Campo y potencial electrostático
de un distribución de cargas (discreta y continua)

$$\vec{E}(\vec{r}) = -\vec{\nabla}\, V(\vec{r})$$

$$\vec{E}(\vec{r}) = \sum_{n=1}^{N} \frac{1}{4\pi\varepsilon_0} q_n \frac{\vec{r}-\vec{r}_n}{|\vec{r}-\vec{r}_n|^3} \qquad V(\vec{r}) = \sum_{n=1}^{N} \frac{1}{4\pi\varepsilon_0} q_n \frac{1}{|\vec{r}-\vec{r}_n|}$$

?

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_V \rho(\vec{r}\,') \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}\, dV' \qquad V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_V \rho(\vec{r}\,') \frac{1}{|\vec{r}-\vec{r}\,'|}\, dV'$$

Conectando distribuciones continuas y discretas δ

Delta de Dirac (1D)

$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

Delta de Dirac (3D)

$$\delta(\vec{r}) = \delta(x)\, \delta(y)\, \delta(z)$$

$$\iiint_{-\infty}^{\infty} \delta(\vec{r})\, dV = 1$$

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Una carga en el origen

$$\rho(\vec{r}') = q\, \delta(\vec{r}')$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_V \rho(\vec{r}') \frac{1}{|\vec{r} - \vec{r}'|} dV'$$

$$= \frac{1}{4\pi\varepsilon_0} \iiint_V q\, \delta(\vec{r}) \frac{1}{|\vec{r} - \vec{r}'|} dV'$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{|\vec{r}|}$$

Conectando distribuciones continuas y discretas δ

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$$\delta(\vec{r}) = \delta(x)\, \delta(y)\, \delta(z)$$

$$\iiint_{-\infty}^{\infty} \delta(\vec{r})\, dV = 1$$

Una carga en $\vec{r_0}$

$$\rho(\vec{r}') = q\, \delta(\vec{r}' - \vec{r_0})$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_V \rho(\vec{r}') \frac{1}{|\vec{r} - \vec{r}'|} dV'$$

$$= \frac{1}{4\pi\varepsilon_0} \iiint_V q\delta(\vec{r}' - \vec{r_0}) \frac{1}{|\vec{r} - \vec{r}'|} dV'$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{|\vec{r} - \vec{r_0}|}$$

Conectando distribuciones continuas y discretas δ

Delta de Dirac (1D)

$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

Delta de Dirac (3D)

$$\delta(\vec{r}) = \delta(x)\, \delta(y)\, \delta(z)$$

$$\iiint_{-\infty}^{\infty} \delta(\vec{r})\, dV = 1$$

Multiples carga q_n en $\vec{r_n}$

$$\rho(\vec{r}') = \sum_{n=1}^{N} q_n\, \delta(\vec{r}' - \vec{r_n})$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_V \rho(\vec{r}') \frac{1}{|\vec{r} - \vec{r}'|} dV'$$

$$= \frac{1}{4\pi\varepsilon_0} \iiint_V \sum_{n=1}^{N} q_n\, \delta(\vec{r}' - \vec{r_n}) \frac{1}{|\vec{r} - \vec{r}'|} dV'$$

$$= \frac{1}{4\pi\varepsilon_0} \sum_{n=1}^{N} q_n \frac{1}{|\vec{r} - \vec{r_n}|}$$

Conectando distribuciones en
volumétricas, **superficiales** y **lineales**

$$\boldsymbol{\rho}(x', y', z') = \boldsymbol{\sigma}(x', y')\,\delta(z') = \boldsymbol{\lambda}(x')\,\delta(y')\,\delta(z')$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_V \boldsymbol{\rho}(\vec{r}') \frac{1}{|\vec{r} - \vec{r}'|} dx'dy'dz'$$

Expansión multipolar

campo y potencial
lejos de la distribución de cargas

El potencial lejos de la distribución de cargas

$|\vec{r}| \gg |\vec{r}'|$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\iiint_V \rho(\vec{r}')\frac{1}{|\vec{r}-\vec{r}'|}dV'$$

$$\hat{r} = \frac{\vec{r}}{r}$$

$$|\vec{r}-\vec{r}'| = \sqrt{|r^2 - 2\,\vec{r}\,\vec{r}' + r'^2|} = r\sqrt{\left|1 - 2\frac{\hat{r}\,\vec{r}'}{r} + \left(\frac{r'}{r}\right)^2\right|}$$

El potencial lejos de la distribución de cargas

$|\vec{r}| \gg |\vec{r}'|$

Teorema del Binomio
(generalizado)

$$(a+x)^s = \sum_{n=0}^{\infty}\frac{s!}{n!\,(s-n)!}x^n a^{s-n}$$

$$(1+x)^s = \sum_{n=0}^{\infty}\frac{s!}{n!\,(s-n)!}x^n$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{r}\frac{1}{\sqrt{\underbrace{\left|1 - 2\frac{\hat{r}\,\vec{r}'}{r} + \left(\frac{r'}{r}\right)^2\right|}_{\varepsilon}}}$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{r}\left(1 - \frac{1}{2}\varepsilon + \frac{3}{8}\varepsilon^2 - \frac{5}{16}\varepsilon^3 + \cdots\right)$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{r}\left[1 - \frac{1}{2}\left(-2\frac{\hat{r}\,\vec{r}'}{r} + \left(\frac{r'}{r}\right)^2\right) + \frac{3}{8}\left(-2\frac{\hat{r}\,\vec{r}'}{r} + \left(\frac{r'}{r}\right)^2\right)^2 + \cdots\right]$$

El potencial lejos de la distribución de cargas

$|\vec{r}| \gg |\vec{r}'|$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{r}\left[1 - \frac{1}{2}\left(-2\frac{\hat{r}\,\vec{r}'}{r} + \left(\frac{r'}{r}\right)^2\right) + \frac{3}{8}\left(-2\frac{\hat{r}\,\vec{r}'}{r} + \left(\frac{r'}{r}\right)^2\right)^2 + \cdots\right]$$

$$= \frac{1}{r}\left[1 + \frac{\hat{r}\,\vec{r}'}{r} - \frac{1}{2}\left(\frac{r'}{r}\right)^2 + \frac{3}{2}\left(\frac{\hat{r}\,\vec{r}'}{r}\right)^2 - \frac{3}{2}\frac{\hat{r}\,\vec{r}'}{r}\left(\frac{r'}{r}\right)^2 + \frac{3}{8}\left(\frac{r'}{r}\right)^4 + \cdots\right]$$

$$= \frac{1}{r}\left[1 + \frac{\hat{r}\,\vec{r}'}{r} + \frac{1}{2}\left(3\left(\frac{\hat{r}\,\vec{r}'}{r}\right)^2 - \left(\frac{r'}{r}\right)^2\right) + \cdots\right]$$

El potencial lejos de la distribución de cargas

$|\vec{r}| \gg |\vec{r}'|$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_V \rho(\vec{r}')\frac{1}{|\vec{r}-\vec{r}'|}\,dV'$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{r}\int_V \rho(\vec{r}')\left[1 + \frac{\hat{r}\,\vec{r}'}{r} + \frac{1}{2}\left(3\left(\frac{\hat{r}\,\vec{r}'}{r}\right)^2 - \left(\frac{r'}{r}\right)^2\right) + \cdots\right]dV'$$

El potencial lejos de la distribución de cargas

Término monopolar

$|\vec{r}| \gg |\vec{r}'|$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r}\int_V \rho(\vec{r}')\left[1 + \frac{\hat{r}\,\vec{r}'}{r} + \frac{1}{2}\left(3\left(\frac{\hat{r}\,\vec{r}'}{r}\right)^2 - \left(\frac{r'}{r}\right)^2\right) + \cdots\right] dV'$$

$$V_{\text{mon}}(\vec{r}) = \frac{1}{4\pi\varepsilon_0\, r}\int_V \rho(\vec{r}')\, dV' = \frac{Q}{4\pi\varepsilon_0\, r}$$

Carga total (escalar)

$$Q = \int_V \rho(\vec{r}')\, dV'$$

$$V_{\text{mon}}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0\, r}$$

El potencial lejos de la distribución de cargas

Término dipolar

$|\vec{r}| \gg |\vec{r}'|$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r}\int_V \rho(\vec{r}')\left[1 + \frac{\hat{r}\,\vec{r}'}{r} + \frac{1}{2}\left(3\left(\frac{\hat{r}\,\vec{r}'}{r}\right)^2 - \left(\frac{r'}{r}\right)^2\right) + \cdots\right] dV'$$

$$V_{\text{dip}}(\vec{r}) = \frac{1}{4\pi\varepsilon_0 r}\int_V \rho(\vec{r}')\,\frac{(\hat{\boldsymbol{r}}\,\vec{\boldsymbol{r}}')}{\boldsymbol{r}}\, dV'$$

$$= \frac{\hat{r}}{4\pi\varepsilon_0 r^2}\int_V \rho(\vec{r}')\,\vec{r}'\, dV'$$

Momento dipolar (vector)

$$\vec{p} = \int_V \rho(\vec{r}')\,\vec{r}'\, dV'$$

$$V_{\text{dip}}(\vec{r}) = \frac{\hat{r}}{4\pi\varepsilon_0 r^2}\vec{p}$$

El potencial lejos de la distribución de cargas

Término cuadrupolar

$|\vec{r}| \gg |\vec{r}\,'|$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r}\int_V \rho(\vec{r}\,')\left[1 + \frac{\hat{r}\,\vec{r}\,'}{r} + \frac{1}{2}\left(3\left(\frac{\hat{r}\,\vec{r}\,'}{r}\right)^2 - \left(\frac{r'}{r}\right)^2\right) + \cdots\right]dV'$$

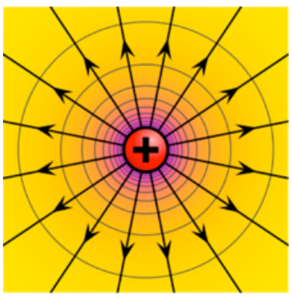
$$V_{\text{quad}}(\vec{r}) = \frac{1}{4\pi\varepsilon_0 r}\int_V \rho(\vec{r}\,')\,\frac{1}{2}\left(3\left(\frac{\hat{r}\,\vec{r}\,'}{r}\right)^2 - \left(\frac{r'}{r}\right)^2\right)dV'$$

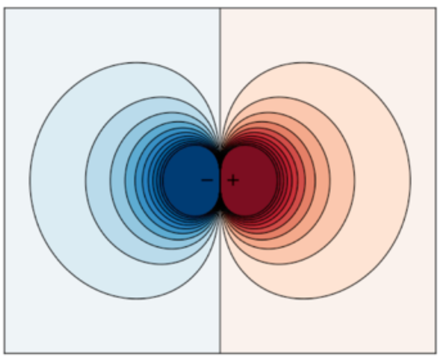
Momento cuadrupolar (tensor)

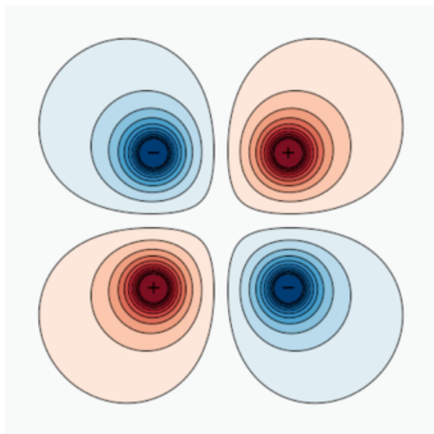
$$Q_{i,j} = \int_V \rho(\vec{r}\,')\,\frac{1}{2}(3\,r_i\,r_j - r\,\delta_{i,j})dV'$$

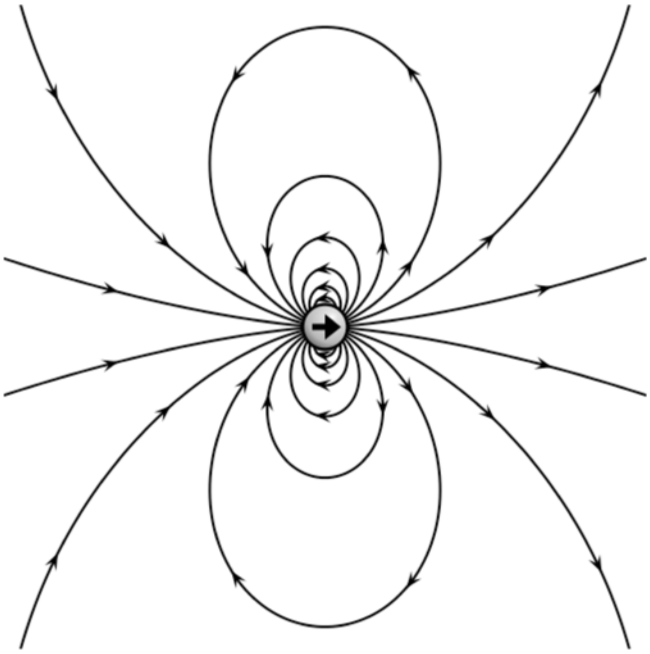
$$V_{\text{quad}}(\vec{r}) = \frac{\hat{r}}{4\pi\varepsilon_0 r^3}\sum_{i,j}\frac{1}{2}Q_{i,j}\,\hat{r}_i\hat{r}_j$$

$$V(\vec{r}) = V_{\text{mon}}(\vec{r}) + V_{\text{dip}}(\vec{r}) + V_{\text{quad}}(\vec{r}) + \cdots$$









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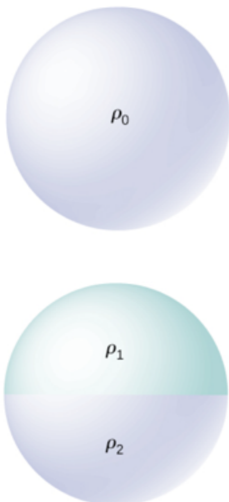

$\rho_1 = -\rho_2$