

# 1. Identidades vectoriales

## 1.1. Productos vectoriales y escalares

### 1.1.1. Producto escalar

Si  $\vec{A}$  y  $\vec{B}$  son vectores que en una base ortonormal  $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$  se escriben como:  $\vec{A} = A_1 \vec{u}_1 + A_2 \vec{u}_2 + A_3 \vec{u}_3$  y  $\vec{B} = B_1 \vec{u}_1 + B_2 \vec{u}_2 + B_3 \vec{u}_3$ , entonces su producto escalar se define:

$$\vec{A} \cdot \vec{B} = A_1 B_1 + A_2 B_2 + A_3 B_3 \quad (1)$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta \quad (2)$$

siendo  $\theta$  el ángulo que forman dichos vectores.

Si  $\vec{A}$  es un vector unitario, entonces  $\vec{A} \cdot \vec{B}$  se corresponde con la proyección de  $\vec{B}$  sobre la dirección definida por el vector unitario  $\vec{A}$ .

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \quad (3)$$

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C} \quad (4)$$

### 1.1.2. Producto vectorial

Si  $\vec{A}$  y  $\vec{B}$  son vectores que en una base ortonormal  $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$  se escriben como:  $\vec{A} = A_1 \vec{u}_1 + A_2 \vec{u}_2 + A_3 \vec{u}_3$  y  $\vec{B} = B_1 \vec{u}_1 + B_2 \vec{u}_2 + B_3 \vec{u}_3$ , entonces su producto vectorial se define:

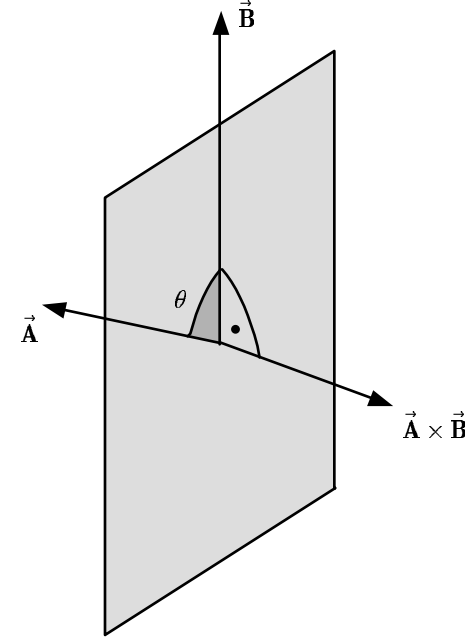
$$\begin{aligned} \vec{A} \times \vec{B} &= \begin{vmatrix} \vec{u}_1 & \vec{u}_2 & \vec{u}_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = \\ &= (A_2 B_3 - A_3 B_2) \vec{u}_1 + (A_3 B_1 - A_1 B_3) \vec{u}_2 + \\ &+ (A_1 B_2 - A_2 B_1) \vec{u}_3 \end{aligned} \quad (5)$$

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta \quad (6)$$

$|\vec{A} \times \vec{B}|$  coincide con el área del paralelogramo definido por ambos vectores.

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \quad (7)$$

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \quad (8)$$



### 1.1.3. Identidades vectoriales

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B} (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{A} \cdot \vec{B}) \quad (9)$$

$$(\vec{A} \times \vec{B}) \times \vec{C} = \vec{B} (\vec{A} \cdot \vec{C}) - \vec{A} (\vec{B} \cdot \vec{C}) \quad (10)$$

$$(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) = (\vec{A} \cdot \vec{C}) (\vec{B} \cdot \vec{D}) - (\vec{A} \cdot \vec{D}) (\vec{B} \cdot \vec{C}) \quad (11)$$

$$\begin{aligned} (\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D}) &= \\ &= \vec{C} [\vec{A} \cdot (\vec{B} \times \vec{D})] - \vec{D} [\vec{A} \cdot (\vec{B} \times \vec{C})] = \\ &= \vec{B} [\vec{A} \cdot (\vec{C} \times \vec{D})] - \vec{A} [\vec{B} \cdot (\vec{C} \times \vec{D})] \end{aligned} \quad (12)$$

## 2. Coordenadas Cartesianas

### 2.1. Transformación de Coordenadas

#### 2.1.1. Desde Coordenadas Cilíndricas

$$x = \rho \cos \varphi \quad (13)$$

$$y = \rho \sen \varphi \quad (14)$$

$$z = z \quad (15)$$

#### 2.1.2. Desde Coordenadas Esféricas

$$x = r \sen \theta \cos \varphi \quad (16)$$

$$y = r \sen \theta \sen \varphi \quad (17)$$

$$z = r \cos \theta \quad (18)$$

## 2.2. Transformación de Vectores

#### 2.2.1. Desde Coordenadas Cilíndricas

$$A_x = A_\rho \cos \varphi - A_\varphi \sen \varphi \quad (19)$$

$$A_y = A_\rho \sen \varphi + A_\varphi \cos \varphi \quad (20)$$

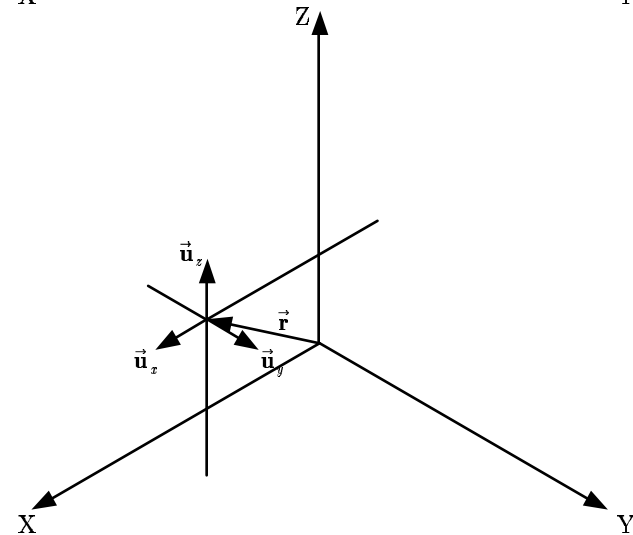
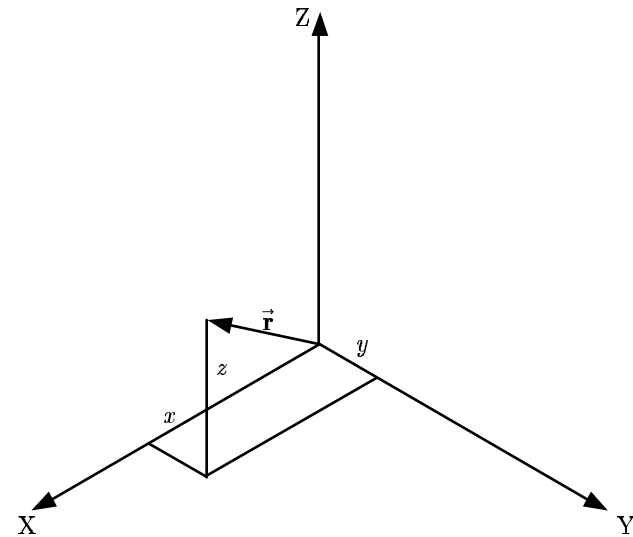
$$A_z = A_z \quad (21)$$

#### 2.2.2. Desde Coordenadas Esféricas

$$A_x = A_r \sen \theta \cos \varphi - A_\varphi \sen \varphi + A_\theta \cos \theta \cos \varphi \quad (22)$$

$$A_y = A_r \sen \theta \sen \varphi + A_\varphi \cos \varphi + A_\theta \cos \theta \sen \varphi \quad (23)$$

$$A_z = A_r \cos \theta - A_\theta \sen \theta \quad (24)$$



$\sin \varphi$

### 3. Coordenadas Cilíndricas

#### 3.1. Transformación de Coordenadas

##### 3.1.1. Desde Coordenadas Cartesianas

$$\rho = \sqrt{x^2 + y^2} \quad (25)$$

$$\operatorname{tg} \varphi = \frac{y}{x} \quad (26)$$

$$z = z \quad (27)$$

##### 3.1.2. Desde Coordenadas Esféricas

$$\rho = r \operatorname{sen} \theta \quad (28)$$

$$\varphi = \varphi \quad (29)$$

$$z = r \operatorname{cos} \theta \quad (30)$$

#### 3.2. Transformación de Vectores

##### 3.2.1. Desde Coordenadas Cartesianas

$$A_\rho = A_x \frac{x}{\sqrt{x^2 + y^2}} + A_y \frac{y}{\sqrt{x^2 + y^2}} \quad (31)$$

$$A_\varphi = -A_x \frac{y}{\sqrt{x^2 + y^2}} + A_y \frac{x}{\sqrt{x^2 + y^2}} \quad (32)$$

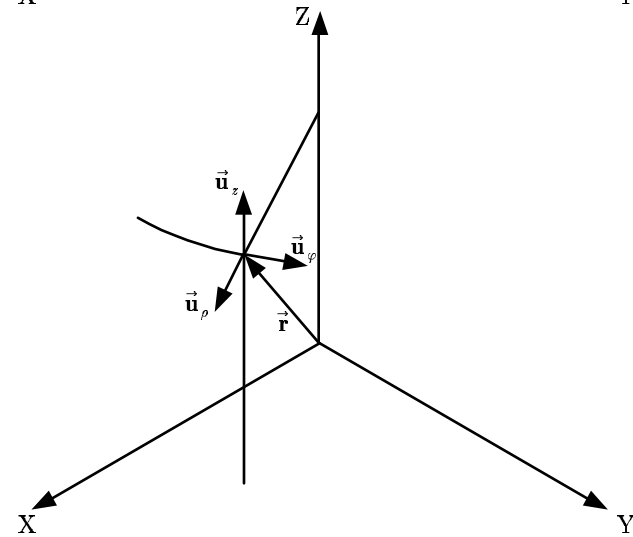
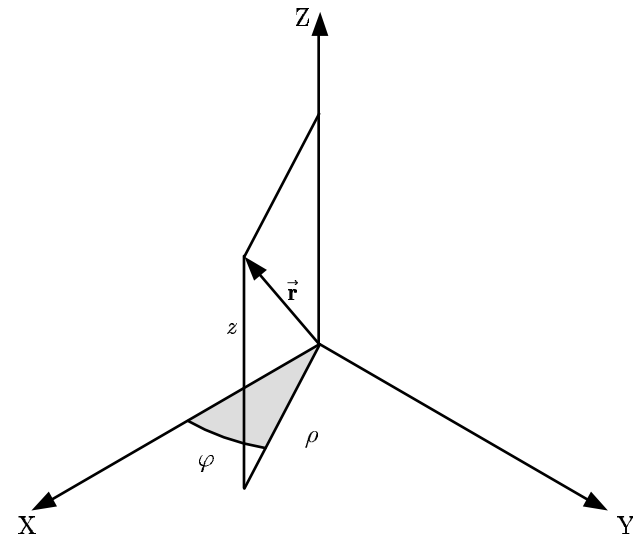
$$A_z = A_z \quad (33)$$

##### 3.2.2. Desde Coordenadas Esféricas

$$A_\rho = A_r \operatorname{sen} \theta + A_\theta \operatorname{cos} \theta \quad (34)$$

$$A_\varphi = A_\varphi \quad (35)$$

$$A_z = A_r \operatorname{cos} \theta - A_\theta \operatorname{sen} \theta \quad (36)$$



## 4. Coordenadas Esféricas

### 4.1. Transformación de Coordenadas

#### 4.1.1. Desde Coordenadas Cartesianas

$$r = \sqrt{x^2 + y^2 + z^2} \quad (37)$$

$$\operatorname{tg} \theta = \frac{\sqrt{x^2 + y^2}}{z} \quad (38)$$

$$\operatorname{tg} \varphi = \frac{y}{x} \quad (39)$$

#### 4.1.2. Desde Coordenadas Cilíndricas

$$r = \sqrt{\rho^2 + z^2} \quad (40)$$

$$\operatorname{tg} \theta = \frac{\rho}{z} \quad (41)$$

$$\varphi = \varphi \quad (42)$$

## 4.2. Transformación de Vectores

### 4.2.1. Desde Coordenadas Cartesianas

$$A_r = A_x \frac{x}{\sqrt{x^2 + y^2 + z^2}} + \quad (43)$$

$$+ A_y \frac{y}{\sqrt{x^2 + y^2 + z^2}} + A_z \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

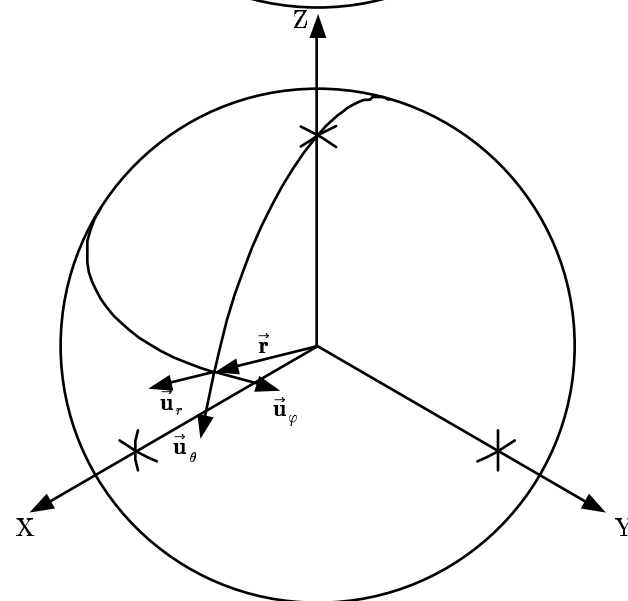
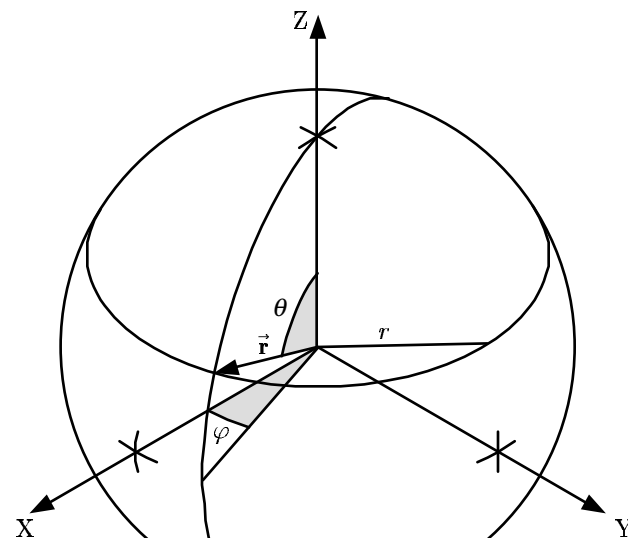
$$A_\varphi = -A_x \frac{y}{\sqrt{x^2 + y^2}} + A_y \frac{x}{\sqrt{x^2 + y^2}} \quad (44)$$

$$A_\theta = A_x \frac{xz}{\sqrt{(x^2 + y^2 + z^2)(x^2 + y^2)}} + \quad (45)$$

$$+ A_y \frac{yz}{\sqrt{(x^2 + y^2 + z^2)(x^2 + y^2)}} - A_z \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}$$

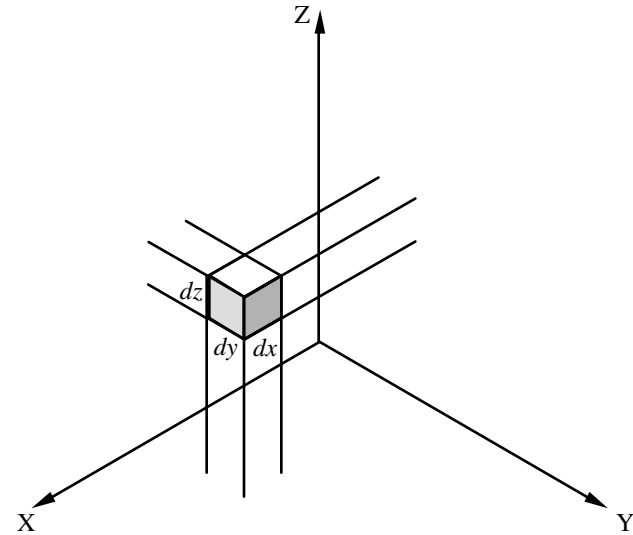
### 4.2.2. Desde Coordenadas Cilíndricas

$$A_r = A_\rho \frac{\rho}{\sqrt{\rho^2 + z^2}} + A_z \frac{z}{\sqrt{\rho^2 + z^2}} \quad (46)$$



$$A_\varphi = A_\varphi \quad (47)$$

$$A_\theta = A_\rho \frac{z}{\sqrt{\rho^2 + z^2}} - A_z \frac{\rho}{\sqrt{\rho^2 + z^2}} \quad (48)$$



## 5. Fórmulas de análisis vectorial

### 5.1. Coordenadas Cartesianas

#### 5.1.1. Elemento de longitud

$$d\vec{r} = dx\vec{u}_x + dy\vec{u}_y + dz\vec{u}_z \quad (49)$$

$$dr = \sqrt{dx^2 + dy^2 + dz^2} \quad (50)$$

#### 5.1.2. Elementos de superficie

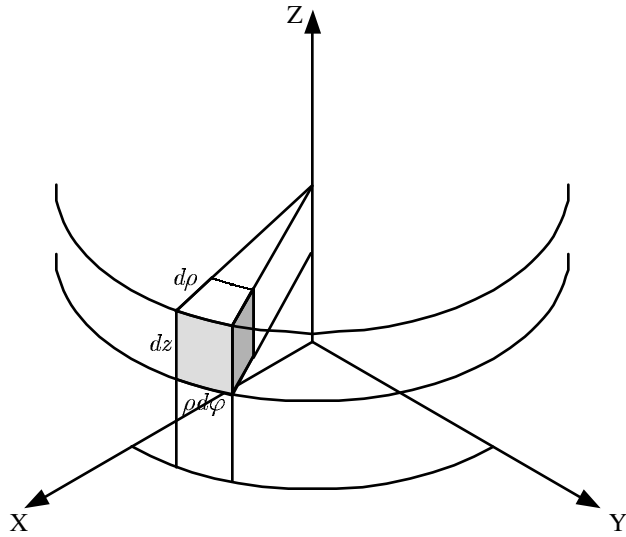
$$d\vec{s}_x = dydz\vec{u}_x \quad (51)$$

$$d\vec{s}_y = dzdx\vec{u}_y \quad (52)$$

$$d\vec{s}_z = dxdy\vec{u}_z \quad (53)$$

#### 5.1.3. Elemento de volumen

$$dv = dxdydz \quad (54)$$



## 5.2. Coordenadas Cilíndricas

### 5.2.1. Elemento de longitud

$$d\vec{r} = d\rho\vec{u}_\rho + \rho d\varphi\vec{u}_\varphi + dz\vec{u}_z \quad (55)$$

$$dr = \sqrt{d\rho^2 + \rho^2 d\varphi^2 + dz^2} \quad (56)$$

### 5.2.2. Elementos de superficie

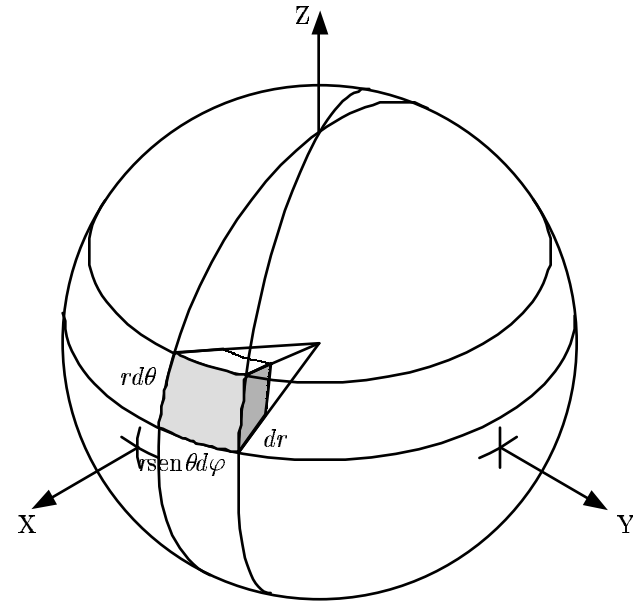
$$d\vec{s}_\rho = \rho d\varphi dz \vec{u}_\rho \quad (57)$$

$$d\vec{s}_\varphi = d\rho dz \vec{u}_\varphi \quad (58)$$

$$d\vec{s}_z = \rho d\rho d\varphi \vec{u}_z \quad (59)$$

### 5.2.3. Elemento de volumen

$$dv = \rho d\rho d\varphi dz \quad (60)$$



## 5.3. Coordenadas Esféricas

### 5.3.1. Elemento de longitud

$$d\vec{r} = dr\vec{u}_r + r \text{sen } \theta d\varphi\vec{u}_\varphi + rd\theta\vec{u}_\theta \quad (61)$$

$$dr = \sqrt{dr^2 + r^2 \text{sen}^2 \theta d\varphi^2 + r^2 d\theta^2} \quad (62)$$

### 5.3.2. Elementos de superficie

$$d\vec{s}_r = r^2 \text{sen } \theta d\varphi d\theta \vec{u}_r \quad (63)$$

$$d\vec{s}_\theta = r \text{sen } \theta dr d\varphi \vec{u}_\theta \quad (64)$$

$$d\vec{s}_\varphi = r dr d\theta \vec{u}_\varphi \quad (65)$$

### 5.3.3. Elemento de volumen

$$dv = r^2 \text{sen } \theta dr d\theta d\varphi \quad (66)$$

## 5.4. Gradiente, Divergencia y Rotacional

### 5.4.1. Coordenadas Cartesianas

$$\vec{\nabla}\Phi = \frac{\partial\Phi}{\partial x}\vec{u}_x + \frac{\partial\Phi}{\partial y}\vec{u}_y + \frac{\partial\Phi}{\partial z}\vec{u}_z \quad (67)$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad (68)$$

$$\begin{aligned} \vec{\nabla}\times\vec{A} &= \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{u}_x + \\ &+ \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{u}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{u}_z \end{aligned} \quad (69)$$

### 5.4.2. Coordenadas Cilíndricas

$$\vec{\nabla}\Phi = \frac{\partial\Phi}{\partial\rho}\vec{u}_\rho + \frac{1}{\rho}\frac{\partial\Phi}{\partial\varphi}\vec{u}_\varphi + \frac{\partial\Phi}{\partial z}\vec{u}_z \quad (70)$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z} \quad (71)$$

$$\begin{aligned} \vec{\nabla}\times\vec{A} &= \left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\vec{u}_\rho + \\ &+ \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\vec{u}_\varphi + \left[\frac{1}{\rho}\frac{\partial(\rho A_\varphi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial\varphi}\right]\vec{u}_z \end{aligned} \quad (72)$$

### 5.4.3. Coordenadas Esféricas

$$\vec{\nabla}\Phi = \frac{\partial\Phi}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\vec{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\varphi}\vec{u}_\varphi \quad (73)$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi} \quad (74)$$

$$\begin{aligned} \vec{\nabla}\times\vec{A} &= \left\{\frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta A_\varphi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\varphi}\right]\right\}\vec{u}_r + \\ &+ \left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial(r A_\varphi)}{\partial r}\right]\vec{u}_\theta + \\ &+ \left\{\frac{1}{r}\left[\frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\right\}\vec{u}_\varphi \end{aligned} \quad (75)$$

### 5.4.4. Identidades

$$\vec{\nabla}\times\vec{\nabla}\Phi = \vec{0} \quad (76)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0 \quad (77)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\Phi) = \nabla^2\Phi \quad (78)$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A} \quad (79)$$

$$\vec{\nabla}(\Phi + \Psi) = \vec{\nabla}\Phi + \vec{\nabla}\Psi \quad (80)$$

$$\vec{\nabla}\cdot(\vec{A} + \vec{B}) = \vec{\nabla}\cdot\vec{A} + \vec{\nabla}\cdot\vec{B} \quad (81)$$

$$\vec{\nabla}\times(\vec{A} + \vec{B}) = \vec{\nabla}\times\vec{A} + \vec{\nabla}\times\vec{B} \quad (82)$$

$$\vec{\nabla}(\Phi\Psi) = \Psi\vec{\nabla}\Phi + \Phi\vec{\nabla}\Psi \quad (83)$$

$$\vec{\nabla}\cdot(\Phi\vec{A}) = \vec{A}\cdot\vec{\nabla}\Phi + \Phi\vec{\nabla}\cdot\vec{A} \quad (84)$$

$$\begin{aligned} \vec{\nabla}(\vec{A}\cdot\vec{B}) &= (\vec{A}\cdot\vec{\nabla})\vec{B} + (\vec{B}\cdot\vec{\nabla})\vec{A} + \\ &+ \vec{A}\times(\vec{\nabla}\times\vec{B}) + \vec{B}\times(\vec{\nabla}\times\vec{A}) \end{aligned} \quad (85)$$

$$\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\vec{\nabla}\times\vec{A} + \vec{A}\vec{\nabla}\times\vec{B} \quad (86)$$

$$\vec{\nabla}\times(\Phi\vec{A}) = \vec{\nabla}\Phi\times\vec{A} + \Phi\vec{\nabla}\times\vec{A} \quad (87)$$

$$\begin{aligned} \vec{\nabla}\times(\vec{A}\times\vec{B}) &= \vec{A}\vec{\nabla}\cdot\vec{B} - \vec{B}\vec{\nabla}\cdot\vec{A} + \\ &+ (\vec{B}\cdot\vec{\nabla})\vec{A} - (\vec{A}\cdot\vec{\nabla})\vec{B} \end{aligned} \quad (88)$$

### 5.4.5. Teoremas

$$\int_v(\vec{\nabla}\cdot\vec{A})dv = \oint_s\vec{A}\cdot d\vec{s} \quad (89)$$

$$\int_s(\vec{\nabla}\times\vec{A})d\vec{s} = \oint_c\vec{A}\cdot d\vec{r} \quad (90)$$

$$\int_v(\vec{\nabla}\times\vec{A})dv = \oint_s d\vec{s}\times\vec{A} \quad (91)$$

$$\int_s d\vec{s}\times\vec{\nabla}\Phi = \oint_c\Phi d\vec{r} \quad (92)$$