_1}\,\hat{r}$$

$$\vec{P}_I = \vec{D}_I - \epsilon_0 \vec{E}_I$$

$$\vec{P}_I = \frac{\epsilon_1 - \epsilon_0}{\epsilon_1}\,\frac{Q_1}{4\pi r^2}\,\hat{r}$$

REGIÓN $\boxed{II}$ $b<r<c$

$$D_{II}(r)\,4\pi r^2 = Q_1 + Q_2$$

$$\vec{D}_{II}(r) = \frac{Q_1+Q_2}{4\pi r^2}\,\hat{r}$$

$$\vec{E}_{II} = \frac{\vec{D}_{II}}{\epsilon_2} = \frac{Q_1+Q_2}{4\pi r^2 \epsilon_2}\,\hat{r}$$

medio LIH

$$\vec{P}_{II} = \frac{\epsilon_2 - \epsilon_0}{\epsilon_2}\,\frac{Q_1+Q_2}{4\pi r^2}\,\hat{r}$$

REGIÓN $\boxed{III}$ $r>c$

$D_{III}(r)4\pi r^2 = Q_1+Q_2+Q_3$

$$\vec{D}_{III} = \frac{Q_1+Q_2+Q_3}{4\pi r^2}\hat{r}$$

$Q_1+Q_2+Q_3 = 0$ DEBIDO A LA CONEXIÓN DE LA BATERÍA QUE SÓLO REDISTRIBUYE CARGAS.

$\Rightarrow \vec{D}_{III} = 0 = \vec{E}_{III} = \vec{P}_{III}$

$$\vec{D} = \begin{cases} 0 & ; \quad r<a \\ \dfrac{Q_1}{4\pi r^2}\hat{r} & ; \quad a<r<b \\ \dfrac{Q_1+Q_2}{4\pi r^2}\hat{r} & ; \quad b<r<c \\ 0 & ; \quad c<r \end{cases}$$

$$\vec{E} = \begin{cases} 0 & ; \quad r<a \\ \dfrac{Q_1}{4\pi r^2 \epsilon_1}\hat{r} & ; \quad a<r<b \\ \dfrac{Q_1+Q_2}{4\pi r^2 \epsilon_2}\hat{r} & ; \quad b<r<c \\ 0 & ; \quad c<r \end{cases}$$

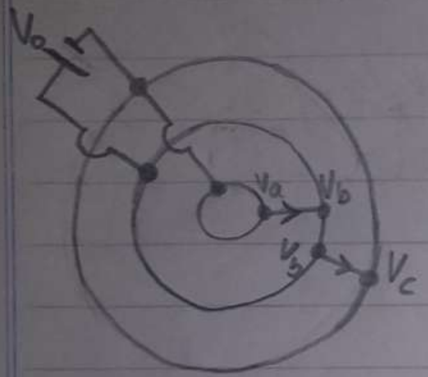
$$\vec{P} = \begin{cases} 0 & ; \quad r<a \\ \dfrac{\epsilon_1-\epsilon_0}{\epsilon_1}\dfrac{Q_1}{4\pi r^2}\hat{r} & ; \quad a<r<b \\ \dfrac{\epsilon_2-\epsilon_0}{\epsilon_2}\dfrac{Q_1+Q_2}{4\pi r^2}\hat{r} & ; \quad b<r<c \\ 0 & ; \quad c<r \end{cases}$$

LAS DENSIDADES DE CARGA LIBRE SUPERFICIAL EN $r=a,b,c$ SON

$$\sigma_a = \frac{Q_1}{4\pi a^2} \qquad \sigma_c = \frac{Q_3}{4\pi c^2}$$

$$\sigma_b = \frac{Q_2}{4\pi b^2}$$

PARA CALCULAR LAS CARGAS USAMOS EL POTENCIAL



$$+V_0 = V_b - V_a = -\int_a^b \vec{E}_I \cdot \vec{d\ell} =$$

$$\vec{d\ell} = dr\,\hat{r}$$

$$V_0 = -\int_a^b \frac{Q_1}{4\pi\epsilon_1}\frac{dr}{r^2}$$

$$V_0 = \frac{Q_1}{4\pi\epsilon_1}\left[\frac{1}{b} - \frac{1}{a}\right] = -\frac{Q_1}{4\pi\epsilon_1}\underbrace{\left[\frac{1}{a} - \frac{1}{b}\right]}_{>0}$$

$$\boxed{Q_1 = \frac{-V_0 4\pi\epsilon_1}{\frac{1}{a} - \frac{1}{b}}}$$

$Q_1 < 0$

$$-V_0 = V_c - V_b = -\int_b^c \vec{E}_{II} \cdot \vec{d\ell} =$$

$$-V_0 = -\frac{Q_1 + Q_2}{4\pi\epsilon_2}\left[\frac{1}{b} - \frac{1}{c}\right]$$

$$Q_1 + Q_2 = \frac{4\pi\epsilon_2 V_0}{\frac{1}{b} - \frac{1}{c}}$$

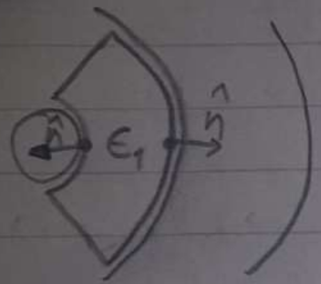
$$\boxed{Q_2 = \underbrace{\frac{4\pi\epsilon_2 V_0}{\frac{1}{b} - \frac{1}{c}}}_{>0} + \underbrace{\frac{4\pi\epsilon_1 V_0}{\frac{1}{a} - \frac{1}{b}}}_{>0}}$$

$Q_2 > 0$

$$\boxed{Q_3 = -(Q_1 + Q_2) = -\frac{4\pi\epsilon_2 V_0}{\frac{1}{b} - \frac{1}{c}}}$$

$Q_3 < 0$

CARGAS DE POLARIZACIÓN



$\hat{n}$: NORMAL EXTERIOR AL MATERIAL ϵ_1

MATERIAL ϵ_1 $\qquad \sigma = \vec{P}\cdot\hat{n}$

$$\sigma_p(r=a^+) = \vec{P}_I(r=a^+)\cdot(-\hat{r}) =$$

$$= -\left(\frac{\epsilon_1-\epsilon_0}{\epsilon_1}\right)\left(\frac{Q_1}{4\pi a^2}\right)$$

$$\sigma_p(r=a^+) = \frac{(\epsilon_1-\epsilon_0)\,V_0}{a^2\left[\frac{1}{a}-\frac{1}{b}\right]} > 0$$

$$\rho_p(a<r<b) = -\vec{\nabla}\cdot\vec{P}_I = -\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 P_I\right] =$$

$$= -\frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2}\,\frac{\epsilon_1-\epsilon_0}{\epsilon_1}\,\frac{Q_1}{4\pi \cancel{r^2}}\right] = 0$$

$$\sigma_p(r=b^-) = \vec{P}_I(r=b^-)\cdot(\hat{r}) =$$

$$= \frac{\epsilon_1-\epsilon_0}{\epsilon_1}\,\frac{Q_1}{4\pi b^2}$$

$$\sigma_p(r=b^-) = -\frac{(\epsilon_1-\epsilon_0)\,V_0}{b^2\left[\frac{1}{a}-\frac{1}{b}\right]} < 0$$

$$Q_p\left(\substack{\text{TOTAL}\\ \text{MAT}\,\epsilon_1}\right) = \sigma_p(r=a)\,4\pi a^2 + \sigma_p(r=b)\,4\pi b^2 =$$

$$= \frac{(\epsilon_1-\epsilon_0)V_0 4\pi}{\frac{1}{a}-\frac{1}{b}} - \frac{(\epsilon_1-\epsilon_0)V_0 4\pi}{\frac{1}{a}-\frac{1}{b}} = 0$$

MATERIAL ϵ_2

$$\sigma_p(r=b^+) = \vec{P}_{II}(r=b^+)\cdot(-\hat{r}) =$$

$$= -\frac{\epsilon_2-\epsilon_0}{\epsilon_2}\,\frac{Q_1+Q_2}{4\pi b^2}$$

$$\boxed{\sigma_p(r=b^+) = -\frac{(\epsilon_2-\epsilon_0)\,V_0}{b^2\left[\frac{1}{b}-\frac{1}{c}\right]}} < 0$$

$$\rho_p(b<r<c) = -\vec{\nabla}\cdot\vec{P}_{II} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2}\,\frac{\epsilon_2-\epsilon_0}{\epsilon_2}\,\frac{Q_1+Q_2}{4\pi \cancel{r^2}}\right]$$

$$\boxed{\rho_p = 0}$$

$$\sigma_p(r=c^-) = \vec{P}_{II}(r=c^-)\,\hat{r}$$

$$= \frac{\epsilon_2-\epsilon_0}{\epsilon_2}\,\frac{Q_1+Q_2}{4\pi c^2}$$

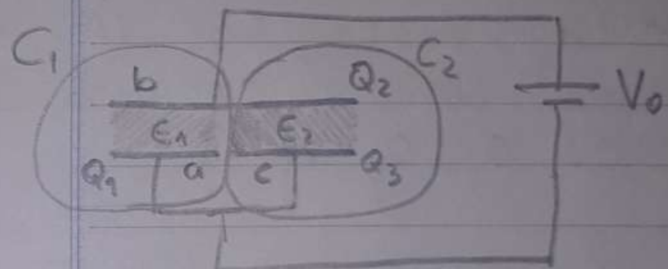
$$\boxed{\sigma_p(r=c^-) = \frac{(\epsilon_2-\epsilon_0)\,V_0}{c^2\left[\frac{1}{b}-\frac{1}{c}\right]}}$$

$$Q_p(\text{TOTAL MAT } \epsilon_2) = \sigma_p(r=b)\,4\pi b^2 + \sigma_p(r=c)\,4\pi c^2 =$$

$$= -\frac{(\epsilon_2-\epsilon_0)V_0 4\pi}{\frac{1}{b}-\frac{1}{c}} + \frac{(\epsilon_2-\epsilon_0)4\pi V_0}{\frac{1}{b}-\frac{1}{c}}$$

$$\boxed{Q_p = 0}$$

PARA CALCULAR LA CAPACIDAD PODEMOS REESCRIBIR EL CIRCUITO PARA ACLARAR IDEAS



AQUÍ VEMOS CLARAMENTE QUE SON DOS CAPACITORES EN PARALELO. LOS CAPACITORES NO SON PLANOS, EL DIBUJO ES SÓLO PARA ESQUEMATIZAR.

$$C_{TOTAL} = C_1 + C_2$$

DONDE $C_{1,2}$ ES LA CAPACIDAD DE UN CAPACITOR ESFÉRICO CON DIELÉCTRICO $\epsilon_{1,2}$

$$C_1 = \frac{4\pi\epsilon_1}{\frac{1}{a}-\frac{1}{b}} \qquad C_2 = \frac{4\pi\epsilon_2}{\frac{1}{b}-\frac{1}{c}} \Longrightarrow$$

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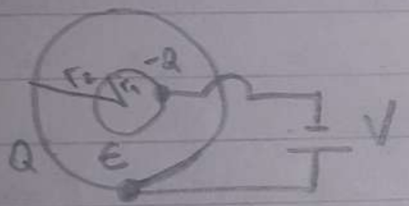
$$\boxed{C_{TOTAL} = \frac{4\pi\epsilon_1}{\frac{1}{a}-\frac{1}{b}} + \frac{4\pi\epsilon_2}{\frac{1}{b}-\frac{1}{c}}}$$

ACLARACIÓN C_1 Y C_2 YA LOS CALCULAMOS CUANDO SACAMOS LAS CARGAS Q_1, Q_2, Q_3.

SI NO LO PUEDEN HACER APARTE.



$$V_2 - V_1 = V = -\int_{r_1}^{r_2} \overbrace{\frac{-Q}{4\pi r^2 \epsilon}\hat{r}}^{\vec{E}}\, dr\,\hat{r}$$

$$V = \frac{Q}{4\pi\epsilon}\left(-\frac{1}{r}\right)\Bigg|_{r_1}^{r_2} = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

$$\boxed{C = \frac{Q}{V} = \frac{4\pi\epsilon}{\frac{1}{r_1}-\frac{1}{r_2}}}$$