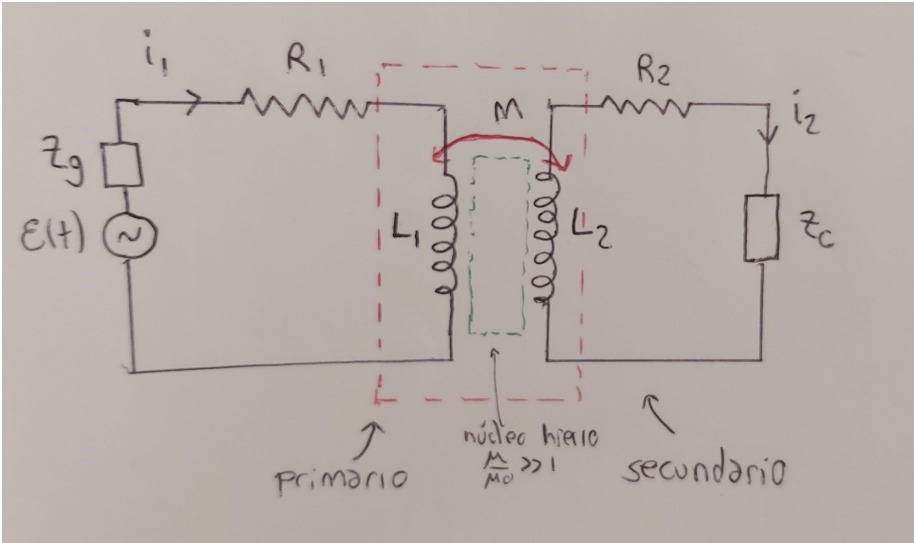


Física 3: Electricidad y Magnetismo

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Clase 22

Transformadores:



$$\Phi_1 = \Phi_{11} + \Phi_{12}$$

$$\Phi_2 = \Phi_{21} + \Phi_{22}$$

$$\Phi_{11} = L_1 i_1, \quad \Phi_{12} = \pm M i_2, \quad \Phi_{21} = \pm M i_1, \quad \Phi_{22} = L_2 i_2$$

$$\Phi_1 = L_1 i_1 \pm M i_2$$

$$\Phi_2 = \pm M i_1 + L_2 i_2$$

$$\varepsilon_{ind1} = -\frac{d\Phi_1}{dt} = -L_1 \frac{di_1}{dt} \mp M \frac{di_2}{dt}$$

$$\varepsilon_{ind2} = -\frac{d\Phi_2}{dt} = \mp M \frac{di_1}{dt} - L_2 \frac{di_2}{dt}$$

Aplicamos esto en los circuitos de alterna,

$$\varepsilon = Z_g I_1 + R_1 I_1 + j\omega L_1 I_1 \pm j\omega M I_2$$

(primario)

$$0 = Z_c I_2 + R_2 I_2 + j\omega L_2 I_2 \pm j\omega M I_1$$

(secundario)

En el primario defino las impedancias

$$Z_{11} = Z_g + R_1 + j\omega L_1, \quad Z_{12} = \pm j\omega M$$

En el secundario defino las impedancias

$$Z_{22} = Z_c + R_2 + j\omega L_2, \quad Z_{21} = \pm j\omega M$$

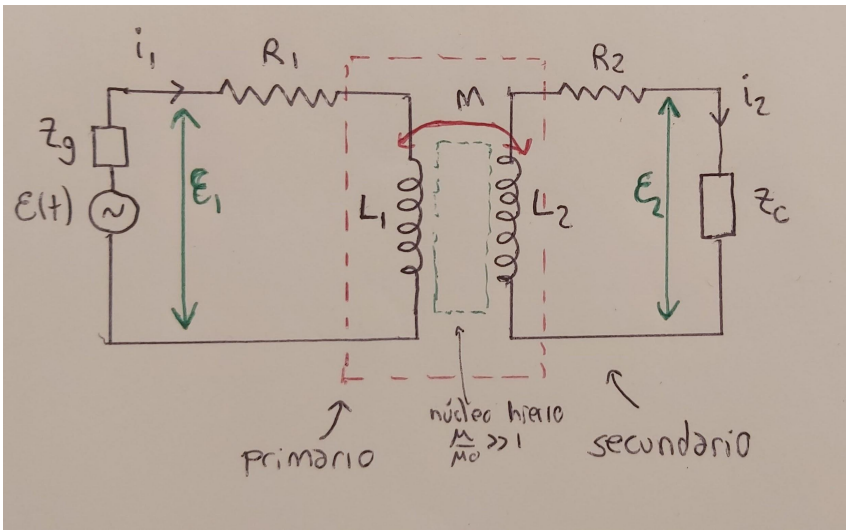
Nos queda el sistema de *ecuaciones acopladas*,

$$\varepsilon = Z_{11} I_1 + Z_{12} I_2$$

$$0 = Z_{21} I_1 + Z_{22} I_2$$

$$\Rightarrow I_1 = \frac{Z_{22}}{D} \varepsilon, \quad I_2 = -\frac{Z_{12}}{D} \varepsilon, \quad D = Z_{11} Z_{22} - Z_{12}^2$$

$$\Rightarrow \boxed{\frac{I_1}{I_2} = -\frac{Z_{22}}{Z_{12}}}$$



$$\varepsilon_1 = \varepsilon - Z_g I_1 = \varepsilon - \frac{Z_g Z_{22}}{D} \varepsilon$$

$$\varepsilon_2 = Z_c I_2 = -\frac{Z_c Z_{12}}{D} \varepsilon$$

$$\Rightarrow \frac{\varepsilon_1}{\varepsilon_2} = \frac{Z_g Z_{22} - D}{Z_c Z_{12}} = \frac{Z_{12}^2 - (R_1 + j\omega L_1) Z_{22}}{Z_c Z_{12}}$$

Transformador "ideal":

$$\omega L_1, \omega L_2 \gg Z_c$$

$$\Rightarrow Z_{11} \approx j\omega L_1, Z_{22} \approx j\omega L_2$$

$$R_1, R_2, |Z_g| \ll |Z_c|$$

Si las bobinas tienen N_1 y N_2 vueltas y la misma relación S/h entre área y altura,

$$L_{1,2} = \frac{\mu S}{h} N_{1,2}^2, \quad M = \frac{\mu S}{h} N_1 N_2 \Rightarrow k = \frac{M}{\sqrt{L_1 L_2}} = 1 \quad \text{acoplamiento perfecto} \rightarrow \frac{L_1}{M} = \frac{M}{L_2} = \frac{N_1}{N_2}$$

$$\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$$

Entonces, en un transformador ideal se tiene,

$$\frac{I_1}{I_2} = -\frac{Z_{22}}{Z_{12}} \approx -\frac{j\omega L_2}{\pm j\omega M} = \mp \frac{L_2}{M} = \mp \frac{N_2}{N_1}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{Z_{12}^2 - (R_1 + j\omega L_1)Z_{22}}{Z_c Z_{12}} = \frac{-\omega^2 M^2 - (R_1 + j\omega L_1)(R_2 + j\omega L_2 + Z_c)}{\pm j\omega M Z_c} \approx \frac{-\omega^2 M^2 - j\omega L_1(j\omega L_2 + Z_c)}{\pm j\omega M Z_c} = \frac{-j\omega L_1 Z_c}{\pm j\omega M Z_c} = \mp \frac{L_1}{M} = \mp \frac{N_1}{N_2}$$

el signo depende de cómo son los arrollamientos en las bobinas

$$\frac{\varepsilon_1}{\varepsilon_2} = \mp \frac{N_1}{N_2}$$

Si $N_1 > N_2 \Rightarrow \varepsilon_1 > \varepsilon_2 \rightarrow$ trafo *bajada*, ej: de 220 V a 110 V

Si $N_1 < N_2 \Rightarrow \varepsilon_1 < \varepsilon_2 \rightarrow$ trafo *subida*, ej: de 110 V a 220 V

$$\frac{I_1}{I_2} = \mp \frac{N_2}{N_1}$$

$\Rightarrow \varepsilon_1 I_1 = \varepsilon_2 I_2$ la energía se conserva !

(esto ocurre porque en un trafo ideal no hay pérdidas)