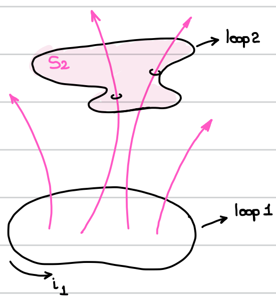
INDUCTANCIA



Supongamos dos circuitos en reposo. Si una corriente estacionaria i_1 circula por el circuito #1 produce un campo $\bar{B}_1(\bar{x})$

$$\bar{B}_1(\bar{x}) = \frac{\mu_0}{4\pi} i_1 \oint \frac{d\bar{l} \times (\bar{r}-\bar{r}_\ell)}{|\bar{r}-\bar{r}_\ell|^3}$$

El flujo de campo magnético a través del area que encierra el circuito #2 es

$$\Phi_2 = \int d\vec{S_2} \cdot \bar{B}_1(\bar{x})$$

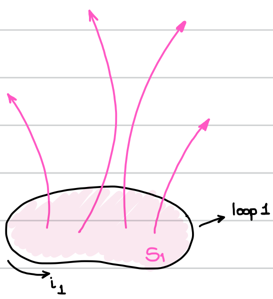
$$= \underbrace{\frac{\mu_0}{4\pi} \int d\vec{S_2} \cdot \left(\oint \frac{d\bar{l} \times (\bar{r}_s-\bar{r}_\ell)}{|\bar{r}_s-\bar{r}_\ell|^3} \right)}_{M_{21}} i_1 \propto i_1$$

$$\boxed{\Phi_2 = M_{21}\, i_1}$$

M_{21}: inductancia mutua

Si i_1 varía en el tiempo

$$-\mathcal{E}_2 = \frac{d\Phi_2}{dt} = M_{21} \frac{d i_1}{dt} \implies \boxed{\mathcal{E}_2 = -M_{21} \frac{d i_1}{dt}}$$



$$\Phi_1 = \int d\vec{S_1}\, \bar{B}_1(\bar{x})$$

$$= \underbrace{\frac{\mu_0}{4\pi} \int d\vec{S_1} \cdot \left(\oint \frac{d\bar{l} \times (\bar{r}_s-\bar{r}_\ell)}{|\bar{r}_s-\bar{r}_\ell|^3} \right)}_{L} i_1 \propto i_1$$

$$\boxed{\Phi_1 = L\, i_1}$$

L: autoinductancia

$$-\mathcal{E}_1 = \frac{d\Phi_1}{dt} = L \frac{d i_1}{dt}$$

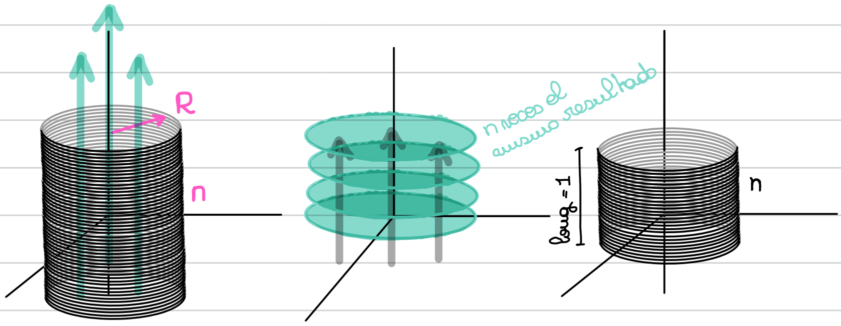
$$\boxed{\mathcal{E}_1 = -L \frac{d i_1}{dt}}$$

73 Calcular la auto-inductancia de:

(a) Un solenoide infinito de radio R y $n = N/\ell$ vueltas por unidad de longitud (exprese el resultado por unidad de longitud).

(b) Un toroide con N vueltas y radio medio R, usando que la diferencia entre el radio exterior e interior es mucho menor que R.

a) $\bar{B}(\bar{x}) = \begin{cases} \mu_0 i n \, \hat{z} & \rho < R \\ 0 & \rho \geq R \end{cases}$



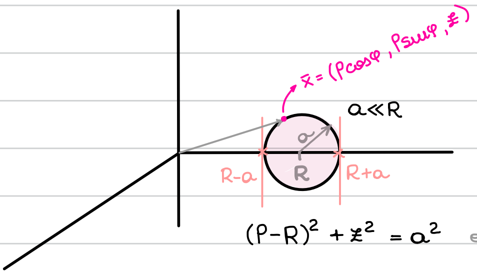
$$\Phi^{(1)} = \int_0^{2\pi} d\varphi \int_0^R \rho \, d\rho \, \mu_0 i n \underbrace{\hat{z} \cdot \hat{z}}_{=1}$$

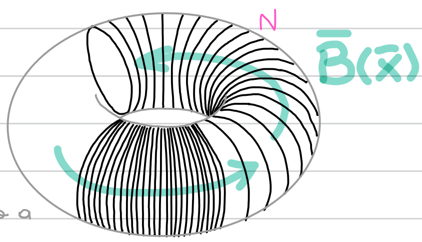
$$= \pi R^2 \mu_0 n \cdot i$$

$$\frac{\Phi}{\ell} = \pi R^2 \mu_0 n^2 \, i \xrightarrow[\Phi = L\, i]{} \boxed{\frac{L}{\ell} = \pi R^2 \mu_0 n^2}$$

b) Toroide flaquito

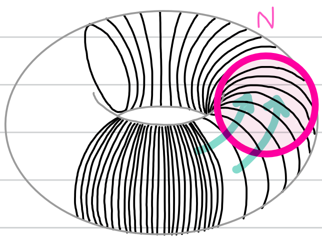
$$\bar{B}(\bar{x}) = \begin{cases} 0 & \text{si} \quad \rho < R-a \\ \frac{\mu_0}{2\pi} NI/\rho \, \hat{\varphi} & \text{si} \quad R-a < \rho < R+a \\ 0 & \text{si} \quad \rho > R+a \end{cases}$$





$$(\rho - R)^2 + z^2 = a^2$$ eq. de un círculo de radio a centrado en $\rho = R$, $z = 0$

$$z = \pm\sqrt{a^2 - (\rho - R)^2}$$



$$\Phi^{(1)} = \int_{R-a}^{R+a} d\rho \int_{-\sqrt{a^2-(\rho-R)^2}}^{\sqrt{a^2-(\rho-R)^2}} dz \; \frac{\mu_0}{2\pi} NI/\rho \; \underbrace{\hat{\varphi} \cdot \hat{\varphi}}_{=1}$$

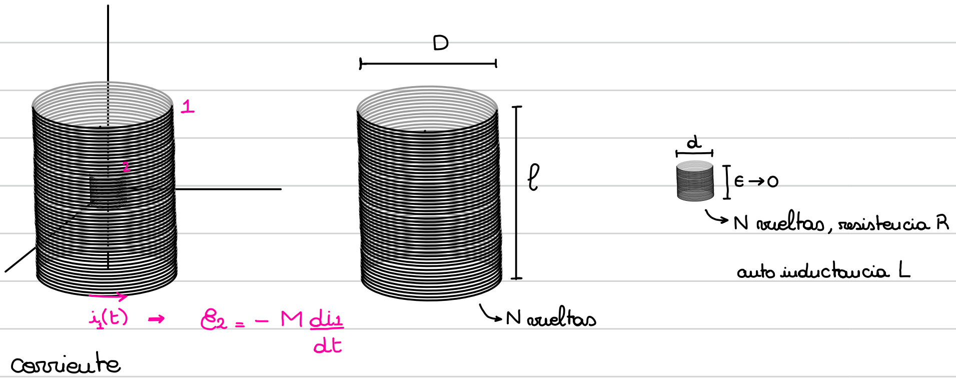
$$= \frac{\mu_0}{2\pi} NI \int_{R-a}^{R+a} d\rho \; \frac{2\sqrt{a^2-(\rho-R)^2}}{\rho}$$

$$= \frac{\mu_0}{\cancel{2\pi}} a^2 NI \, \cancel{2\pi} \underbrace{(R - \sqrt{R^2 - a^2})} \longrightarrow \Phi^{TOT} = \frac{\mu_0}{2} \frac{a^2 N^2}{R} I \xrightarrow[\Phi = L I]{} \boxed{L = \frac{\mu_0}{2} \frac{a^2 N^2}{R}}$$

Haciendo Taylor en torno a $a = 0$

$$= \frac{a^2}{2R}$$

72 Un solenoide tiene 1000 vueltas, 20 cm de diámetro y 40 cm de largo. En su centro se ubica coaxialmente otro solenoide de 1000 vueltas, 4 cm de diámetro y longitud despreciable, cuya resistencia vale 50Ω. Inicialmente circulan 5 A por el solenoide exterior, luego se reduce linealmente la corriente a 1 A en 0,5 s. Calcular la corriente que se induce en el solenoide interior, cuya auto-inductancia es L.



$i_1(t) = i_0 + \alpha t$

$i_1(0{,}5s) = \frac{i_0}{5} \rightarrow i_0 + \frac{\alpha}{2} = \frac{i_0}{5}$

$-\frac{4}{5} i_0 = \frac{\alpha}{2}$

$i_1(t) = i_0\left(1 - \frac{8}{5}t\right)$

$\mathcal{E}_2 = -M\frac{di_1}{dt}$ con $M = \frac{\Phi_2}{i_1}$ $(\Phi_2 = M i_1)$

Para poder hallar $\mathcal{E}_2$, calculamos Φ_2 (flujo de campo $\bar{B}_1(\bar{x})$ a través del solenoide interno)

$\Phi_2 = \int d\vec{S}_2 \cdot \bar{B}_1(\bar{x})$

Si el solenoide tiene altura ϵ despreciable $\ell \gg \epsilon$ y entonces puedo despreciar efectos de borde (usar el $\bar{B}_1(\bar{x})$ generado por un solenoide ∞)

$$\bar{B}_1(\bar{x}) = \begin{cases} \mu_0 i_1 \overset{\nearrow n}{\frac{N}{\ell}}\, \hat{z} & \text{si } \rho < \frac{D}{2} \\ 0 & \text{si } \rho \geq D/2 \end{cases}$$

$$\Phi_2^{(1)} = \int_0^{2\pi} d\varphi \int_0^{d/2} \rho\, d\rho \left(\mu_0 i_1 \frac{N}{\ell}\overbrace{\hat{z}) \cdot \hat{z}}^{=1}$$

$$= \pi\left(\frac{d}{2}\right)^2 \mu_0 \frac{N}{\ell} i_1$$

$i_1(t) = i_0\left(1 - \frac{8}{5}t\right) \Rightarrow \frac{di_1}{dt} = -\frac{8}{5} i_0$

$$\Phi_2^{TOT} = \pi\left(\frac{d}{2}\right)^2 \mu_0 \frac{N^2}{\ell} i_1$$

$$\mathcal{E}_2 = -M\frac{di_1}{dt} = -\frac{\Phi_2}{i_1}\frac{di_1}{dt} = -\pi\left(\frac{d}{2}\right)^2 \mu_0 \frac{N^2}{\ell}\frac{di_1}{dt} = \pi \frac{8}{5} i_0 \frac{d^2}{4} \mu_0 \frac{N^2}{\ell}$$

$$\boxed{\mathcal{E}_2 = \frac{2}{5}\pi \mu_0 i_0 d^2 \frac{N^2}{\ell}}$$

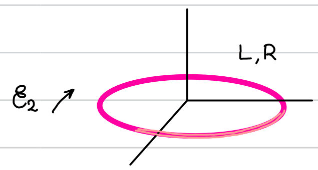
$$\mathcal{E}_2 + \mathcal{E}_2' + \Delta V = 0$$

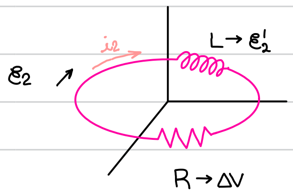
$$\mathcal{E}_2 - L\frac{di_2}{dt} - R\,i_2 = 0$$

fem inducida por la variación de i_1

fem inducida por la variación de i_2

Caída de potencial debida a la resistencia del solenoide





Llamo $x = \frac{\mathcal{E}_2}{R} - i_2 \Rightarrow dx = -di_2$

$$Rx + L\frac{dx}{dt} = 0$$

$$\frac{dx}{x} = -\frac{R}{L}\,dt$$

$$x = Ae^{-\frac{R}{L}t}$$

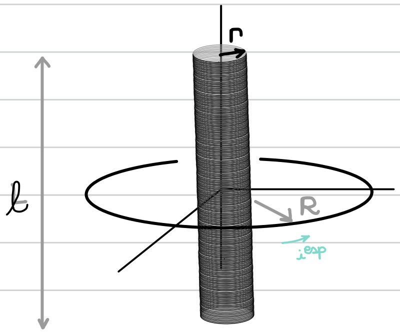
$$\frac{\mathcal{E}_2}{R} - i_2(t) = Ae^{-R/L\,t}$$

$$i_2(t) = \frac{\mathcal{E}_2}{R} - Ae^{-R/L\,t}$$

$$i_2(0) = 0 \longrightarrow A = \frac{\mathcal{E}_2}{R}$$

$$\boxed{i_2(t) = \frac{\mathcal{E}_2}{R}\left(1 - e^{-R/L\,t}\right)} \quad ; \text{ con } \mathcal{E}_2 = \frac{2}{5}\pi i_0 \mu_0 \frac{d^2N^2}{\ell}$$

77 Calcule M_{12} y M_{21} entre una espira circular de radio R y un solenoide finito de longitud L y radio r (suponga $r \ll L$ y $r \ll R$), dispuestos de tal forma que los centros y los ejes de ambos son coincidentes. Utilice las aproximaciones que crea necesarias y diga cuál de los dos resultados es más confiable cuando ℓ es chico con respecto a R.



$r \ll L$, $r \ll R$

$L < R$

$$\Phi_2^{esp} = M_{21}\, i_1^{sol} \Rightarrow \Phi_2^{esp} = \int d\vec{S}_{esp}\cdot \vec{B}_1^{sol}(\vec{x})$$

$$\Phi_1^{sol} = M_{12}\, i_2^{esp} \Rightarrow \Phi_1^{sol} \text{ “}= \int d\vec{S}_{sol}\cdot \vec{B}_2^{esp}(\vec{x})\text{”}$$

Campos que sabemos calcular

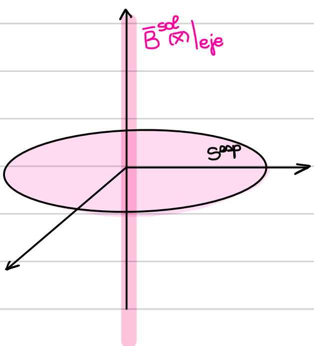
$$\cdot\ \vec{B}^{esp}(\vec{x})\Big|_{eje} = \frac{\mu_0}{2}\, i^{esp} \frac{R^2}{(R^2+z^2)^{3/2}}\,\hat{z} \qquad (58)$$

$$\cdot\ \vec{B}^{sol}(\vec{x})\Big|_{eje} = \frac{\mu_0}{2}\frac{r\, i^{sol} N}{\ell}\left[\frac{z+\ell/2}{\sqrt{r^2+(z+\ell/2)^2}} - \frac{z-\ell/2}{\sqrt{r^2-(z+\ell/2)^2}}\right]\hat{z} \qquad (59)$$

$$\cdot\ \vec{B}^{sol\,\infty}(\vec{x}) = \begin{cases} \mu_0 i^{sol} N/\ell\ \hat{z} & \text{adentro} \\ 0 & \text{afuera} \end{cases} \qquad (57.c)$$

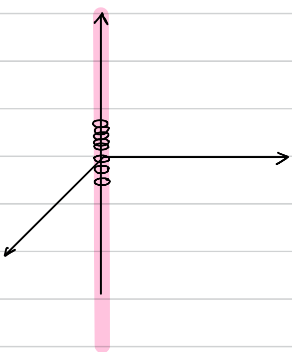
OPCIÓN #1

$\Phi_2^{esp} = M_{21}\, i_1^{sol}$

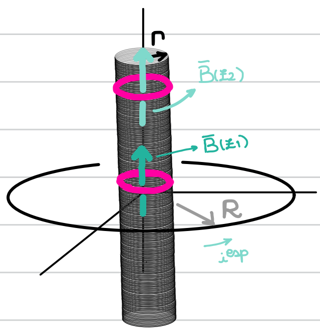


OPCIÓN #2

$\Phi_1^{sol} = M_{12}\, i_2^{esp}$



$$\vec{B}^{esp}(\vec{x})\Big|_{eje} = \frac{\mu_0}{2}\, i_2^{esp} \frac{R^2}{(R^2+z^2)^{3/2}}\,\hat{z}$$



$$\Phi_1^{sol(1)}(z) = \int_0^{2\pi} d\varphi \int_0^{r} \rho\, d\rho\ \hat{z}\ \frac{\mu_0}{2}\, i_2^{esp} \frac{R^2}{(R^2+z^2)^{3/2}}\,\hat{z}$$

$$\Phi_1^{sol(1)}(z) = \pi r^2 \frac{\mu_0}{2}\, i_2^{esp} \frac{R^2}{(R^2+z^2)^{3/2}}$$

$$\Phi_1^{sol} = \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} dz \; n(z) \; \Phi_1^{sol(1)}(z)$$

$$= \frac{N}{\ell} \pi r^2 \frac{\mu_0}{2} i_2^{esp} \underbrace{\int_{-\frac{\ell}{2}}^{\ell/2} dz \frac{R^2}{(R^2+z^2)^{3/2}}}_{\frac{\ell}{\sqrt{R^2+(\ell/2)^2}}}$$

$$\Phi_1^{sol} = \frac{\mu_0}{2} \pi r^2 \frac{N}{\sqrt{R^2+(\ell/2)^2}} \; i_2^{esp}$$

$$\Phi_1^{sol} = M_{12} \; i_2^{esp} \Rightarrow \boxed{M_{12} = \frac{\mu_0}{2}\pi \frac{N r^2}{\sqrt{R^2+(\ell/2)^2}}}$$

Caso en que calculo el flujo de campo $\bar{B}^{sol}(\bar{x})$ aproximado por $\bar{B}^{sol}(\bar{x})\Big|_{eje}$, a través de la espira

$$M_{21} = \frac{\mu_0}{2}\pi \frac{NR^2}{\sqrt{r^2+(\ell/2)^2}}$$

Aproximando $B_1^{sol\,\infty}(x) = \begin{cases} \frac{N}{\ell}\mu_0 i_1^{sol} \hat{z} & \rho < r \\ 0 & \rho \geq r \end{cases}$

$$\Phi_2^{esp} = \int_0^{2\pi} d\varphi \int_0^{r} \rho \, d\rho \; \frac{N}{\ell}\mu_0 i_1^{sol} = \pi r^2 \frac{N}{\ell}\mu_0 i_1^{sol} \Rightarrow M_{21} = \frac{\mu_0}{2}\pi \cdot \frac{2r^2 N}{\ell}$$

Mirar con cuidado