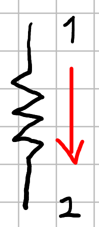
Clase 12/9 - Circuitos de corriente variable: Transitorio y Corriente alterna

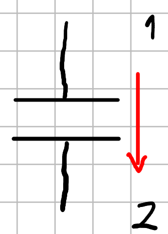
Repaso transitorio

Elementos de un circuito (pasivos)



$\Delta V = V_2 - V_1$

$\Delta V = -iR$



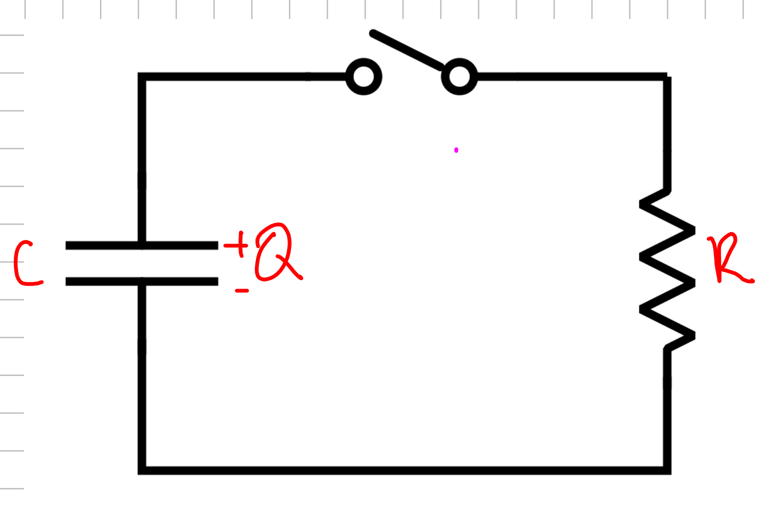
$|\Delta V| = \left|\frac{Q}{C}\right|$

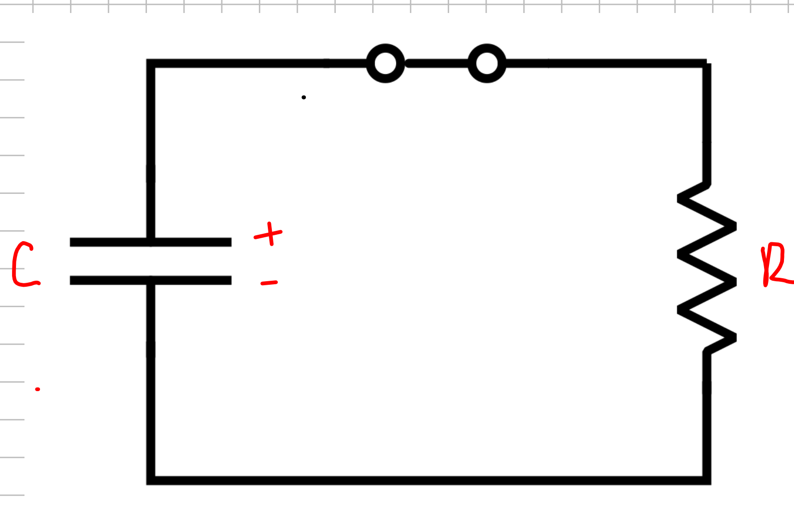
$\Delta V = -L\frac{di}{dt}$ → si hay corriente continua no hay caída de tensión

i

y $-\frac{dq}{dt} = i$

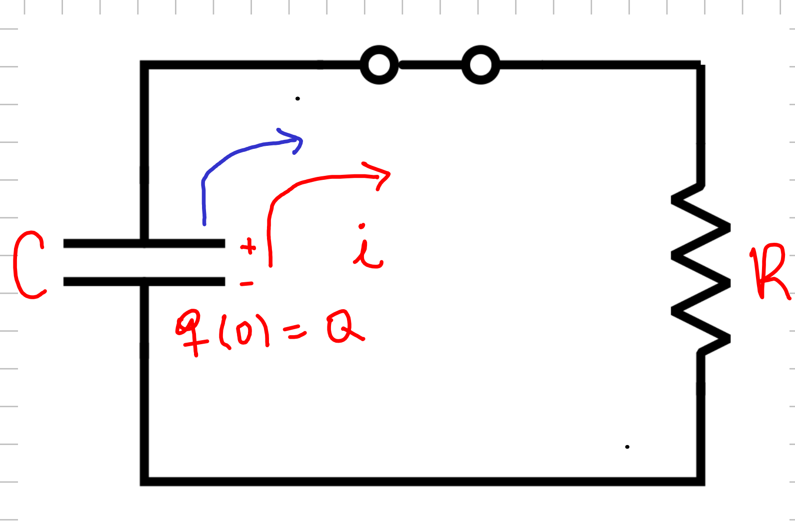
Carga y descarga de un capacitor





Eventualmente toda la energía que almacenaba el capacitor la disipa la resitencia y el sistema queda con Q = 0 e I = 0

¿Cómo varía la carga del capacitor? (y la corriente)



Ecuación de la malla

$$\frac{q}{C} - iR = 0$$

Pero la corriente no es constante y la carga del capacitor tampoco. Entonces:

$$i = -\frac{dq}{dt}$$

$$\frac{q(t)}{C} + R\frac{dq(t)}{dt} = 0$$

$$\frac{dq}{dt} = -\frac{q(t)}{RC}$$

$$\int_{Q}^{q(t)} \frac{dq'}{q'} = \int_{0}^{t} \frac{-dt'}{RC} \longrightarrow \ln\left(\frac{q(t)}{Q}\right) = -t/RC \rightarrow q(t) = Q\,e^{-t/RC}$$

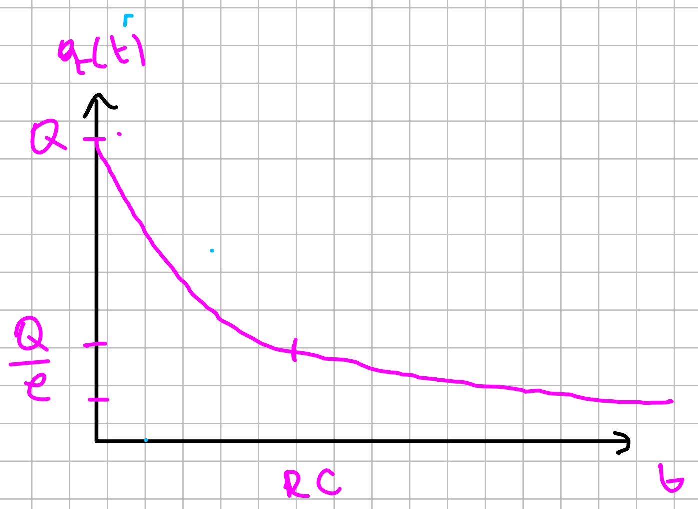
$$e^{\left(\ln\left(\frac{q(t)}{Q}\right)\right)} = e^{-t/RC}$$

$$q(t) \xrightarrow[t \to \infty]{} 0$$

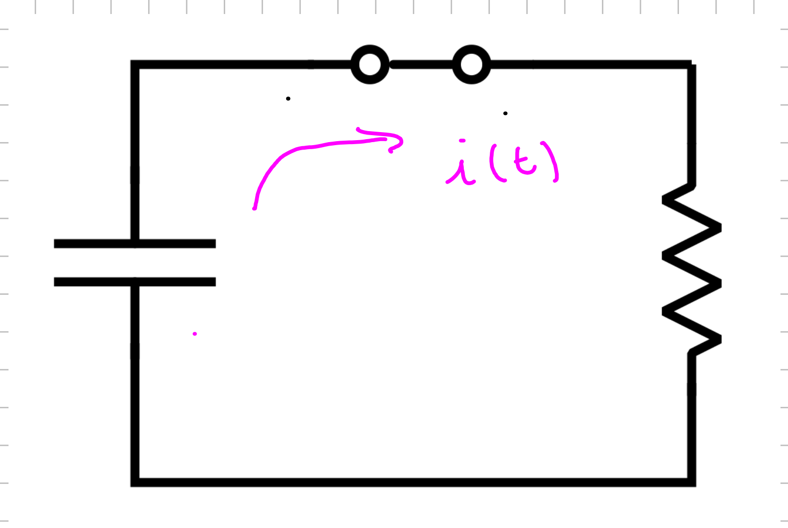
RC : tiempo caract

$$RC = \tau_c \qquad q(\tau_c) = \frac{Q}{e}$$

$$[RC] = \frac{V}{A}\,\frac{C}{V} = \frac{C}{A} = s$$

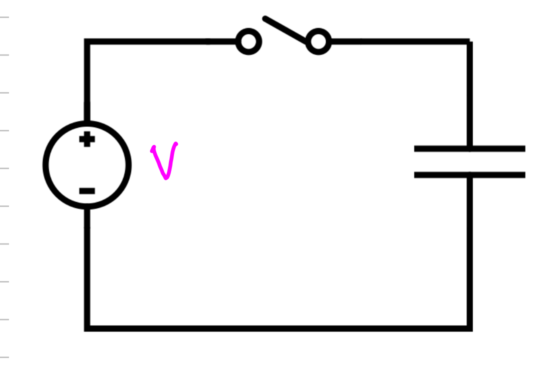


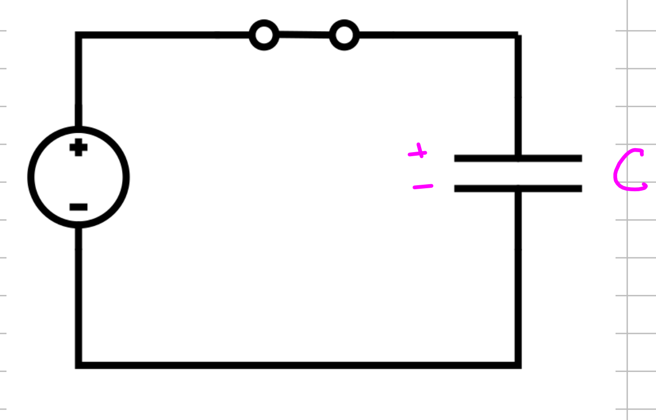
¿Y la corriente?



$$i(t) = -\frac{dq}{dt} = \frac{Q}{RC}\,e^{-t/RC} \qquad i(t) \xrightarrow[t \to \infty]{} 0$$

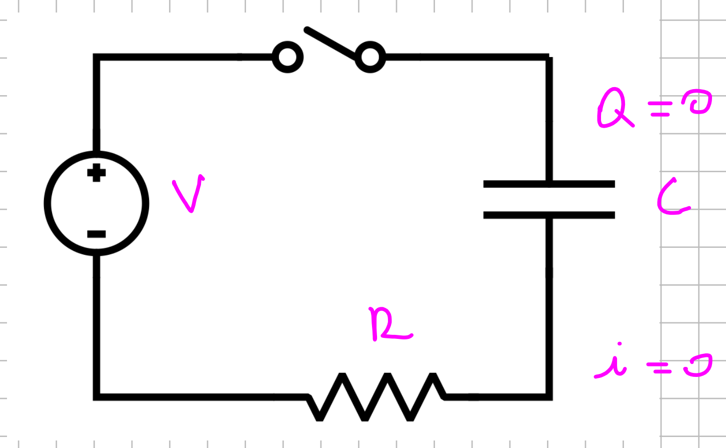
Carga y descarga de un capacitor

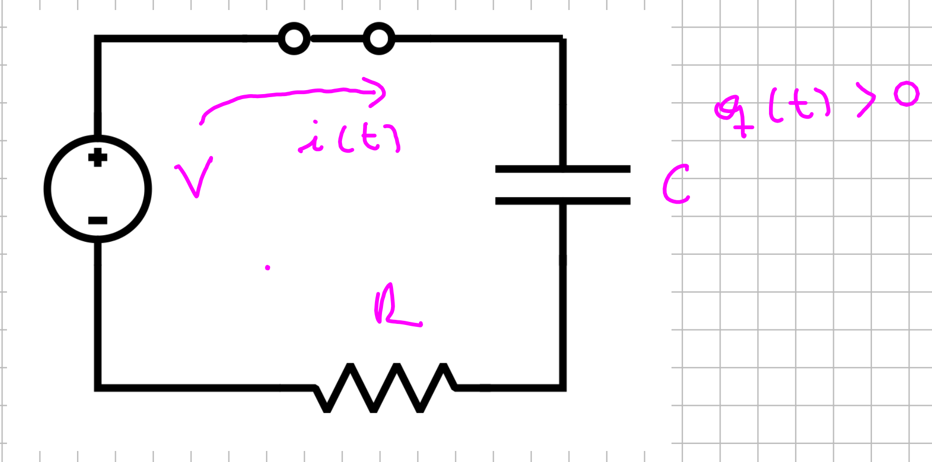




$$\frac{q(t)}{C} - V = 0 \longrightarrow q(t) = CV = \text{cte}$$

Más ejemplos: Capacitor, resistencia y fuente de voltaje





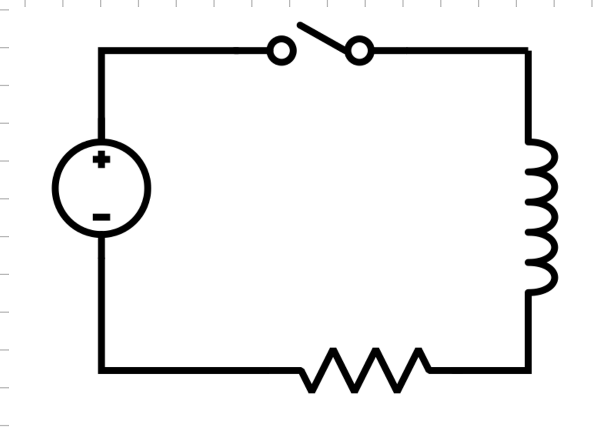
$$V - \frac{q}{C} - iR = 0 \quad (\text{guía } 3)$$

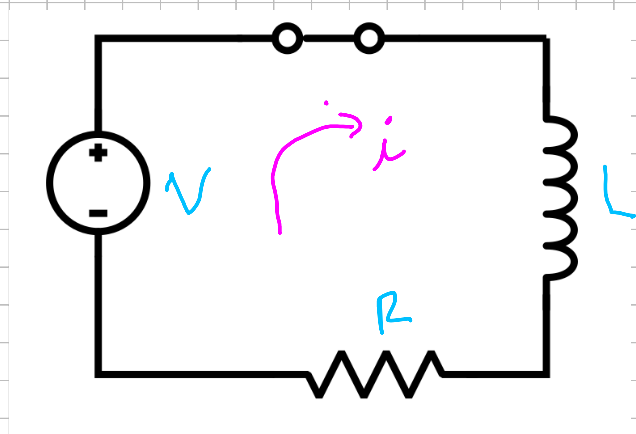
$$\underset{\text{cte}}{\overset{\shortparallel}{V}} - \frac{q}{C} + \frac{dq}{dt} R = 0$$

$$\frac{dq}{(VC - q)} = -\frac{dt}{RC} \longrightarrow q(t) = VC\,(1 - \underbrace{e^{-t/RC}}_{\substack{\to 0 \\ t \to \infty}}) \longrightarrow VC$$

$$i(t) = -\frac{dq}{dt} = \frac{V}{R}\, e^{-t/RC}$$

Inductancias





ecuación de mallas

$$V - \frac{di}{dt}L - iR = 0$$

Igual que con el circuito RC, resolvemos tipo mallas y nos queda una ecuación diferencial

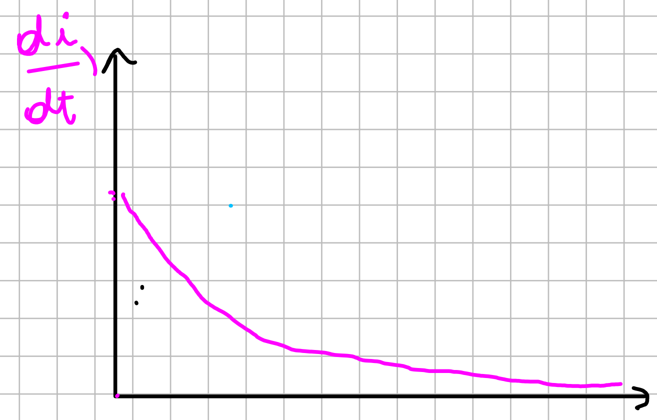
$$i(t) = \frac{V}{R}\left(1 - e^{-tR/L}\right)$$

$\frac{L}{R}$: tiempo caract

$$[L/R] = \frac{\cancel{A}}{\cancel{V}} \cdot \frac{\cancel{V}s}{\cancel{A}} \longrightarrow \left[L\frac{di}{dt}\right] \quad V$$

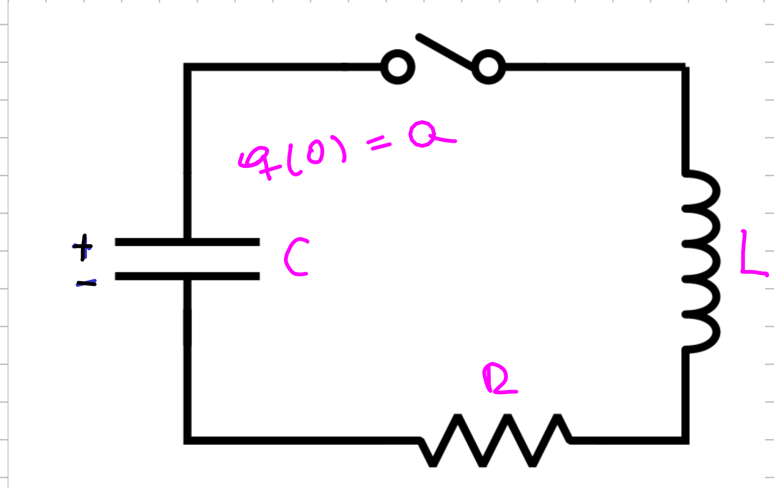
$$[L]\left[\frac{di}{dt}\right] = V$$

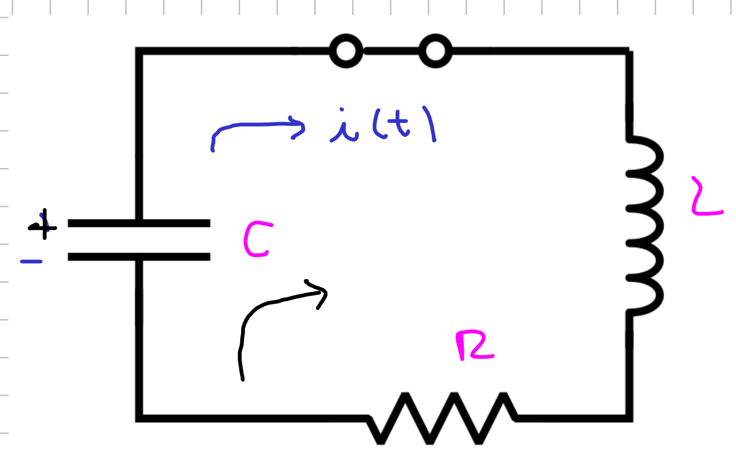
$$\frac{Vs}{A} \qquad \frac{A}{s}$$



Eventualmente la caída de tensión en L es 0

Circuito RLC





$$\frac{q}{C} - iR - L\frac{di}{dt} = 0 \longrightarrow \frac{q}{C} + \frac{dq}{dt}R + L\frac{d^2q}{dt^2} = 0$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0 \longrightarrow \frac{d^2q}{dt^2} + \gamma\frac{dq}{dt} + \omega_0^2 q = 0 \quad (1)$$

γ = cte de amortiguamiento $\quad [\gamma] = 1/s$

ω_0 = frecuencia natural del cir $\quad [\omega_0] = 1/s = Hz$

Soluciones

$$Q(t) = A e^{\alpha t} \qquad \frac{dQ(t)}{dt} = A\alpha e^{\alpha t} = \alpha Q(t) \qquad \frac{d^2Q(t)}{dt^2} = A\alpha^2 e^{\alpha t} = \alpha^2 Q(t)$$

$$(1) \longrightarrow \underbrace{(\alpha^2 + \gamma\alpha + \omega_0^2)}_{=0} Q(t) = 0$$

$$\alpha_{1,2} = \frac{-\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2}}{2} \qquad \gamma^2 - \text{cosas posit} < \gamma^2$$

$$Q(t) = A e^{\alpha_1 t} + B e^{\alpha_2 t} \longrightarrow Re[Q] = q$$

Casos: 1) Sobre amortiguado

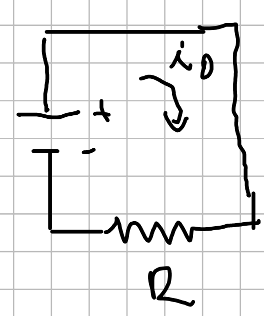
$$\gamma^2 > 4\omega_0^2 \qquad \left(\frac{\gamma}{2}\right)^2 > \omega_0^2 \qquad \alpha_1, \alpha_2 \in \mathbb{R}$$

$$\left.\begin{array}{ll} q(t) = A e^{\alpha_1 t} + B e^{\alpha_2 t} & q(0) = Q \\ \frac{dq}{dt}(t) = A\alpha_1 e^{\alpha_1 t} + B\alpha_2 e^{\alpha_2 t} & i(0) = i_0 = \frac{Q}{RC} \end{array}\right\} \longrightarrow A, B$$

$$i_0 = \frac{Q}{RC}$$

$$q \xrightarrow[t\to\infty]{} 0 \qquad i \xrightarrow[t\to\infty]{} 0 \qquad \text{porque } \alpha_1, \alpha_2 < 0$$

$$V = \frac{Q}{C}$$



Casos: 2) Crítico

$$\alpha_{1,2} = \frac{-\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2}}{2} \longrightarrow \alpha_1 = \alpha_2 \longrightarrow \frac{\gamma}{2} = \omega_0$$

Casos: 3) Subamortiguado

$$\alpha_{1,2} = \frac{-\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2}}{2} \qquad \gamma^2 < 4\omega_0^2$$

$$\alpha_{1,2} = \frac{-\gamma \pm i \overbrace{\sqrt{4\omega_0^2 - \gamma^2}}^{\omega}}{2} \qquad \left\{\begin{array}{l} e^{\alpha_1 t} = e^{-\gamma t/2} e^{i\omega t} \\ e^{\alpha_2 t} = e^{-\gamma t/2} e^{-i\omega t} \end{array}\right.$$

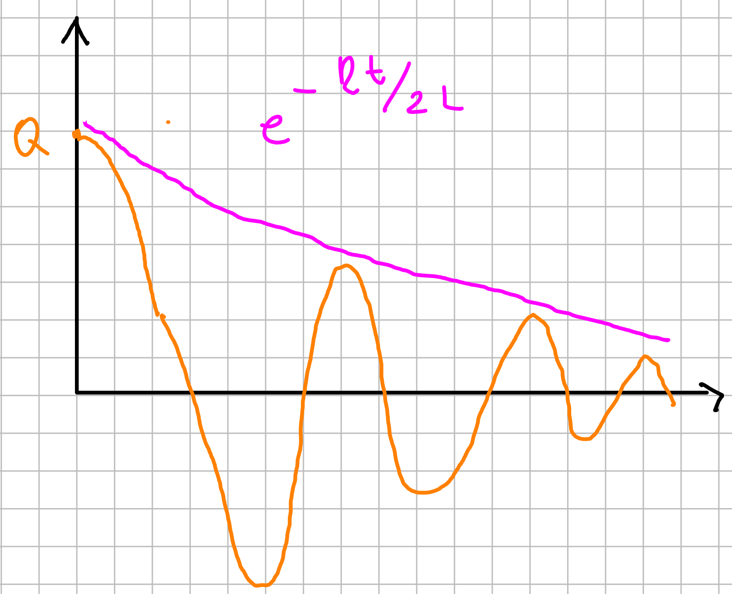
$Q(t) \longrightarrow q(t) = A_2\, e^{-\gamma t/2} \cos(\omega t) + B_2\, e^{-\gamma t/2} \operatorname{sen}(\omega t)$

$\cdot)\ q(0) = Q = A_2$

$\cdot)\ \dfrac{dq(0)}{dt} = -i_0 = -\dfrac{\gamma}{2} A_2 + \omega B_2 \longrightarrow B_2 = \dfrac{\frac{\gamma}{2} Q - i_0}{\omega}$

$$q(t) = Q\, e^{-\frac{Rt}{2L}} \cos(\omega t) + \left(\frac{R}{4L} Q - \frac{Q}{RC}\right) \frac{e^{-\frac{Rt}{2L}}}{\omega} \operatorname{sen}(\omega t)$$

$$\omega = \frac{\sqrt{\frac{4}{LC} - \frac{R^2}{L^2}}}{2} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$



$L \quad q(t) = Q\, e^{-\frac{Rt}{2L}} \cos(\omega t)$

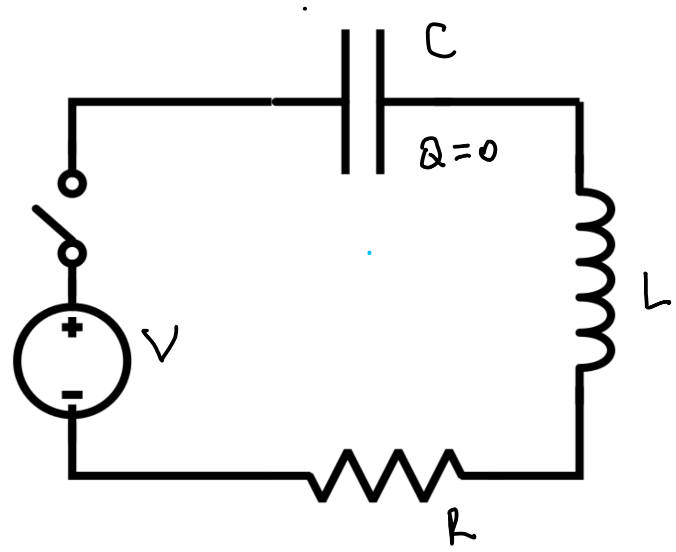
80 Una fuente de 400 V se conecta en $t = 0$ a un circuito serie formado por $L = 2\,\mathrm{H}$, $R = 20\,\Omega$ y $C = 8\,\mu\mathrm{F}$.

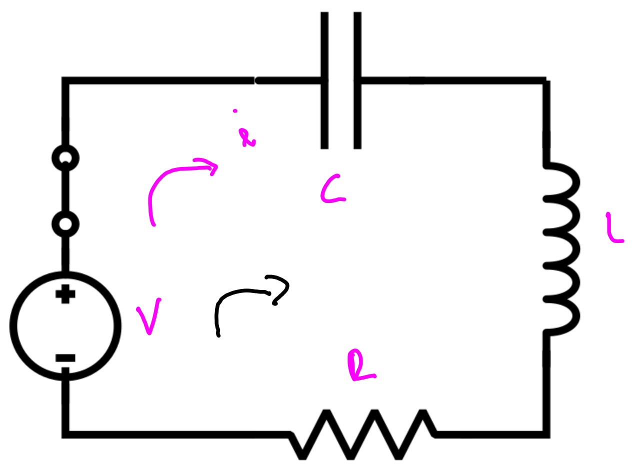
(a) Demostrar que el proceso de carga es oscilatorio y calcular la frecuencia de las oscilaciones. Comparar esta frecuencia con el valor de $(LC)^{-1/2}$.

(b) Calcular la derivada temporal inicial de la corriente.

(c) Hallar, en forma aproximada, la máxima tensión sobre C.

(d) ¿Qué resistencia debe agregarse en serie para que el amortiguamiento del circuito sea crítico?





a) Hacemos mallas. Es como el RLC pero ahora en lugar de tener el capacitor cargado, está descargado y conectamos la fuente V

$$V - \frac{q}{C} - L\frac{di}{dt} - iR = 0 \qquad \frac{q}{C} + L\frac{d^2q}{dt^2} + R\frac{dq}{dt} = V$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{CL} = V$$

$$\gamma = \frac{R}{L} \qquad w_0^2 = \frac{1}{LC}$$

$$\frac{d^2q}{dt^2} + \gamma\frac{dq(t)}{dt} + w_0^2 q = \frac{V}{L} \quad (1)$$

$$q(t) = q_H(t) + q_p$$

$$q_H(t) \longrightarrow \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{CL} = 0$$

$$q_H(t) = A e^{\alpha_1 t} + B e^{\alpha_2 t} \qquad \alpha_{1,2} = -\frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{4} - w_0^2}$$

$$q_p = q_0$$

$$q_p \text{ en } (1) \longrightarrow 0 + 0 + \frac{1}{LC} q_0 = \frac{V}{L} \longrightarrow q_0 = CV$$

$$q(t) = A e^{\alpha_1 t} + B e^{\alpha_2 t} + CV$$

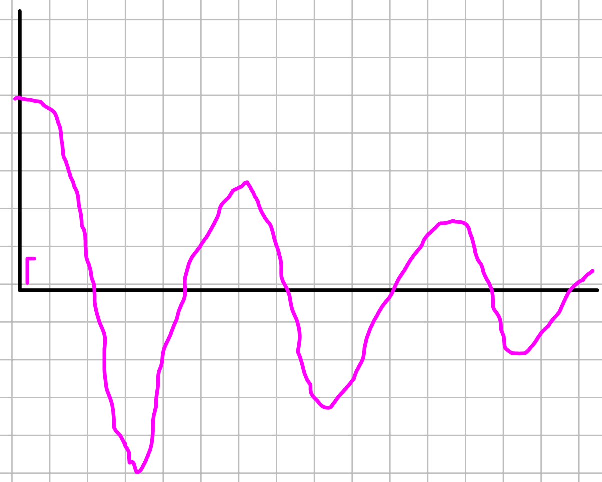
Pide demostrar que es oscilatorio. Para eso tenemos que ver en qué regimen estamos.

$$R = 20\,\Omega \qquad L = 2\,H \qquad C = 8\,\mu F$$

$$\frac{\gamma^2}{4} = \frac{R^2}{L^2}\frac{1}{4} = \frac{20^2}{2^2}\frac{1}{s}\frac{1}{4} = 25\frac{1}{s^2}$$

$$w_0^2 = \frac{1}{LC} = \frac{1}{2H\, 8\, 10^{-6} F} = \frac{1}{16}10^6\frac{1}{s^2} = (250)^2\ Hz^2 \longrightarrow w_0 = 250\ Hz$$

$$\boxed{w_0^2 >> \frac{\gamma^2}{4}} \Rightarrow$$ 3) subamortiguado

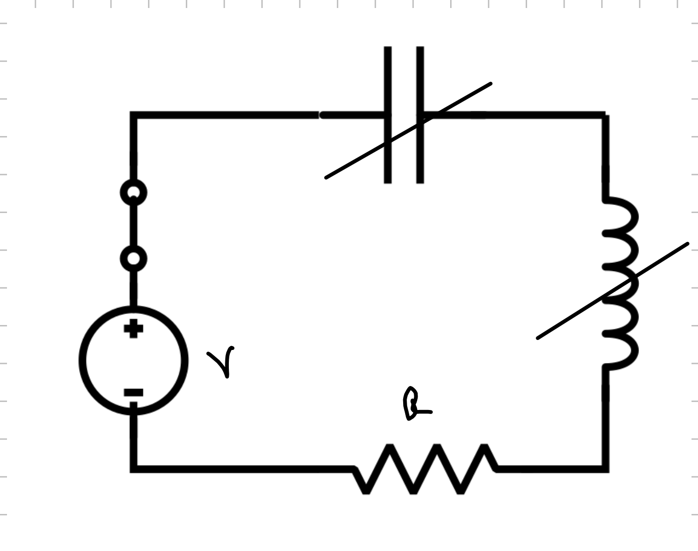


$$- \ w = \sqrt{w_0^2 - \frac{\gamma^2}{4}} \simeq 249{,}95\ Hz$$

$$w \simeq w_0 = 250\ Hz$$

$$q(t) = A\operatorname{sen}(wt)\, e^{-t\gamma/2} + B\cos(wt)\, e^{-t\gamma/2} + CV$$

$$\frac{dq}{dt}(t) = -A\operatorname{sen}(wt)\frac{\gamma}{2}e^{-t\gamma/2} + A\cos(wt)\, w\, e^{-t\gamma/2} - B\cos(wt)\frac{\gamma}{2}e^{-\frac{t\gamma}{2}} - wB\operatorname{sen}(wt)\, e^{\frac{-\gamma t}{2}}$$
$$= -i(t)$$



$$q(0) = 0 = B + CV \Rightarrow B = -CV$$

$$i_0 = i(t=0) = \frac{V}{R}$$

$$Aw - B\frac{\gamma}{2} = -\frac{V}{R} = -i_0$$

$$A, B \longrightarrow$$

$$B = -CV \qquad A = \left(\frac{B\gamma}{2} - \frac{V}{R}\right)\frac{1}{w}$$

b) $$\left.\frac{di}{dt}\right|_{t=0} = \frac{V\left(4w^2(CRw - 1) + \gamma^2(CRw + 1)\right)}{2\gamma R}$$

c) $$q(t) = \left[3\ 10^{-3} - e^{-5t}\left[3\ 10^{-3}\cos(wt) + 8\ 10^{-2}\operatorname{sen}(wt)\right]\right. \text{Coul}$$

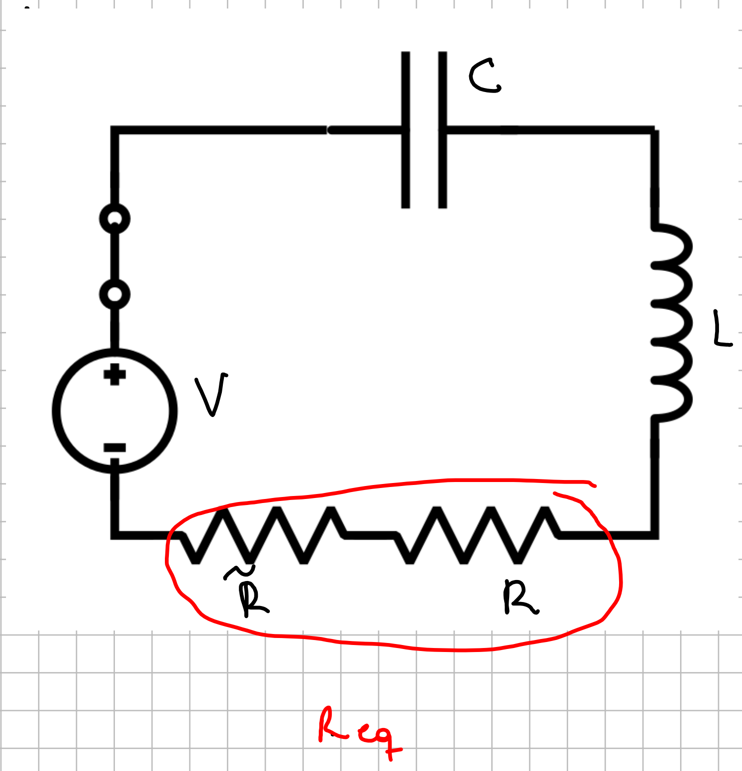
números
(aprox)

$$\Delta V_{max} = \frac{q_{max}}{C}$$ q max $\to$ q / sen(wt) es max $\longrightarrow = 1 \Rightarrow \cos = 0$

$$\frac{q_{max}}{c} = \frac{\left[3\ 10^{-3} - e^{-\frac{\pi}{2w}\cdot\frac{\gamma}{2}}\ 8.\right]}{C}\cdot V$$

(ó $$\left.\frac{dq}{dt}\right|_{t\,max} = 0$$ y $q(t_{max}) = q_{max}$)

d)



¿ $\tilde{R}$ / crítico ?

condición crítica $\quad \omega_0^2 = \frac{\gamma^2}{4}$

$\Rightarrow \quad R \rightsquigarrow R_{eq} = R + \tilde{R}$

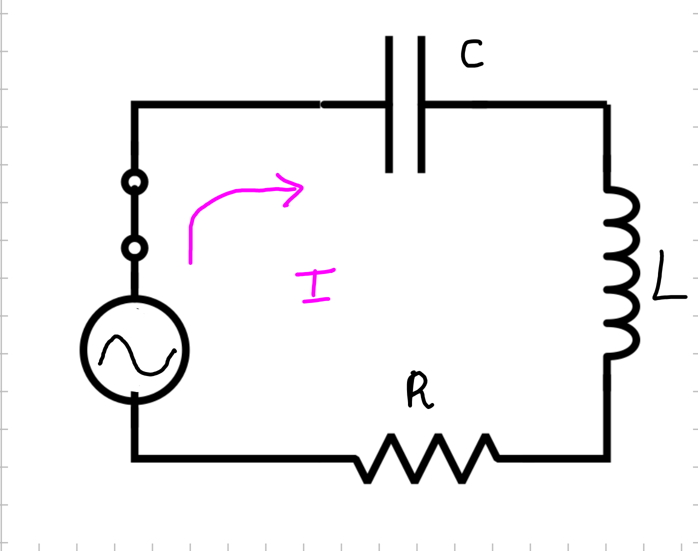
$$\omega_0^2 = \frac{\gamma^2}{4}$$

$$\frac{1}{LC} = \frac{1}{4} \frac{(R+\tilde{R})^2}{L^2}$$

$$R + \tilde{R} = \sqrt{\frac{4L}{C}}$$

$$\tilde{R} = \underbrace{\sqrt{\frac{4L}{C}}}_{\sim 10^3 \Omega} - \underbrace{R}_{= 20\Omega}$$

Corriente alterna: formalismo complejo



Tengo una fuente de tensión sinusoidal con una dada frecuencia

$$V(t) - \frac{q}{C} - \frac{di}{dt}L - iR = 0$$

$$\frac{q}{C} + iR + \frac{di}{dt}L = V(t) \qquad (1)$$

$$q(t) = \underbrace{q_H(t)}_{\text{transitorio}} + \underbrace{q_V(t)}_{\text{estacionario}}$$

$V=0$ $\xrightarrow[t\to\infty]{} 0$

$q_V(t) \propto \sin(\omega t), \cos(\omega t)$

ω : frec de la fuente

$$I(t) = \underbrace{I_0 e^{i\varphi}}\,\underbrace{e^{i\omega t}} \qquad Re(I(t)) = i(t)$$

$$\frac{dI}{dt} = i\omega I_0 e^{i\varphi} e^{i\omega t} = i\omega I(t)$$

$$Q = \int I(t) = \frac{1}{i\omega} \underbrace{I_0 e^{i\varphi}}_{I} e^{i\omega t} = \frac{1}{i\omega} I(t)$$

$$\frac{I(t)}{i\omega C} + I(t)\, i\omega L + R I(t) = V_0 e^{i\omega t}$$

$$\underbrace{\frac{I}{i\omega C}}_{Z_C} + I \underbrace{i\omega L}_{Z_L} + \underbrace{R}_{Z_R} I = V_0$$

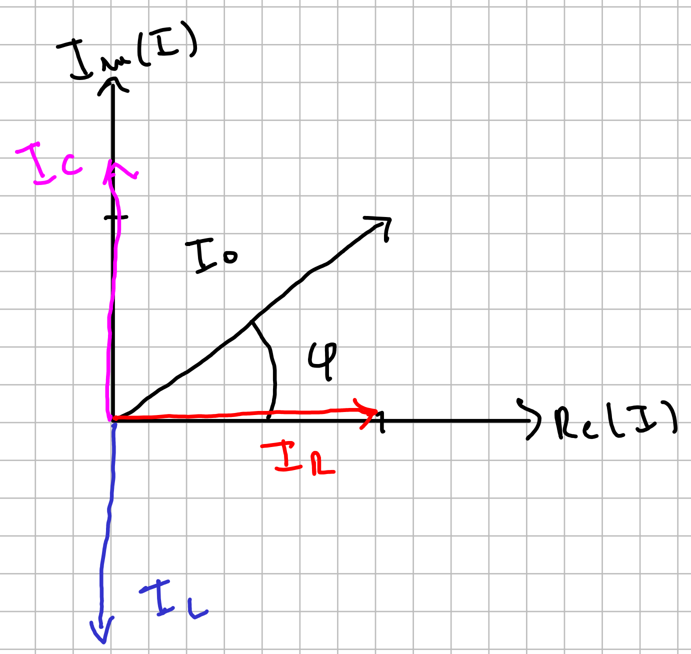
$$(Z_C + Z_L + Z_R)\, I = V_0$$

de im

$$I = \frac{V_0}{(Z_C+Z_L+Z_R)} = Re\left[\frac{V_0}{(Z_C+Z_L+Z_R)}\right] + i\, Im\left[\frac{V_0}{(Z_C+Z_L+Z_R)}\right]$$

$$I = \frac{V_0}{[Z_C+Z_L+Z_R]} = I_0 e^{i\varphi} \Rightarrow I_0 = \left|\frac{V_0}{(Z_C+Z_L+Z_R)}\right|$$

$$\varphi = \arctan\left[\frac{Im(\text{cociente})}{Real(\text{cociente})}\right]$$

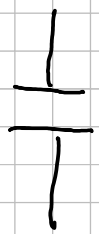


con solo R → $I_R = \frac{V_0}{R}$ real

con solo C → $I_C = \frac{V_0}{\frac{1}{i\omega C}} = i V_0 \omega C = e^{i\frac{\pi}{2}} V_0 \omega C$

con solo L → $I_L = \frac{V_0}{i\omega L} = -\frac{V_0 i}{\omega L} = e^{-i\frac{\pi}{2}} \cdot \frac{V_0}{\omega L}$

⇒ Impedancia

 C $\longrightarrow$ $Z_C = \dfrac{1}{iWC} = -\dfrac{i}{WC} = \dfrac{e^{-i\pi/2}}{WC}$ cambia la fase

 R $\longrightarrow$ $Z_I = R = R = e^{i0} R$ no cambia la fase

 L $\longrightarrow$ $Z_L = iWL = iWL = e^{i\pi/2} WL$ cambia la fase

Impedancias en serie

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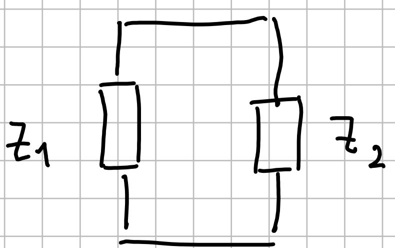
$Z_{eq} = Z_1 + Z_2$

Impedancias en paralelo



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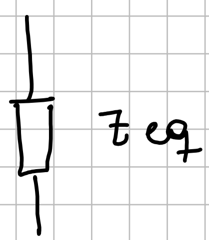


$\dfrac{1}{Z_{eq}} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2}$ (*)

Admitancia

$\dfrac{1}{Z} = Y$ $\longrightarrow$ admitancia

Ojo que son números complejos e invertirlos no es así nomás

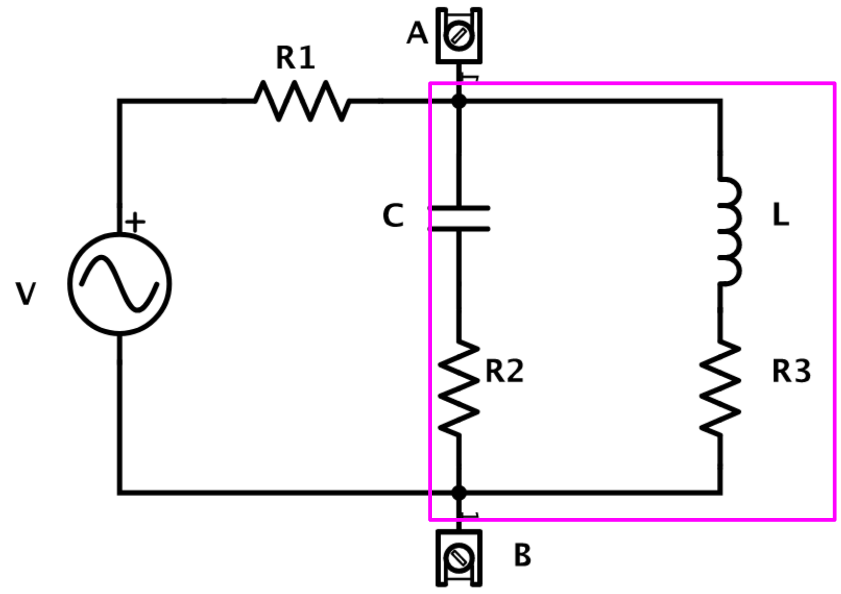
$Y_{eq} = Y_1 + Y_2$ (*)

Reactancia

$Z = R + i\underbrace{X}_{\equiv \text{Reactancia}}$

$X = Im(Z)$

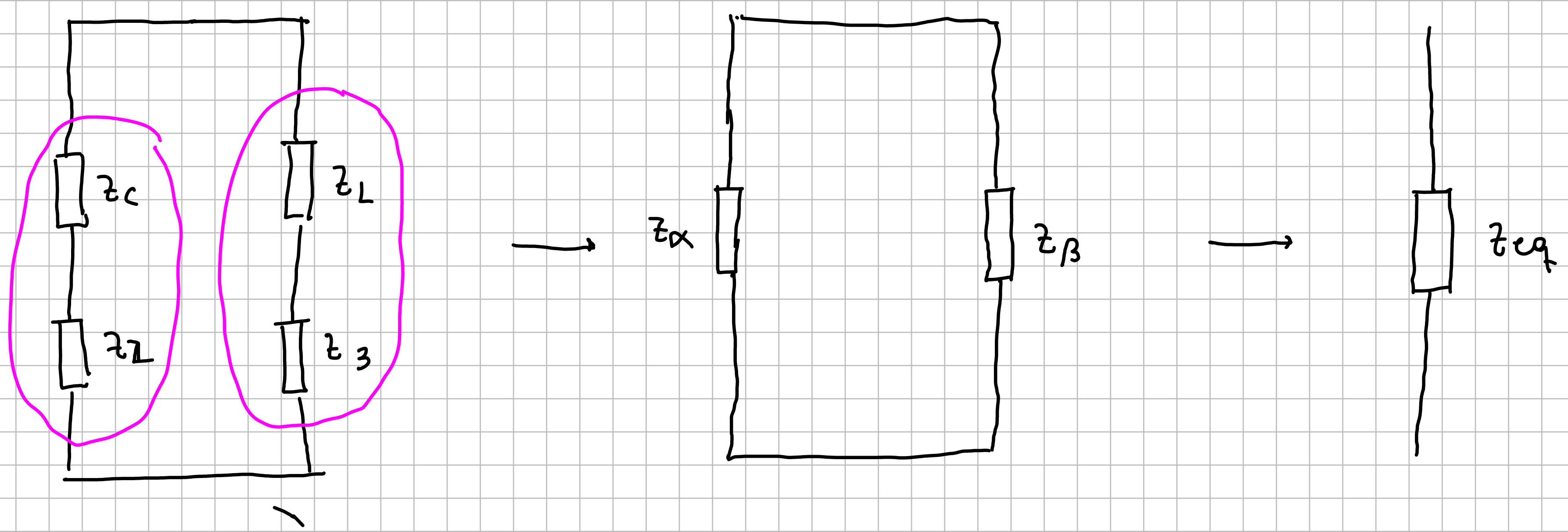
83 En el circuito indicado la fuente de tensión V tiene 100 V y 50 Hz, $C = 20\,\mu\mathrm{F}$, $L = 0{,}25\,\mathrm{H}$, $R_1 = R_2 = R_3 = 10\,\Omega$.



(a) Calcular la impedancia equivalente a la derecha de los puntos A y B.

(b) Calcular la corriente que circula por cada resistencia.

(c) Construir el diagrama vectorial del circuito.



$$Z_\alpha = R + \frac{1}{i\omega C} \qquad Z_\beta = R + i\omega L$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_\alpha} + \frac{1}{Z_\beta} \qquad Y_{eq} = Y_\alpha + Y_\beta$$

$$Z_{eq} = \frac{Z_\alpha Z_\beta}{Z_\alpha + Z_\beta} = \frac{\left(R - \frac{i}{\omega C}\right)\left(R + i\omega L\right)}{R - \frac{i}{\omega C} + R + i\omega L} = \frac{\left(R^2 + \frac{L^2}{C^2}\right) + i\left(R\omega L - \frac{R}{\omega C}\right)}{2R + i\left(\omega L - \frac{1}{\omega C}\right)}$$

$$Z_{eq} = \frac{\left[\left(R^2 + \frac{L^2}{C^2}\right) + i\left(R\omega L - R/\omega C\right)\right]\left[2R - i\left(\omega L - 1/\omega C\right)\right]}{4R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

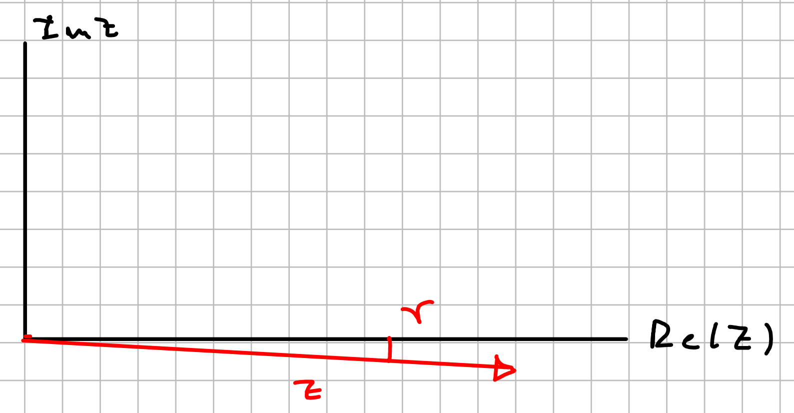
$$R[Z_{eq}] = \frac{R + CR\left(C - L + 2CR^2\right)\omega^2 + C^3 L R \omega^4}{1 + C\omega^2\left(4CR^2 + L\left(CL\omega^2 - 2\right)\right)}$$

$$Im[Z_{eq}] = \frac{C\omega\left[1 - R^2 + C\left(2CR^2 - L\left(1 + R^2\right)\right)\omega^2\right]}{1 + C\omega^2\left(4CR^2 + L\left(C L \omega^2 - 2\right)\right)}$$

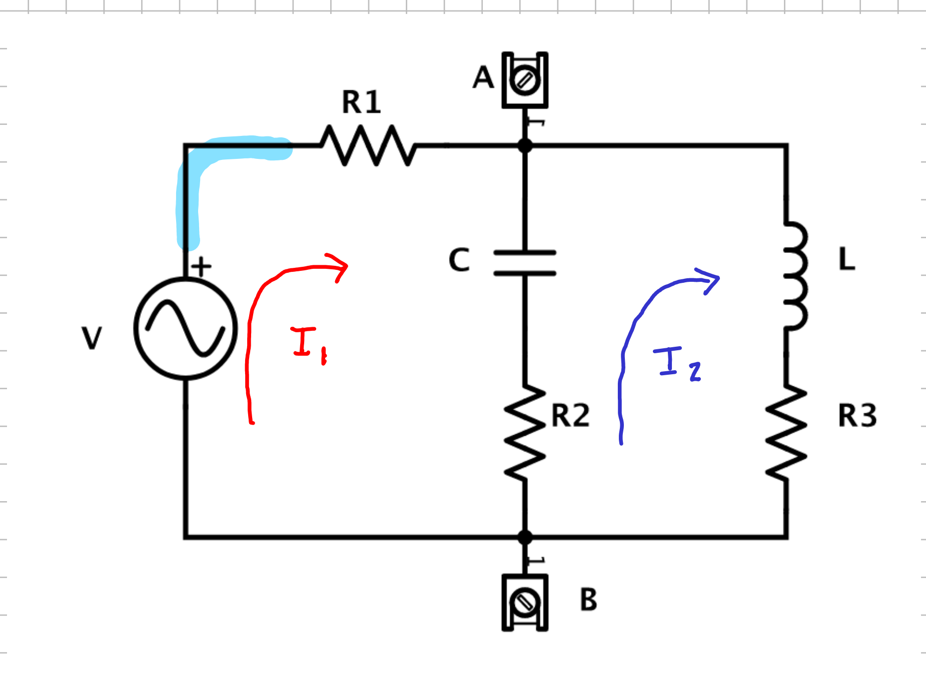
$Z = (10,125 - 0,1\, i)\ \Omega \leftarrow$

$|Z| \simeq 10,125\ \Omega$ $\qquad z = |z|\, e^{i\gamma}$

$\gamma = -0,01$ rad $= 0,6°$

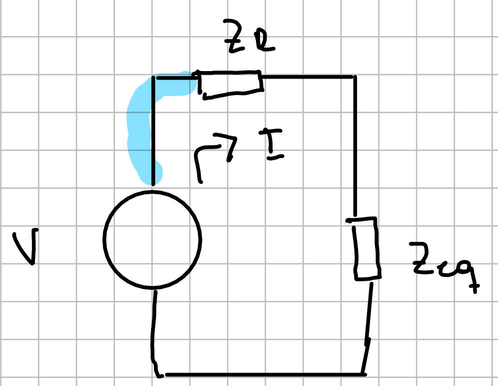


b)



$$\begin{cases} V_0 - I_1 z_R - I_1 z_C - I_1 z_R + I_2 z_C + I_2 z_R = 0 \\ -z_R I_2 - z_C I_2 + z_R I_1 + z_C I_1 - z_L I_2 - z_R I_2 = 0 \end{cases}$$

$I_1, I_2 \to V_0, z$



$I = I_1$ $\qquad V_0 - (z_R + z_{eq}) I = 0$

$$I = I_1 = \frac{V_0}{z_R + z_{eq}}$$

$z_R = 10\,\Omega \Rightarrow z_R + z_{eq} = (20,125 - 0,1\, i)\,\Omega$

$|z_R + z_{eq}| \simeq 20,125$

$\gamma \simeq -0,005$ rad

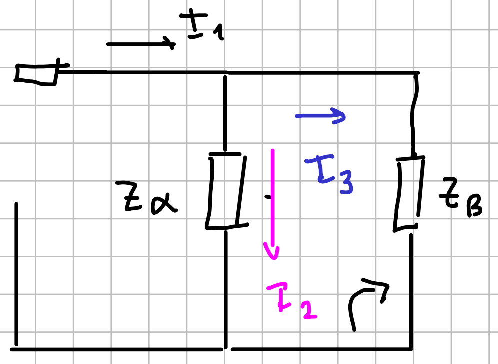
$Z_{eq} = (10,125 - 0,1\, i)\ \Omega \leftarrow$

$|Z|_{eq} \simeq 10,125\ \Omega$ $\qquad z_{eq} = |z|_{eq}\, e^{i\gamma}$

$\gamma = -0,01$ rad $= -0,6°$

$$I = \frac{V_0}{20\,\Omega \cdot e^{+i\gamma}} = \frac{100\,V}{20\,\Omega} e^{-i\gamma} = 5\,A\, e^{-i\gamma} \Rightarrow I(t) = I_1(t) = 5\,A\, e^{i(\omega t - \gamma)}$$

$$i_1(t) = Re[I_1(t)] = 5\,A \cos(\omega t - \gamma)$$



$I_2 + I_3 = I_1$

$I_2 z_\alpha - I_3 z_\beta = 0$ $\qquad z_\alpha = R - \frac{i}{\omega C}$ $\qquad z_\beta = R + i\omega L$

$$\frac{I_2}{I_3} = \frac{z_\beta}{z_\alpha} \longrightarrow I_2 + \frac{z_\alpha}{z_\beta} I_2 = I_1$$

$$\Rightarrow \rightarrow I_2 = I_1 \frac{1}{\left(1 + \frac{Z_\alpha}{Z_\beta}\right)} \qquad \frac{Z_\alpha}{Z_\beta} = \frac{R - i/wC}{R + iwL} \leftarrow$$

$$\rightarrow I_3 = I_2 \frac{Z_\alpha}{Z_\beta}$$

$$\boxed{I_1(t) = 5 A\, e^{i(wt-\gamma)}}$$

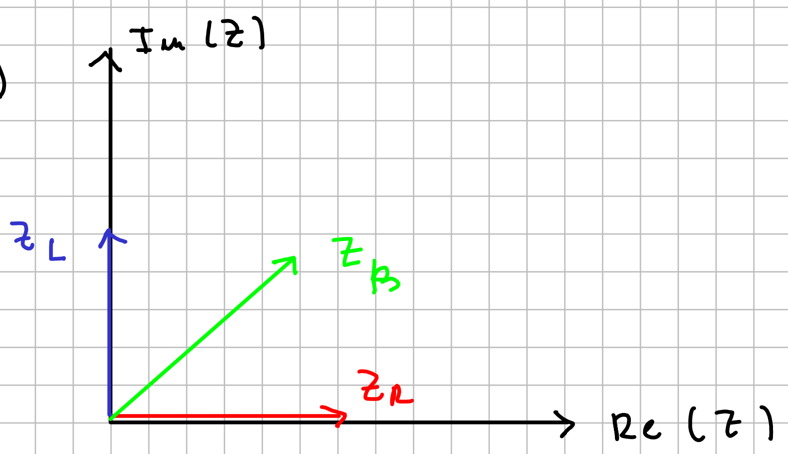
$$\left.\begin{array}{l} i_1(t) = Re[I_1(t)] \\ i_2(t) = Re[I_2(t)] \\ i_3(t) = R[I_3(t)] \end{array}\right\}$$

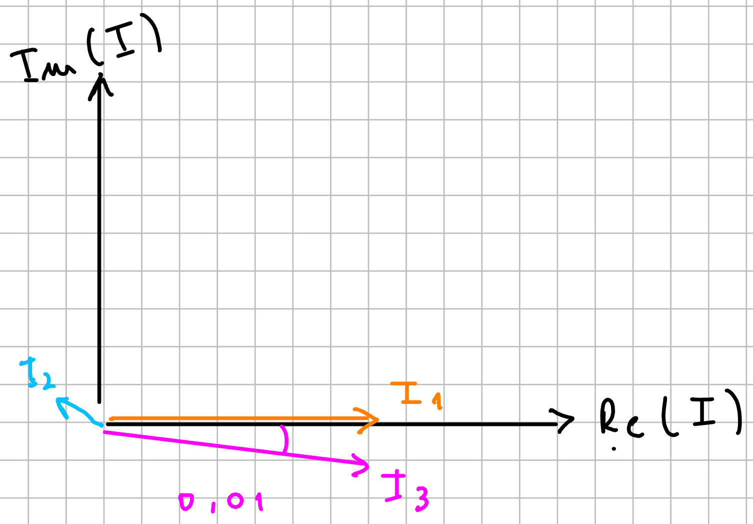
$$I_2(t) = 0{,}081\, e^{i(wt-\gamma+2.44)}\ A = -0{,}06 + i\, 0{,}05 \qquad I_1 = 5\ A$$

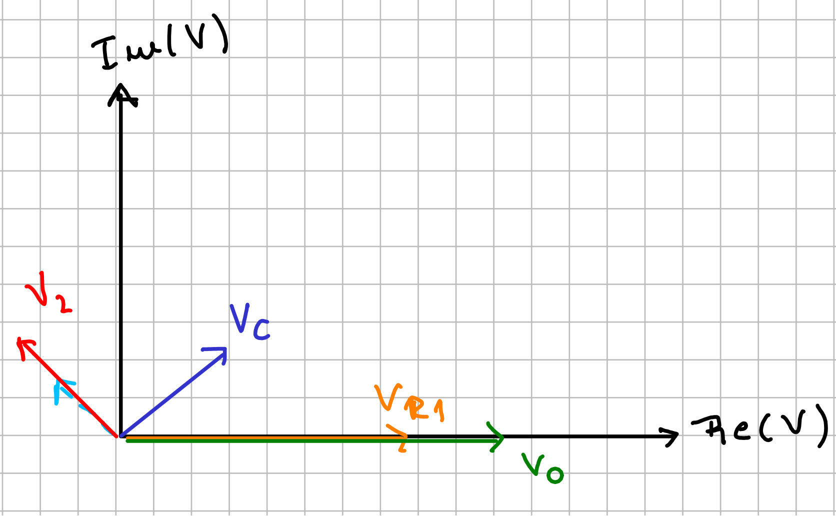
$$I_3(t) = 5{,}1\, e^{i(wt-\gamma-0{,}01)}\ A = 5{,}1 - i\, 0{,}05$$

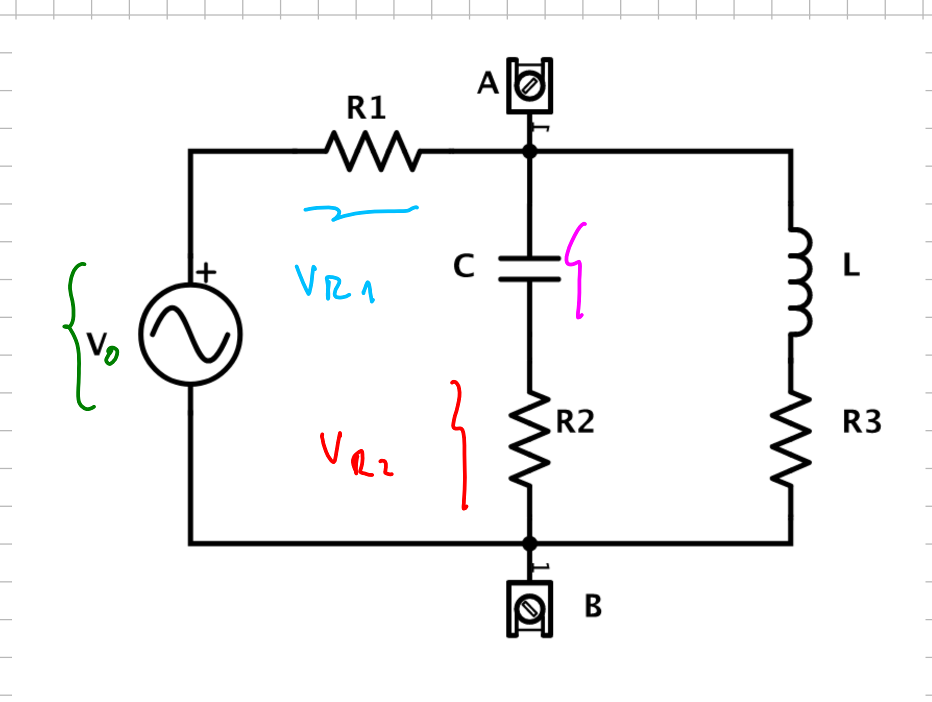
c)

Diagrama de las impedancias









$V_{R1} = R I_1$

$V_C = \frac{I_2}{iwC}$

$V_C = \frac{I_2}{wC} \cdot e^{-i\pi/2}$

$V_{R2} = R\, I_2$