

CHAPTER IX.

WAVES FROM MOVING SOURCES.

Adagio. Andante. Allegro moderato.

§ 450. The following story is true. There was a little boy, and his father said, "Do try to be like other people. Don't frown." And he tried and tried, but could not. So his father beat him with a strap; and then he was eaten up by lions.

Reader, if young, take warning by his sad life and death. For though it may be an honour to be different from other people, if Carlyle's dictum about the 30 millions be still true, yet other people do not like it. So, if you are different, you had better hide it, and pretend to be solemn and wooden-headed. Until you make your fortune. For most wooden-headed people worship money; and, really, I do not see what else they can do. In particular, if you are going to write a book, remember the wooden-headed. So be rigorous; that will cover a multitude of sins. And do not frown.

There is a time for all things: for shouting, for gentle speaking, for silence; for the washing of pots and the writing of books. Let now the pots go black, and set to work. It is hard to make a beginning, but it must be done.

Electric and magnetic force. May they live for ever, and never be forgot, if only to remind us that the science of electromagnetics, in spite of the abstract nature of the theory, involving quantities whose nature is entirely unknown at present, is really and truly founded upon the observation of real Newtonian forces, electric and magnetic respectively.

Figura 1: Vol 3, Electromagnetic theory by Heaviside, Oliver, 1850-1925.

Luego de esto, Heaviside procede a introducir el análisis vectorial.

1 Identidades del cálculo vectorial

En las siguientes líneas, ϕ y ψ representan escalares, mientras que \mathbf{a} , \mathbf{b} y \mathbf{c} son vectores.

Triple productos y productos mixtos

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) \quad (\equiv [\mathbf{a}, \mathbf{b}, \mathbf{c}])$$

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$$

Reglas de producto

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a}) + (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a}$$

$$\nabla \cdot (\phi\mathbf{a}) = \phi\nabla \cdot \mathbf{a} + \mathbf{a} \cdot \nabla\phi$$

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$$

$$\nabla \times (\phi\mathbf{a}) = \phi\nabla \times \mathbf{a} - \mathbf{a} \times \nabla\phi$$

$$\nabla \times (\mathbf{a} \times \mathbf{b}) = (\mathbf{b} \cdot \nabla)\mathbf{a} - (\mathbf{a} \cdot \nabla)\mathbf{b} + \mathbf{a}\nabla \cdot \mathbf{b} - \mathbf{b}\nabla \cdot \mathbf{a}$$

$$\nabla \times (\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (\nabla \times \mathbf{b}) - \mathbf{b} \times (\nabla \times \mathbf{a}) - (\mathbf{a} \times \nabla) \times \mathbf{b} + (\mathbf{b} \times \nabla) \times \mathbf{a}$$

Derivadas segundas

$$\nabla \cdot (\nabla \times \mathbf{a}) = 0$$

$$\nabla \times (\nabla\phi) = 0$$

$$\nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2\mathbf{a}$$

Teoremas integrales fundamentales

$$\int_{\mathbf{r}_1}^{\mathbf{r}_2} (\nabla\phi) \cdot d\mathbf{l} = \phi(\mathbf{r}_2) - \phi(\mathbf{r}_1)$$

$$\int_V (\nabla \cdot \mathbf{a}) d^3r = \int_{S(V)} \mathbf{a} \cdot d\mathbf{s}$$

$$\int_{S(C)} (\nabla \times \mathbf{a}) \cdot d\mathbf{s} = \oint_C \mathbf{a} \cdot d\mathbf{l}$$

Otros teoremas integrales

$$\int_V (\nabla\phi) d^3r = \int_{S(V)} d\mathbf{s} \phi$$

$$\int_V (\nabla \times \mathbf{a}) d^3r = \int_{S(V)} d\mathbf{s} \times \mathbf{a}$$

2 Operadores vectoriales en distintos sistemas de coordenadas

Coordenadas cartesianas

$$d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}$$

$$d^3r = dx dy dz$$

$$\nabla\phi = \partial_x\phi \hat{\mathbf{x}} + \partial_y\phi \hat{\mathbf{y}} + \partial_z\phi \hat{\mathbf{z}}$$

$$\nabla \cdot \mathbf{a} = \partial_x a_x + \partial_y a_y + \partial_z a_z$$

$$\nabla \times \mathbf{a} = (\partial_y a_z - \partial_z a_y) \hat{\mathbf{x}} + (\partial_z a_x - \partial_x a_z) \hat{\mathbf{y}} + (\partial_x a_y - \partial_y a_x) \hat{\mathbf{z}}$$

$$\nabla^2\phi = \partial_x^2\phi + \partial_y^2\phi + \partial_z^2\phi$$

Coordenadas esféricas

$$d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + r \operatorname{sen} \theta d\varphi \hat{\boldsymbol{\varphi}}$$

$$d^3r = r^2 \operatorname{sen} \theta dr d\theta d\varphi$$

$$\nabla\phi = \partial_r\phi \hat{\mathbf{r}} + r^{-1}\partial_\theta\phi \hat{\boldsymbol{\theta}} + (r \operatorname{sen} \theta)^{-1}\partial_\varphi\phi \hat{\boldsymbol{\varphi}}$$

$$\nabla \cdot \mathbf{a} = r^{-2}\partial_r(r^2 a_r) + (r \operatorname{sen} \theta)^{-1}\partial_\theta(\operatorname{sen} \theta a_\theta) + (r \operatorname{sen} \theta)^{-1}\partial_\varphi a_\varphi$$

$$\nabla \times \mathbf{a} = (r \operatorname{sen} \theta)^{-1}[\partial_\theta(\operatorname{sen} \theta a_\varphi) - \partial_\varphi a_\theta] \hat{\mathbf{r}} + r^{-1}[(\operatorname{sen} \theta)^{-1}\partial_\varphi a_r - \partial_r(r a_\varphi)] \hat{\boldsymbol{\theta}} \\ + r^{-1}[\partial_r(r a_\theta) - \partial_\theta a_r] \hat{\boldsymbol{\varphi}}$$

$$\nabla^2\phi = r^{-2}\partial_r(r^2\partial_r\phi) + (r^2 \operatorname{sen} \theta)^{-1}\partial_\theta(\operatorname{sen} \theta \partial_\theta\phi) + (r^2 \operatorname{sen}^2\theta)^{-1}\partial_\varphi^2\phi$$

Coordenadas cilíndricas

$$d\mathbf{l} = dr \hat{\mathbf{r}} + r d\varphi \hat{\boldsymbol{\varphi}} + dz \hat{\mathbf{z}}$$

$$d^3r = r dr d\varphi dz$$

$$\nabla\phi = \partial_r\phi \hat{\mathbf{r}} + r^{-1}\partial_\varphi\phi \hat{\boldsymbol{\varphi}} + \partial_z\phi \hat{\mathbf{z}}$$

$$\nabla \cdot \mathbf{a} = r^{-1}\partial_r(r a_r) + r^{-1}\partial_\varphi a_\varphi + \partial_z a_z$$

$$\nabla \times \mathbf{a} = [r^{-1}\partial_\varphi a_z - \partial_z a_\varphi] \hat{\mathbf{r}} + [\partial_z a_r - \partial_r a_z] \hat{\boldsymbol{\varphi}} + r^{-1}[\partial_r(r a_\varphi) - \partial_\varphi a_r] \hat{\mathbf{z}}$$

$$\nabla^2\phi = r^{-1}\partial_r(r\partial_r\phi) + r^{-2}\partial_\varphi^2\phi + \partial_z^2\phi$$